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Research Article

Minimal Length Effect on Thermodynamics and Weak Cosmic Censorship Conjecture in Anti-de Sitter Black Holes via Charged Particle Absorption

Benrong Mu,[1,2] **Jun Tao**,[2] **and Peng Wang**[2]

[1]*Physics Teaching and Research Section, College of Medical Technology, Chengdu University of Traditional Chinese Medicine, Chengdu 611137, China*
[2]*Center for Theoretical Physics, College of Physical Science and Technology, Sichuan University, Chengdu 610064, China*

Correspondence should be addressed to Jun Tao; taojun@scu.edu.cn

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In this paper, we investigate the minimal length effects on the thermodynamics and weak cosmic censorship conjecture in a RN-AdS black hole via charged particle absorption. We first use the generalized uncertainty principle (GUP) to investigate the minimal length effect on the Hamilton-Jacobi equation. After the deformed Hamilton-Jacobi equation is derived, we use it to study the variations of the thermodynamic quantities of a RN-Ads black hole via absorbing a charged particle. Furthermore, we check the second law of thermodynamics and the weak cosmic censorship conjecture in two phase spaces. In the normal phase space, the second law of thermodynamics and the weak cosmic censorship conjecture are satisfied in the usual and GUP-deformed cases, and the minimal length effect makes the increase of entropy faster than the usual case. After the charge particle absorption, the extremal RN-AdS black hole becomes nonextremal. In the extended phase space, the black hole entropy can either increase or decrease. When $T > 2Pr_+$, the second law is satisfied. When $T < 2Pr_+$, the second law of thermodynamics is violated for the extremal or near-extremal black hole. Finally, we find that the weak cosmic censorship conjecture is legal for extremal and near-extremal RN-Ads black holes in the GUP-deformed case.

1. Introduction

The classical theory of black holes predicts that nothing, including light, could escape from black holes. However, Stephen Hawking first showed that quantum effects could allow black holes to emit particles [1]. Since then, people have begun to study the thermodynamic properties of black holes as a thermodynamic system and have made a lot of achievements. For example, using the semiclassical method, Kraus and Wilczek have modeled Hawking radiation as a tunneling effect [2, 3]. Hawking radiation-related problems are studied in depth by using the null geodesic method and the Hamilton-Jacobi method [4–9]. Analogous to the four laws of thermodynamics, Bardeen et al. proposed four laws for black holes [10]. The research on black holes and gravitational waves also has made important progress. The first gravitational wave signal GW150914 was directly detected on September 14, 2015. The signal confirms an important prediction of general relativity that there are binary black hole systems in the universe, and they could combine to form a larger black hole [11].

General relativity predicts that the final product of the gravitational collapse of a star could lead to a singularity of spacetime. To avoid destructions caused by the singularity, Penrose first proposed the weak cosmic censorship conjecture (WCCC), which states that naked singularities cannot be formed in a real physical process from regular initial conditions [12, 13]. In other words, the singularity is always hidden behind the horizon, and an observer at the infinite distance can never observe the existence of the singularity. To test the validity of the weak cosmic censorship conjecture, Wald first tried to overcharge/overspin an extremal

Kerr-Newman black hole by throwing a test particle with charge/angular momentum into the event horizon [14]. It was found that near-extremal charged/rotating black holes could be overcharged/overspun by absorbing the test particle with charge/angular momentum [15–19]. However, considering the back reaction and self-force effects, the study suggests that the weak cosmic censorship conjecture may be still satisfied [20–25]. Since there is a lack of universal evidence for the weak cosmic censorship conjecture, its validity has been tested in various black holes [26–33]. Recently, the validity of the weak cosmic censorship conjecture through absorption of charged particles has been tested in the extended phase space, where the cosmological constant is treated as a thermodynamic variable. The results showed that the first law of thermodynamics and the weak cosmic censorship conjecture are satisfied, while the second law of thermodynamics is violated for the extremal and near-extremal black holes [34–43].

Since the singularity is a point where general relativity fails, we need a broader theory to describe the gravity and quantum behavior of black holes, especially the singularity of spacetime. On the other hand, various theories of quantum gravity, such as loop quantum gravity, string theory, quantum geometry, and Doubly Special Relativity, imply the existence of a minimal observable length [44–48]. The generalized uncertainty principle (GUP) [49] is one of the simple models to realize this minimal observable length. The GUP can be derived from the deformed fundamental commutation relation [50]:

$$[X, P] = i\hbar\left(1 + \beta P^2\right), \tag{1}$$

where $\beta = \beta_0/m_p^2$ is the deformation parameter, β_0 is a dimensionless number, and m_p is the Planck mass. The minimal observable length is $\Delta_{\min} = \hbar\sqrt{\beta}$. The value range of β_0 is constrainted as $1\beta_0 < 10^{36}$ [51, 52]. For a review of GUP, see [53]. GUP is one of the simplest models of effective quantum gravity, and many interesting results in the study of black hole physics have been produced [54–71]. Specifically, the authors of [72] discussed the effect of quantum gravity on the weak cosmic censorship conjecture and showed that the second law of thermodynamics and the cosmic censorship conjecture are violated owing to the rainbow effect.

In this paper, we will discuss the effects of quantum gravity on black hole thermodynamics and the weak cosmic censorship conjecture in the framework of GUP. The rest of this paper is organized as follows. In Section 2, we derive the GUP-deformed Hamilton-Jacobi equation for a particle in the RN-Ads spacetime and discuss its motion around the black hole horizon. In Section 3, the minimal length effect on the thermodynamics of the black hole is discussed in the extended phase space. In Section 4, we investigate the minimal length effect on the validity of the weak cosmic censorship conjecture. We summarize our results in Section 5. For simplicity, we set $G = \hbar = c = k_B = 1$ in this paper.

2. Deformed Hamilton-Jacobi Equation in a RN-AdS Black Hole

In this section, we first review the thermodynamic properties of RN-AdS black holes. Then, the GUP-deformed Hamilton-Jacobi equation is derived, and the motion of a charge particle near the horizon of the black hole is discussed.

The metric of a Reissner-Nordström anti-de Sitter (RN-AdS) black hole in $(3 + 1)$ curved spacetime is given by

$$ds^2 = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{2}$$

with the electromagnetic potential

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0\right), \tag{3}$$

where

$$h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}, \tag{4}$$

where l is the AdS radius and M and Q are the ADM mass and charge of the black hole, respectively. The AdS radius l is related to the cosmological constant as $\Lambda = -3/l^2$. When the black hole is nonextremal, the equation $h(r) = 0$ has two positive real roots r_+ and r_-, where the maximum root r_+ represents the radius of the event horizon. When the black hole is extremal, $h(r) = 0$ only possesses a single root r_+. The mass of the RN-AdS black hole can be expressed in terms of r_+

$$M = \frac{1}{2}\left[r_+ + \frac{r_+^3}{l^2} + \frac{Q^2}{r_+}\right]. \tag{5}$$

The Hawking temperature of the AdS-RN black hole is given by

$$T = \frac{h'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right). \tag{6}$$

Moreover, the Bekenstein–Hawking entropy and the electric potential are

$$S = \frac{A}{4} = \pi r_+^2, \qquad \Phi = \frac{Q}{r_+}, \tag{7}$$

respectively, where $A = 4\pi r_+^2$ is the horizon area.

As a stable thermodynamic system, black holes can be discussed in two phase spaces. In the normal phase where the cosmological constant is a constant, the state parameters satisfy the first law of thermodynamics

$$dM = TdS + \Phi dQ. \tag{8}$$

However, in contrast to the usual first law of thermodynamics, the VdP term is missing in eqn. (8). Inspired by this, the cosmological constant can been taken as the pressure of the black hole [73, 74]. The expression between the cosmological constant and the pressure is given as follows:

$$P \equiv -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}. \tag{9}$$

The first law of thermodynamics in the extended phase space is

$$dM = TdS + \Phi dQ + VdP, \tag{10}$$

where the volume is given by

$$V = \frac{4}{3}\pi r_+^3. \tag{11}$$

The mass of the black hole M is defined as its enthalpy [75, 76]

$$M = U + PV. \tag{12}$$

In [60], we have already derived the Hamilton-Jacobi equations for a scalar particle and a fermion in a curved spacetime background under an electric potential A_μ and showed that these Hamilton-Jacobi equations have the same form:

$$(\partial^\mu I - qA^\mu)(\partial_\nu I - qA_\nu) + m^2 = 0, \tag{13}$$

where I is the action, A_μ is the electromagnetic potential, and m and q are the mass and the charge of particle, respectively. In the RN-AdS metric (2), the Hamilton-Jacobi (equation (13)) reduces to

$$-\frac{1}{h(r)}\left(\frac{\partial I}{\partial t} - qA_t\right)^2 + h(r)(\partial_r I)^2 + \frac{(\partial_\theta I)^2}{r^2} + \frac{(\partial_\phi I)^2}{r^2 \sin^2\theta} + m^2 = 0. \tag{14}$$

Taking into account the symmetries of the spacetime, we can employ the following ansatz:

$$I = -Et + W(r,\theta) + P_\phi \phi, \tag{15}$$

where E and P_ϕ are the energy and the z-component of angular momentum of emitted particles, respectively. The magnitude of the angular momentum of the particle L can be expressed in terms of

$$L^2 = P_\theta^2 + \frac{P_\phi^2}{\sin^2\theta}, \tag{16}$$

where $P_\theta = \partial_\theta W$. Plugging eqns. (15) and (16) into eqn. (14), we get

$$E = q\Phi + \sqrt{[P^r(r_+)]^2 + \left(m^2 + \frac{L^2}{r^2}\right)h(r)}, \tag{17}$$

where $P^r(r) = h(r)P_r(r) = h(r)\partial_r W$ is the radial momentum of the particle. Since the energy of the particle is required to be a positive value [77, 78], we choose the positive sign in front of the square root. P^r is finite and nonzero at event horizon $r = r_+$, which accounts for the Hawking radiation modeled as a tunneling process [79]. At the horizon $r = r_+$, eqn. (17) reduces to

$$E = q\Phi + |P^r(r_+)|, \tag{18}$$

which relates the energy of the particle to its momentum and potential energy near the event horizon $r = r_+$.

To implement deformed fundamental commutation relation (1), one defines

$$\begin{aligned} X_i &= x_i, \\ P_i &= p_i\left(1 + \beta p^2\right) = p_i f\left(\beta p^2\right), \end{aligned} \tag{19}$$

where $p^2 = \sum_i p_i p_i$ and $f(x) = 1 + x$ for the Brau reduction [80, 81]. The operators x_i and p_i are the conventional momentum and position operators satisfying

$$\begin{aligned} \left[x_i, p_j\right] &= i\hbar\delta_{ij}, \left[x_i, x_j\right] = \left[p_i, p_j\right] = 0, \\ x_i &= x_i, \\ p_i &= \frac{\hbar}{i}\frac{\partial}{\partial x_i}. \end{aligned} \tag{20}$$

Using the WKB method, the deformed Hamilton-Jacobi equation in the RN-AdS metric has been obtained [81–83]

$$\frac{1}{h(r)}\left(\frac{\partial I}{\partial t} - qA_t\right)^2 - \chi f^2(\beta\chi) = m^2, \tag{21}$$

where

$$\chi = h(r)(\partial_r I)^2 + \frac{(\partial_\theta I)^2}{r^2} + \frac{(\partial_\phi I)^2}{r^2 \sin^2\theta}. \tag{22}$$

To solve the deformed Hamilton-Jacobi (equation (21)), we plug the ansatz eqn. (15) into eqn. (21) and find that

$$(E + qA_t)^2 = h(r)\chi f^2(\beta\chi) + m^2 h(r), \tag{23}$$

where

$$\chi = \frac{[P^r(r)]^2}{h(r)} + \frac{L^2}{r^2}. \tag{24}$$

To relate E to $P^r(r)$, we might want to evaluate eqn. (23)

at the horizon $r = r_+$. However, GUP is an effective model, which is untrustworthy around the Planck scale. The degrees of freedom within a few Planck lengths away from the horizon are usually trans-Planckian. Moreover, the effective number of these degrees of freedom is very small. Therefore, we evaluate eqn. (23) at the stretched horizon located at $r = r_+ + m_p$ instead of the horizon. We then find that

$$E = P^r(r_+) f\left(\frac{\beta_0}{4\pi T}\frac{[P^r(r_+)]^2}{m_p}\right) + q\Phi. \tag{25}$$

where we use eqn. (6) to express $h'(r_+)$ in terms of T. When $E < q\Phi$, the total energy of the RN-AdS black hole flows out the event horizon, and it means the superaddition has happened. When $E \geq q\Phi$, the total energy of the RN-AdS black hole flows in the event horizon. In this paper, we will study the minimal length effect on the thermodynamics of RN-AdS black holes in the $E \geq q\Phi$ case. When $\beta \longrightarrow 0$, $f((\beta_0/4\pi T)([P^r(r_+)]^2/m_p)) \longrightarrow 1$ and eqn. (25) reduces to eqn. (18).

3. Minimal Length Effect on Thermodynamics of a RN-AdS Black Hole via Charged Particle Absorption

In this section, we use eqn. (25) to investigate minimal length effects on thermodynamics of a RN-AdS black hole in the normal and extended phase spaces via charged particle absorption. For the convenience of the later discussion, we first give the following formulas:

$$\begin{aligned} \left.\frac{\partial h}{\partial r}\right|_{r=r_+} &= 4\pi T, \\ \left.\frac{\partial h}{\partial M}\right|_{r=r_+} &= -\frac{2}{r_+}, \\ \left.\frac{\partial h}{\partial Q}\right|_{r=r_+} &= \frac{2Q}{r_+^2} = \frac{2\Phi}{r_+}, \\ \left.\frac{\partial h}{\partial P}\right|_{r=r_+} &= \frac{8\pi r_+^2}{3} = \frac{2V}{r_+}. \end{aligned} \tag{26}$$

3.1. Normal Phase Space. When the black hole absorbs a charged particle with the mass m, the energy E, and the charge q, the mass M and charge Q of the black hole are varied due to the conservation law. Other thermodynamic variables of the RN-AdS black bole would change accordingly. To check whether the changes of the RN-AdS black hole thermodynamic variables obeys the second law of thermodynamics in the normal phase space. The initial and final states of the RN-AdS black hole are represented by (M, Q, r_+) and $(M + dM, Q + dQ, r_+ + dr_+)$, respectively, where dM, dQ, and dr_+ denote the increases of the mass, charge, and radius of the RN-AdS black hole. The functions $h(M + dM, Q + dQ, r_+ + dr_+)$ and $h(M, Q, r_+)$ satisfy

$$\begin{aligned} h(M + dM, Q + dQ, r_+ + dr_+) &= h(M, Q, r_+) + \left.\frac{\partial h}{\partial M}\right|_{r=r_+} dM \\ &+ \left.\frac{\partial h}{\partial Q}\right|_{r=r_+} dQ + \left.\frac{\partial h}{\partial r}\right|_{r=r_+} dr_+. \end{aligned} \tag{27}$$

In the normal phase space, the black hole mass can be regarded as the internal energy. After the black hole absorbs the particle at the event horizon, the change of the internal energy and charge of the black hole satisfies the following relation:

$$\begin{aligned} dM &= E, \\ dQ &= q. \end{aligned} \tag{28}$$

For a test particle, it is assumed that its energy E and charge q are small compared to the corresponding physical quantity of the black hole,

$$\begin{aligned} q &= dQ \ll Q, \\ E &= dM \ll U. \end{aligned} \tag{29}$$

When the black hole absorbs a charged particle, the mass M and charge Q are varied. The initial outer horizon radius r_+ moves to the final outer horizon radius $r_+ + dr_+$, which leads to

$$h(M, Q, r_+) = h(M + dM, Q + dQ, r_+ + dr_+) = 0. \tag{30}$$

From eqns. (26), (27), and (30), the infinitesimal changes in r_+, M, and Q are related by

$$4\pi T dr_+ - \frac{2}{r_+}E + \frac{2\Phi}{r_+}q = 0. \tag{31}$$

Combining eqn. (31) with eqns. (7) and (25), we get

$$dS = \frac{P^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{T} > 0. \tag{32}$$

Since $f((\beta_0/4\pi T)([P^r(r_+)]^2/m_p)) > 0$, the minimal length effect does not change the sign of dS, which shows that the second law of thermodynamics holds in the normal phase space. For the Brau reduction with $f(x) = 1 + x$, the GUP effect makes the increase of the entropy faster than the usual case.

3.2. Extended Phase Space. In the extended phase space, the cosmological constant can be treated as the pressure of the black hole. So, the initial and final states of the RN-AdS black hole are represented by (M, Q, P, r_+) and

$(M+dM, Q+dQ, P+dP, r_+ + dr_+)$, respectively. The functions $h(M+dM, Q+dQ, P+dP, r_+ + dr_+)$ and $h(M, Q, P, r_+)$ satisfy

$$h(M+dM, Q+dQ, r_+ + dr_+) = h(M, Q, P, r_+) + \left.\frac{\partial h}{\partial M}\right|_{r=r_+} dM + \left.\frac{\partial h}{\partial Q}\right|_{r=r_+} dQ + \left.\frac{\partial h}{\partial P}\right|_{r=r_+} dP + \left.\frac{\partial h}{\partial r}\right|_{r=r_+} dr_+. \tag{33}$$

In the extended phase space, the black hole mass can be regarded as the gravitational enthalpy which relates to the internal energy as in eqn. (12). After the black hole absorbs a particle with the energy E and charge q at the event horizon, the change of internal energy and charge of the black hole satisfy

$$\begin{aligned} d(M - PV) &= E, \\ dQ &= q. \end{aligned} \tag{34}$$

The initial outer horizon radius r_+ moves to the final outer horizon radius $r_+ + dr_+$, which leads to

$$h(M, Q, P, r_+) = h(M+dM, Q+dQ, P+dP, r_+ + dr_+) = 0. \tag{35}$$

From eqns. (26), (33), and (35), the infinitesimal changes in r_+, M, Q, and P are related by

$$4\pi T dr_+ - \frac{2}{r_+}E - \frac{2P}{r_+}dV + \frac{2\Phi}{r_+}q = 0. \tag{36}$$

Combining eqn. (36) with eqns. (7) and (25), we get

$$dS = \frac{P^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{T - 2Pr_+}. \tag{37}$$

Since $f((\beta_0/4\pi T)([P^r(r_+)]^2/m_p)) > 0$, the minimal length effect changes the rate of dS without changing the sign of dS. When $T > 2Pr_+$, the sign of the denominator in eqn. (37) is always positive. It means that the second law is satisfied in the extended phase space when the black hole is far enough from extremity. When $T < 2Pr_+$, the sign of the denominator in eqn. (37) is always negative. It implies that the second law of thermodynamics is not satisfied for the extremal or near-extremal black hole.

4. Minimal Length Effect on Weak Cosmic Censorship Conjecture in a RN-AdS Black Hole

In this section, we investigate the minimal length effect on the weak cosmic censorship conjecture by charge particle absorption in the normal and extended phase spaces. In the test particle limit, the black hole needs to start out close to extremal to have the chance to become a naked singularity. So, we assume that the initial RN-AdS black hole is a near-extreme black hole. Between two event horizons, there is one and only one minimum point at $r = r_{\min}$ with $h'(r_{\min}) = 0$. In the near-extremal and extremal cases, the minimum value of $h(r)$ is not greater than zero,

$$\sigma \equiv h(r_{\min}) \le 0, \tag{38}$$

where $\sigma \equiv 0$ corresponds to the extremal case. If there is a positive real root in the final state $h(r_{\min} + dr_{\min}) = 0$, the weak cosmic censorship conjecture is valid. Otherwise, the weak cosmic censorship conjecture is violated. Again, we first give the following partial derivative formulas:

$$\begin{aligned} \left.\frac{\partial h}{\partial r}\right|_{r=r_{\min}} &= 0, \\ \left.\frac{\partial h}{\partial M}\right|_{r=r_{\min}} &= -\frac{2}{r}, \\ \left.\frac{\partial h}{\partial Q}\right|_{r=r_{\min}} &= \frac{2Q}{r_+^2}, \\ \left.\frac{\partial h}{\partial P}\right|_{r=r_{\min}} &= \frac{8\pi r^2}{3}. \end{aligned} \tag{39}$$

4.1. *Normal Phase Space.*

After absorbing a charged particle with the mass m, the energy E, and the charge q, the physical parameters of the black hole change from the initial state (M, Q, r_+) to the final state $(M+dM, Q+dQ, r_+ + dr_+)$. The final state value of $h(r)$ at $r = r_{\min} + dr_{\min}$ is given by

$$h(M+dM, Q+dQ, r_{\min} + dr_{\min}) = \sigma + \left.\frac{\partial h}{\partial M}\right|_{r=r_{\min}} dM + \left.\frac{\partial h}{\partial Q}\right|_{r=r_{\min}} dQ + \left.\frac{\partial h}{\partial r}\right|_{r=r_{\min}} dr_{\min}. \tag{40}$$

Using eqns. (28), (32), and (39), eqn. (40) reduces to

$$\begin{aligned} h(M+dM, Q+dQ, r_{\min} + dr_{\min}) &= \sigma - \frac{P^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{r_{\min}} \\ &\quad - \frac{2Qq}{r_{\min}}\left(\frac{1}{r_+} - \frac{1}{r_{\min}}\right). \end{aligned} \tag{41}$$

When the initial black hole is an extremal RN-Ads black hole, $h(r) = 0$ has only one solution, in this case, $r_+ = r_{\min}$ and $\sigma = 0$. Therefore, eqn. (41) reduces to

$$h(M+dM, Q+dQ, r_{\min} + dr_{\min}) = -\frac{P^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{r_{\min}}, \tag{42}$$

which means that an extremal RN-Ads black hole becomes a nonextremal one after the absorption of a charged particle in the normal phase space.

For a near-extremal RN-Ads black hole, by introducing infinitesimal parameters ε, we can define the relationship between $r_{\min}$ and r_+ as follows:

$$r_{\min} = r_+(1-\varepsilon), \tag{43}$$

where $\varepsilon \ll 1$. So, σ is suppressed by ε in the near-extremal limit. Moreover, the second term of eqn. (41) is only suppressed by the test particle limit, and the third term is suppressed by both the near-extremal limit and the test particle limit, and hence can be neglected. Therefore, eqn. (41) leads to

$$\begin{aligned} &h(M+dM, Q+dQ, r_+ + dr_+) \\ &= \sigma - \frac{P^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{r_{\min}} < 0, \end{aligned} \tag{44}$$

which means that a near-extremal black hole stays near-extremal after the absorption of a charged test particle in the normal phase space. In the normal phase space, we find that the weak cosmic censorship conjecture is legal for extremal and near-extremal RN-Ads black holes.

4.2. Extended Phase Space. In this section, we investigate the minimal length effect on the weak cosmic censorship conjecture via charged particle absorption in the extended phase space. After absorbing a charged particle, the physical parameters of the black hole change from the initial state (M, Q, P, r_+) to the final state $(M+dM, Q+dQ, P+dP, r_+ + dr_+)$. The final state value of $h(r)$ at $r = r_{\min} + dr_{\min}$ is given by

$$\begin{aligned} &h(M+dM, Q+dQ, P+dP, r_{\min}+dr_{\min}) \\ &= \sigma + \left.\frac{\partial h}{\partial M}\right|_{r=r_{\min}} dM + \left.\frac{\partial h}{\partial Q}\right|_{r=r_{\min}} dQ \\ &+ \left.\frac{\partial h}{\partial P}\right|_{r=r_{\min}} dP + \left.\frac{\partial h}{\partial r}\right|_{r=r_{\min}} dr_{\min}. \end{aligned} \tag{45}$$

Using eqns. (34), (37), and (39), eqn. (45) reduces to

$$\begin{aligned} &h(M+dM, Q+dQ, P+dP, r_{\min}+dr_{\min}) \\ &= \sigma - \frac{2TP^r(r_+) f\left((\beta m_p/4\pi T)(p^r)^2\right)}{(T-2Pr_+) r_{\min}} \\ &- \frac{2Qq}{r_{\min}}\left(\frac{1}{r_+} - \frac{1}{r_{\min}}\right) - \frac{8\pi dP}{r_{\min}}\left(r_+^3 - r_{\min}^3\right). \end{aligned} \tag{46}$$

When the initial black hole is an extremal RN-Ads black hole, $h(r)=0$ has only one solution. In this case, $r_+ = r_{\min}$, $T=0$, and $\sigma = 0$. Therefore, eqn. (46) reduces to

$$h(M+dM, Q+dQ, P+dP, r_{\min}+dr_{\min}) = 0, \tag{47}$$

which means that an extremal RN-Ads black hole stays an extremal state after the absorption of a charged particle.

For a near-extremal RN-Ads black hole, the second term of eqn. (46) is only suppressed by the test particle limit; however, the third and fourth terms of eqn. (46) are suppressed by both the near-extremal limit and the test particle limit, and hence can be neglected. Therefore, eqn. (46) leads to

$$\begin{aligned} &h(M+dM, Q+dQ, P+dP, r_{\min}+dr_{\min}) \\ &= \sigma - \frac{2TP^r(r_+) f\left((\beta_0/4\pi T)\left([P^r(r_+)]^2/m_p\right)\right)}{(T-2Pr_+) r_{\min}} < 0, \end{aligned} \tag{48}$$

which means that a near-extremal black hole stays near-extremal after the absorption of the charged test particle. In the extended phase space, we find that the weak cosmic censorship conjecture is legal for extremal and near-extremal RN-Ads black holes.

5. Conclusion

In this paper, we investigated the minimal length effect on the weak cosmic censorship conjecture in a RN-AdS black bole via the charged particle absorption. We first introduced thermodynamics of RN-Ads black holes in the normal and extended phase space. Then, we employed GUP to investigate the effect of the minimal length on the Hamilton-Jacobi equation. Specifically, we derived the deformed Hamilton-Jacobi equation which has the same form both for scalar and fermionic particles and used it to study the variations of the thermodynamic quantities of a RN-Ads black hole via absorbing a charged particle. Furthermore, we checked the second law of thermodynamics and the weak cosmic censorship conjecture in a RN-Ads black hole. In the normal phase space, the second law of thermodynamics is satisfied, and the GUP effect made the increase of entropy faster than the usual case. We found that the weak cosmic censorship conjecture is satisfied for the extremal and near-extremal RN-AdS black holes. After the charge particle absorption, an extremal RN-AdS black hole becomes nonextremal. In the extended phase space, where the cosmological constant is treated as pressure, the black hole entropy can either increase or decrease depending on the states of the black hole. When $T > 2Pr_+$, the second law is satisfied in the extended phase space. When $T < 2Pr_+$, the second law of thermodynamics is not satisfied. Finally, we found that the weak cosmic censorship conjecture is always legal for extremal and near-extremal RN-Ads black holes.

To discuss the validity of the weak cosmic censorship, we considered the minimum value of the metric function $h(r)$ of an extremal or near-extremal black hole after a charged particle is absorbed by the black hole. It was found that the minimum value stays negative, which means the event horizon of the black hole always exists, and hence the weak cosmic censorship is satisfied. However, apart from the existence of the horizon, the analysis did not provide the information about how the position of the horizon changes. On the other hand, the test of the second law may shed light on how the horizon radius changes. The horizon expands when the second law is valid, while the horizon shrinks when the second law is violated (e.g., for an extremal or near-extremal black hole in the extended phase space). In other words, the validities of the weak cosmic censorship and the second law crucially depend on the existence and the position of the horizon, respectively, after the black hole absorbs the particle.

In [72], the second law of thermodynamics and the weak cosmic censorship conjecture of the RN rainbow black hole have been investigated. The authors considered only the normal phase space case. Different from [72], we have chosen another quantum gravity model, namely GUP, to study the second law of thermodynamics and the weak cosmic censorship conjecture in the normal and extended phase space. When $\beta \longrightarrow 0$, $f((\beta_0/4\pi T)([P^r(r_+)]^2/m_p)) \longrightarrow 1$ and the GUP-deformed case reduces to the usual case, and our result is consistent with those in [29, 31, 36].

Data Availability

The process data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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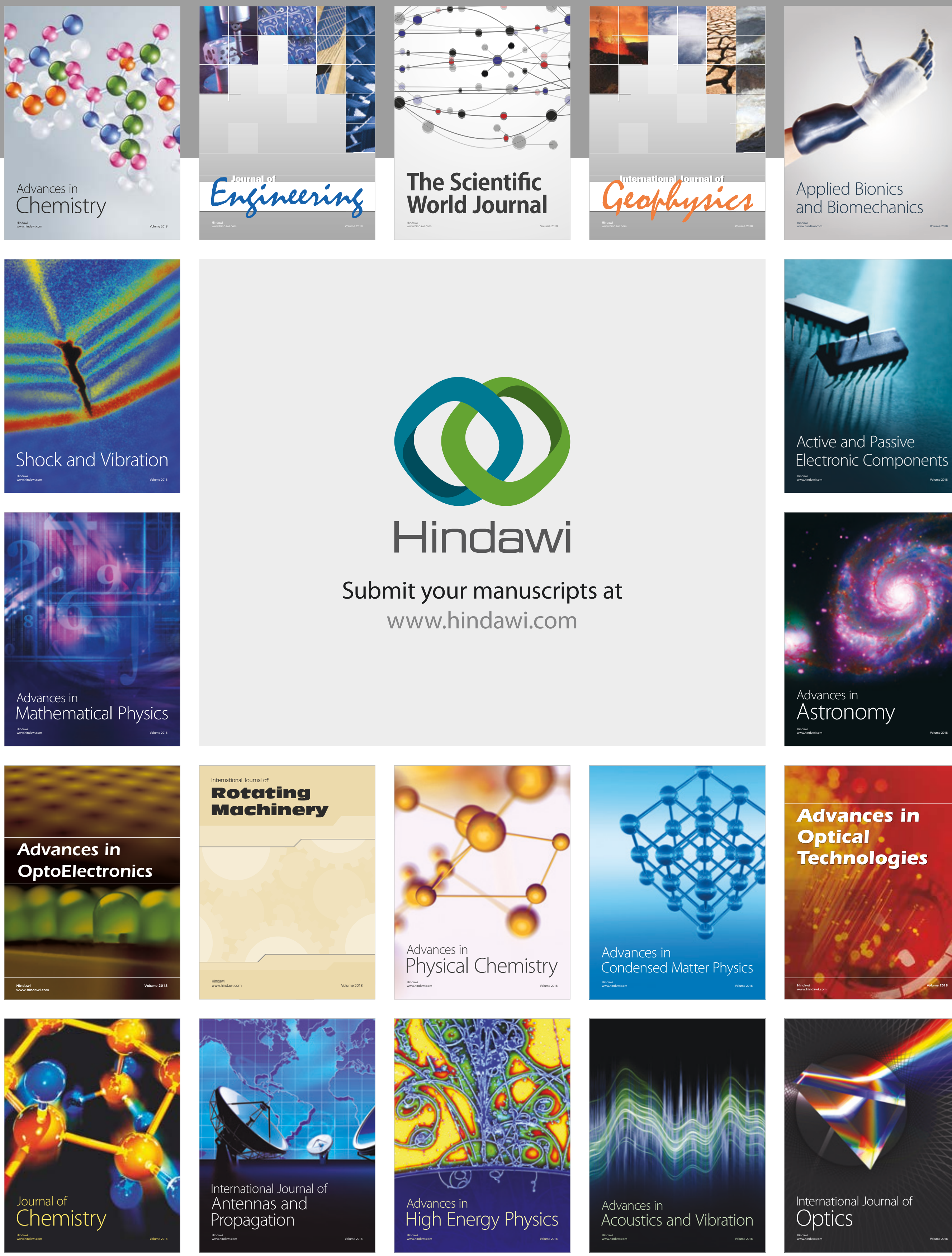
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