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Research Article

The Propagation of Thermoelastic Waves in Different Anisotropic Media Using Matricant Method

Nurlybek A. Ispulov,[1] Almar Zh. Zhumabekov,[1] Abdul Qadir ,[2] Almas A. Kurmanov,[1] Sholpan N. Sarymova,[1] Kairat R. Dossumbekov,[1] and Erkin Arinov[3]

[1] *Institute Physics, Department of Mathematics and Instrumentation, Toraighyrov University, 64 Lomov St, Pavlodar 140000, Kazakhstan*
[2] *Sukkur IBA University, Sindh, Pakistan*
[3] *O.A. Baikonyrov Zhezkazgan University, Kazakhstan*

Correspondence should be addressed to Abdul Qadir; aqadir@iba-suk.edu.pk

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Thermoelasticity is a generalization of classical theories of elasticity and thermal conductivity and describes a wide range of phenomenon. The theory can precisely predict the propagation of thermoelastics waves in case of an isotropic medium. However, the propagation of thermoelastic waves in the anisotropic medium is not fully understood. In this case, the theory of elasticity employs an approximate theory of temperature stress which does not take into consideration the interactions of temperature and deformations. In this paper, an analytical study has been carried out by using method of matricant to investigate the propagation of longitudinal elastic and heat waves in the anisotropic medium of a monoclinic, trigonal, hexagonal, and cubical crystal systems. In this article, a solution to the problem of the propagation of thermal waves and the propagation of a thermal wave along z-axis has been obtained. The attenuation coefficient and phase velocity of thermal waves for various materials are determined. Specifically, the problem of propagation of heat waves in one dimension has been solved.

1. Introduction

Thermodynamics of irreversible processes, developed in the last century, also made it possible to solve the problems of the irreversible deformation and gave a unified interpretation of mechanical and thermal processes [1, 2].

Similarly, Nowacki [3, 4] studied the harmonic wave propagation in a thermoelastic layer. Due to the weak coupling of the temperature and strain field, characterized by a thermal and mechanical parameter, the approximate frequency equation is solved by the perturbation method.

The poroelasticity equations formulated by Biot formed the basis for solving wave propagation problems in the poroelasticity region. Biot also presented the relationship between stress and strain in case of anisotropic poroelastic solid. Based on the Biot equations, Sharma [5, 6] investigated the reflection and transmission at the liquid and porous solid interface; however, a porous solid is considered anisotopic with arbitrary symmetry. Similar research has been carried out in the following areas: problems are considered and the solution of three differential equations by introducing the elastic and thermo-elastic potentials [7], the photothermal transport process [8], the effect of variable thermal conductivity of pPhotothermal diffusion (PTD) [9], an electromagnetothermoelastic coupled problem for a homogeneous, isotropic, thermally and electrically conducting half-space solid [10].

The wave propagation in the thermoelastic anisotropic medium have been investigated by using a matricant method, for instance wave propagation waves in liquid crystals and in thermoelastic medium [11–17]. In the paper [18], authors consider the propagation of Rayleigh surface waves in a functionally graded isotropic thermoelastic half-space. In the work presented, the Stroh analysis of the Rayleigh waves for a general class of anisotropic thermoelastic materials [19].

The theory has also been employed to investigate the interaction of free harmonic waves with multilayer medium, using a combination of the method of linear transformation and transfer matrix [20]. The paper [21] considers the surface of a semi-infinite magnetothermoelastic solid body from which P and SV waves are reflected. The authors of [22] research thermophotoelectric interactions using a new mathematical model of thermoelasticity. This model made it possible to research the interaction between the processes of thermos-elastic plasma.

Most of the classical sources on electrodynamics of the anisotropic medium, including those proposed, do not use the matricant method. In our work, we use the analytical matricant method, which first clarifies the structure of solutions in the form of matrices based on comparing the elements of exponential series (in our case). Further, for some restrictions, dependencies are obtained between the characteristics of waves and material medium. This method was proposed by professor Tleukenov, based on the works of Brillouin and Parodi [23].

In this paper, we have employed a new matricant method to investigate the propagation of thermal waves in case of one dimension. The solutions obtained are in accordance with the known classical solution [24]. Moreover, the results also coincide with poroelasticity equation. The matricant method makes it possible to research wave processes (elastic and electromagnetic) in the isotropic and anisotropic medium.

2. The Research Method

In this paper, we have used the matricant method [25], which allows to obtain accurate analytical solutions of differential equations describing the related processes in medium with piezoelectric, piezomagnetic, thermoelastic, and thermopiezoelectric properties.

This analytical study is based on the development of matrix methods for analyzing the dynamics of the elastic stratified medium.

The method deals with reducing the initial equations of motion by separation of variable method (representation of the solution in the form of plane waves) to the equivalent system of first order ordinary differential equations with variable coefficients and the constructing the matricant structure, i.e., normalized matrix of fundamental solutions.

The advantage of matricant method is that it allows formulating the wave propagation for wider class of medium. Moreover, another advantage of the method is that the expressions so obtained have a very compact form. This proves to be convenient both for analytical and for numerical calculations.

This method has been tested and the results obtained are in consistent with previously known results in various publications.

The main advantage of the matricant method is the uniform description of wave propagation in various effects for instance, thermoelastic, magnetoelastic, piezoelectric, and piezomagnetic effects [26–28].

3. Basic Equation and Formulation of Problem

The study of the propagation of thermoelastic waves in anisotropic medium is based on the simultaneous solution of equations of motion in elastic medium [3, 4]:

$$\frac{\partial \sigma_{XX}}{\partial X} + \frac{\partial \sigma_{XY}}{\partial Y} + \frac{\partial \sigma_{XZ}}{\partial Z} = \rho \frac{\partial^2 U_X}{\partial t^2}, \tag{1}$$

$$\frac{\partial \sigma_{XY}}{\partial X} + \frac{\partial \sigma_{YY}}{\partial Y} + \frac{\partial \sigma_{YZ}}{\partial Z} = \rho \frac{\partial^2 U_Y}{\partial t^2}, \tag{2}$$

$$\frac{\partial \sigma_{XZ}}{\partial X} + \frac{\partial \sigma_{YZ}}{\partial Y} + \frac{\partial \sigma_{ZZ}}{\partial Z} = \rho \frac{\partial^2 U_Z}{\partial t^2}. \tag{3}$$

The equations of heat conductivity proposed by Fourier in case of anisotropic medium is as follows:

$$\lambda_{ij} \frac{\partial \theta}{\partial x_j} = -q_i, \tag{4}$$

and the heat inflow equation without the influence of heat source is given by

$$\frac{\partial q_i}{\partial x_i} = -i\omega \beta_{ij} \varepsilon_{ij} - i\omega \frac{c_\varepsilon}{T_0} \theta, \tag{5}$$

where σ_{ij} represents the components of stress tensor, ρ is the density of medium, λ_{ij} is the components of the heat conductivity tensor, q_i are the components of the heat flow vector, ω is the angular frequency, β_{ij} are the thermomechanical parameters of medium, ε_{ij} are the components of the tensor of small Cauchy deformation, c_ε is the heat capacity under constant deformation, and $\theta = T - T_0$ is the temperature augments compared with natural state temperature T_0 (T_0 is the temperature of the natural state without deformations). For the case when the deformation is small, $|\theta/T_0|\langle\langle 1$.

The equations between stress and strain can be described by Duhamel–Neumann relationships as

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - \beta_{ij} \theta, \tag{6}$$

where c_{ijkl} is the elastic constants, $\alpha = ij$, $\beta = kl$ and β_{ij} is the thermomechanical parameters of the medium.

Here, equations (1)–(6) show that the relationship between temperature and stress generated in a mechanical process as a function of the heat field and deformation in a medium, whereas they are independent variables.

For the monoclinic system, the matrix of elastic constants c_{ijkl} can be written as

$$c_{\alpha\beta} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{54} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{pmatrix}. \tag{7}$$

The thermomechanical parameters of the body are β_{ij}, and they depend on both mechanical and heat properties of the body and for an anisotropic medium of a monoclinic system they are given as follows:

$$\beta_{ij} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{12} & \beta_{22} & 0 \\ \beta_{13} & 0 & \beta_{33} \end{pmatrix}. \tag{8}$$

By using separation of variables method, equations (1)–(6) can be reduced to a system of ordinary differential equations, where the heterogeneity of medium is assumed to be along Z axis, i.e., axis $Z \| A_2$ where A_2 is the second order symmetry axis

$$\frac{d\vec{\mathbf{W}}}{dz} = \mathbf{B}\vec{\mathbf{W}}, \tag{9}$$

where vector $\vec{\mathbf{W}}$ has the form

$$\vec{\mathbf{W}}(x, y, z, t) = \left[u_z(z), \sigma_{zz}, u_x(z), \sigma_{xz}, u_y(z), \sigma_{yz}, \theta, q_z\right]^t \exp\left(i\omega t - i\,mx - i\,ny\right). \tag{10}$$

Here, $\vec{\mathbf{W}}$ is a column vector, which includes the boundary conditions of the problem; $u_z(z)$, $u_x(z)$, and $u_y(z)$ represent the projection of displacement vector on the corresponding coordinates, and $m = k_x$, $n = k_y$, and $l = k_z$, shows the x, y, and z components of a wave vector k, respectively.

The coefficients matrix is given as

$$\mathbf{B} = \mathbf{B}\left[c_{ijkl}(z), \beta_{ij}(z), \theta, \omega, m, n, l\right], \tag{11}$$

it shows the functional dependence of matrix $\mathbf{B}$, for example as $f = f\,(x, y, z, t)$.

Here, the elements of coefficients matrix $\mathbf{B}$ as given in Equation (11) contain the information of wave propagation in the medium. In this paper, we have analyzed the coefficients of matrix $\mathbf{B}$ to determine the polarization of the waves and the relationship among them diverges under the influence of the thermomechanical effect.

Earlier in work [12], a system of differential equations (9) describing the propagation of coupled elastic and thermal waves in anisotropic medium of rhombic, tetragonal, and hexagonal syngony was formulated.

In the monoclinic system, there is a specific direction or a designated plane, or both. Since the direction determines the plane perpendicular to it so direction is chosen along vertical axis. It is usually denoted by c, z, or x_3; the remaining two coordinate axes can freely be positioned in the horizontal plane. All three axes can be of arbitrary length. An anisotropic monoclinic medium is characterized by a second-order symmetry axis. If the $z \| A_2$ inhomogeneity depends on z, then the structure of the matrix $\mathbf{B}$ in (9) will have the following form:

$$\mathbf{B} = \begin{bmatrix} 0 & b_{12} & b_{13} & 0 & b_{15} & 0 & b_{17} & 0 \\ b_{21} & 0 & 0 & b_{24} & 0 & b_{26} & 0 & 0 \\ b_{24} & 0 & 0 & b_{34} & 0 & b_{36} & 0 & 0 \\ 0 & b_{13} & b_{43} & 0 & b_{45} & 0 & b_{47} & 0 \\ b_{26} & 0 & 0 & b_{36} & 0 & b_{56} & 0 & 0 \\ 0 & b_{15} & b_{45} & 0 & b_{65} & 0 & b_{67} & b_{77} \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{77} & b_{78} \\ 0 & -i\omega\, b_{17} & -i\omega\, b_{47} & 0 & -i\omega\, b_{67} & 0 & b_{87} & b_{77} \end{bmatrix}, \tag{12}$$

where b_{ij} represents the components of coefficient matrix in case of monoclinic syngony and are given as follows:

$$\begin{gathered} b_{12} = \frac{1}{c_{33}}\,; b_{13} = \frac{c_{13}}{c_{33}} im\,; b_{15} = \frac{c_{36}}{c_{33}} im + \frac{c_{23}}{c_{33}} \text{in}\,; b_{17} = \frac{\beta_{33}}{c_{33}}, \\ b_{21} = -\omega^2\rho\,; b_{24} = im\,; b_{26} = \text{in}\,; b_{34} = \frac{c_{44}}{c_{44}c_{55} - c_{45}^2}\,; b_{36} = \frac{c_{45}}{c_{44}c_{55} - c_{45}^2}, \\ b_{43} = \frac{c_{11}c_{33} - c_{13}^2}{c_{33}} m^2 + \frac{c_{33}c_{66} - c_{36}^2}{c_{33}} n^2 + \frac{2(c_{16}c_{33} - c_{13}c_{36})}{c_{33}} mn - \omega^2\rho, \\ b_{45} = \frac{c_{16}c_{33} - c_{13}c_{36}}{c_{33}} m^2 + \frac{c_{26}c_{33} - c_{23}c_{36}}{c_{33}} n^2 + \frac{c_{12}c_{33} + c_{33}c_{66} - c_{13}c_{23} + c_{36}^2}{c_{33}} mn, \\ b_{47} = \left(\frac{c_{13}}{c_{33}}\beta_{33} - \beta_{11}\right) im + \frac{c_{36}}{c_{33}}\beta_{33}\text{in}\,; b_{56} = \frac{1}{c_{44}}, \\ b_{65} = \left(c_{66} - \frac{c_{23}^2}{c_{33}}\right) n^2 + \frac{c_{33}c_{66} - c_{36}^2}{c_{33}} m^2 + \frac{2(c_{26}c_{33} - c_{23}c_{36})}{c_{33}} mn, \\ b_{67} = \frac{c_{36}}{c_{33}}\beta_{33} im + \left(\frac{c_{23}}{c_{33}}\beta_{33} - \beta_{12} - \beta_{22}\right)\text{in}\,; b_{77} = \frac{\lambda_{13}}{\lambda_{33}} im + \frac{\lambda_{23}}{\lambda_{33}} \text{in}, \\ b_{78} = -\frac{1}{\lambda_{33}}\,; b_{87} = -i\omega c_{\varepsilon}. \end{gathered} \tag{13}$$

In this paper, the propagation of thermal waves in an anisotropic medium of monoclinic, trigonal, hexagonal, and cubic crystal system have been considered in the presence of even order symmetry axis.

4. Solution of the Problem

The equations of motion as given by (1) for the case of longitudinal elastic wave propagating along one of spatial coordinates in an anisotropic layer can be written as

$$\frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 U_z}{\partial t^2}, \tag{14}$$

where $\sigma_z = c_{33}(\partial Uz/\partial z)$ is the z-component of the stress tensor σ_{ij}, ρ is the medium density, U_z is the z-component of the displacement vector of medium, and c_{33} is the isothermal elastic moduls.

By using separation of variables method, we get in case of harmonic waves:

$$[U_z ; \sigma_z] = [U_i(z), \sigma_{ij}(z)] e^{i\omega t}. \tag{15}$$

The system of equations (1)–(6) is reduced to a system of differential equations of second order, describing the propagation of harmonic waves (9).

The result is system of first order differential equation (9):

$$\left.\begin{aligned} \frac{dU_z}{dz} &= \frac{1}{c_{33}} \sigma_z \\ \frac{d\sigma_z}{dz} &= -\omega^2 \rho U_z \end{aligned}\right\} \Longrightarrow \frac{d}{dz}\begin{pmatrix} U_z \\ \sigma_z \end{pmatrix} = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}\begin{pmatrix} U \\ \sigma \end{pmatrix}. \tag{16}$$

Condition for the existence of nontrivial solutions is the vanishing of the following determinant [19]:

$$\det |\mathbf{B} - \lambda \mathbf{E}| = 0, \tag{17}$$

where $\mathbf{B}$ represents coefficient matrix whose elements contain the parameters of the medium, in which an elastic longitudinal wave propagates. The elements of this matrix are contained in (16) and have the following form:

$$b_{12} = \frac{1}{c_{33}} ; b_{21} = -\omega^2 \rho, \tag{18}$$

this results in obtaining the characteristic equation (17):

$$\lambda^2 = \pm i\omega \sqrt{\frac{\rho}{c_{33}}}. \tag{19}$$

The last relation leads to the conclusion that the wave spectrum is equal to:

$$k_{1,2} = \pm i\omega \sqrt{\frac{\rho}{c_{33}}}. \tag{20}$$

This problem can be solved as follows:

$$\phi = Ae^{\lambda_1 z} + Be^{\lambda_2 z} \Longrightarrow \phi = Ae^{i\omega\sqrt{(\rho/c_{33})}z} + Be^{-i\omega\sqrt{(\rho/c_{33})}z}. \tag{21}$$

Let us take the abovementioned as an example to consider the heat wave propagation in an anisotropic medium of the monoclinic syngony.

Let us assume that harmonic thermal expansion waves with the angular frequency ω occur in an unlimited thermoelastic medium.

The one-dimension equation of heat conductivity is as follows:

$$c_\varepsilon \frac{\partial \theta}{\partial t} = \lambda_{33} \frac{\partial^2 \theta}{\partial z^2}, \tag{22}$$

This can be written in matrix form as follows:

$$\frac{d}{dz}\begin{pmatrix} \theta \\ q_z \end{pmatrix} = \begin{pmatrix} 0 & b_{78} \\ b_{87} & 0 \end{pmatrix}\begin{pmatrix} \theta \\ q_z \end{pmatrix}, \tag{23}$$

where c_ε shows heat capacity at constant strain, $\theta = T - T_0$ temperature increase compared to the temperature T_0 of the natural state, λ_{33} is heat conductivity tensor, and q_z represents components of heat vector [2].

The coefficients of the matrix in (23) have the form:

$$b_{78} = -\frac{1}{\lambda_{33}} ; b_{87} = -i\omega c_\varepsilon. \tag{24}$$

In this case, the characteristic equation (17) can be represented as follows:

$$\delta^2 - i\omega \frac{c_\varepsilon}{\lambda_{33}} = 0, \tag{25}$$

hence, it follows that

$$\delta_{1,2} = \pm \sqrt{\frac{i\omega}{a}}, \tag{26}$$

where $a = \lambda / c_p \rho$ is coefficient of heat conductivity, λ represents coefficient of thermal conductivity, c_p is specific heat capacity, and ρ shows density of the substance.

The roots of (26) can be represented as follows:

$$\begin{aligned} \delta_1 &= \sqrt{\frac{\omega}{a}} e^{i(\pi/4)} ; \delta_2 = \sqrt{\frac{\omega}{a}} e^{i(\pi/4)+\pi}, \Longrightarrow \delta_1 \\ &= \sqrt{\frac{\omega}{a}}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{\frac{\omega}{2a}}(1+i), \end{aligned} \tag{27}$$

$$\delta_2 = -\sqrt{\frac{\omega}{2a}}(1+i). \tag{28}$$

Table 1

Substance	λ, coefficient of thermal conductivity (W/(m∗K))	c_p, specific heat capacity (J/(kg∗K))	ρ, density (kg/m^3)
Quartz	11.3	750	2650
Calcite	4.98	800	2710
Bismuth	6.65	123.5	9750
Graphite	89	840	2150
Aluminum	208	897	2700
Copper	410	385	8950

Subtraction (27) from (28), results in

$$\delta_2 - \delta_1 = -\sqrt{\frac{\omega}{2a}}(1+i), \tag{29}$$

then

$$\delta_1 = -\delta_2 \Longrightarrow \delta_2 = -\delta_1. \tag{30}$$

The solution of the heat wave propagating in case of one dimension is as follows:

$$\mathbf{T}_m = \frac{\mathbf{B} - \delta_2 \mathbf{E}}{\delta_1 - \delta_2} e^{\delta_1 z} + \frac{\mathbf{B} - \delta_1 \mathbf{E}}{\delta_2 - \delta_1} e^{\delta_2 z}. \tag{31}$$

In papers [12, 13], the general form a matricant structure is similar to the (31) and the exact solution of the system of differential equations (9), which describes the propagation of thermoelastic waves in anisotropic medium was formulated. The system of differential equations is as described (23). The formulation of exact solution is given by (31).

Numerator on the right side of (31), by using (29) and (30), becomes

$$\frac{\mathbf{B} - \delta_1 \mathbf{E}}{2\delta_2} = \begin{pmatrix} \dfrac{1}{2} & \dfrac{1}{2\sqrt{(2\omega/a)}(1+i)} \\ \dfrac{i\omega\, c_\varepsilon}{2\sqrt{(2\omega/a)}} & \dfrac{1}{2} \end{pmatrix}. \tag{32}$$

Coefficients of matrix $\mathbf{B}$ can be represented as follows:

$$\mathbf{B} = \begin{pmatrix} \dfrac{1}{2} & \dfrac{1-i}{4\sqrt{(2\omega/a)}} \\ \dfrac{\omega\, c_\varepsilon}{4\sqrt{(2\omega/a)}} & \dfrac{1}{2} \end{pmatrix} + \begin{pmatrix} 0 & -\dfrac{1}{4\sqrt{(2\omega/a)}} \\ \dfrac{\omega\, c_\varepsilon}{4\sqrt{(2\omega/a)}} & 0 \end{pmatrix}. \tag{33}$$

Consequently, the coefficient matrix B is separated into the real and the imaginary parts:

$$\mathbf{B} = \operatorname{Re}\mathbf{B} + \operatorname{Im}\mathbf{B}, \tag{34}$$

this corresponds to heat wave propagation in a solid medium.

For the general case, considering the above relations, the solution of equation (31) can be represented as follows:

$$\mathbf{T}_m = \operatorname{Re}\mathbf{B} e^{-\sqrt{(\omega/2a)}z} Cos\sqrt{\frac{\omega}{2a}}z + \operatorname{Im}\mathbf{B} e^{-\sqrt{(\omega/2a)}z} Sin\sqrt{\frac{\omega}{2a}}z. \tag{35}$$

The solution of the heat wave propagation problem in the one-dimensional case coincides with the classic solution, which is as follows [20]:

$$f = e^{-\sqrt{(\omega/2a)}z} e^{i\left(\omega t - \sqrt{(\omega/2a)}z\right)}. \tag{36}$$

For physical reasons, from the two roots δ_1, δ_2, it is necessary to retain the root, which includes the negative real part.

Consequently, the solution for the heat wave is obtained as follows:

$$\theta = \theta_0 e^{-\sqrt{(\omega/2a)}z} Cos\omega\left(t - \frac{z}{\sqrt{2a\omega}}\right), \tag{37}$$

where $v = \sqrt{2a\omega}$ is the phase velocity and it depends on frequency of the heat wave.

Expression (21) is a purely elastic plane harmonic wave propagating along z-axis. This wave has neither damping nor dispersion. Expression (37) corresponds to a purely thermal plane harmonic wave, which has an attenuation characterized by the coefficient $q = \sqrt{\omega/2a}$, and variance due to the fact that the phase velocity is a function of frequency:$v = \sqrt{2a\omega}$.

The attenuation coefficient and the phase velocity of the heat wave have the form $q = \sqrt{\omega/2a}$ and $v = \sqrt{2a\omega}$, respectively. The coefficient of thermal conductivity is expressed by the ratio $a = \lambda/c_p\rho$.Consider the following substances and their parameters as given in Table 1 [24].

It can be seen from the Figure 1 that q and v depend on ω in the same way, because $q \sim \sqrt{\omega}$ and $v \sim \sqrt{\omega}$. It can also be seen that the increase in q and v, depending on the increase in ω, increase according to a parabolic law.

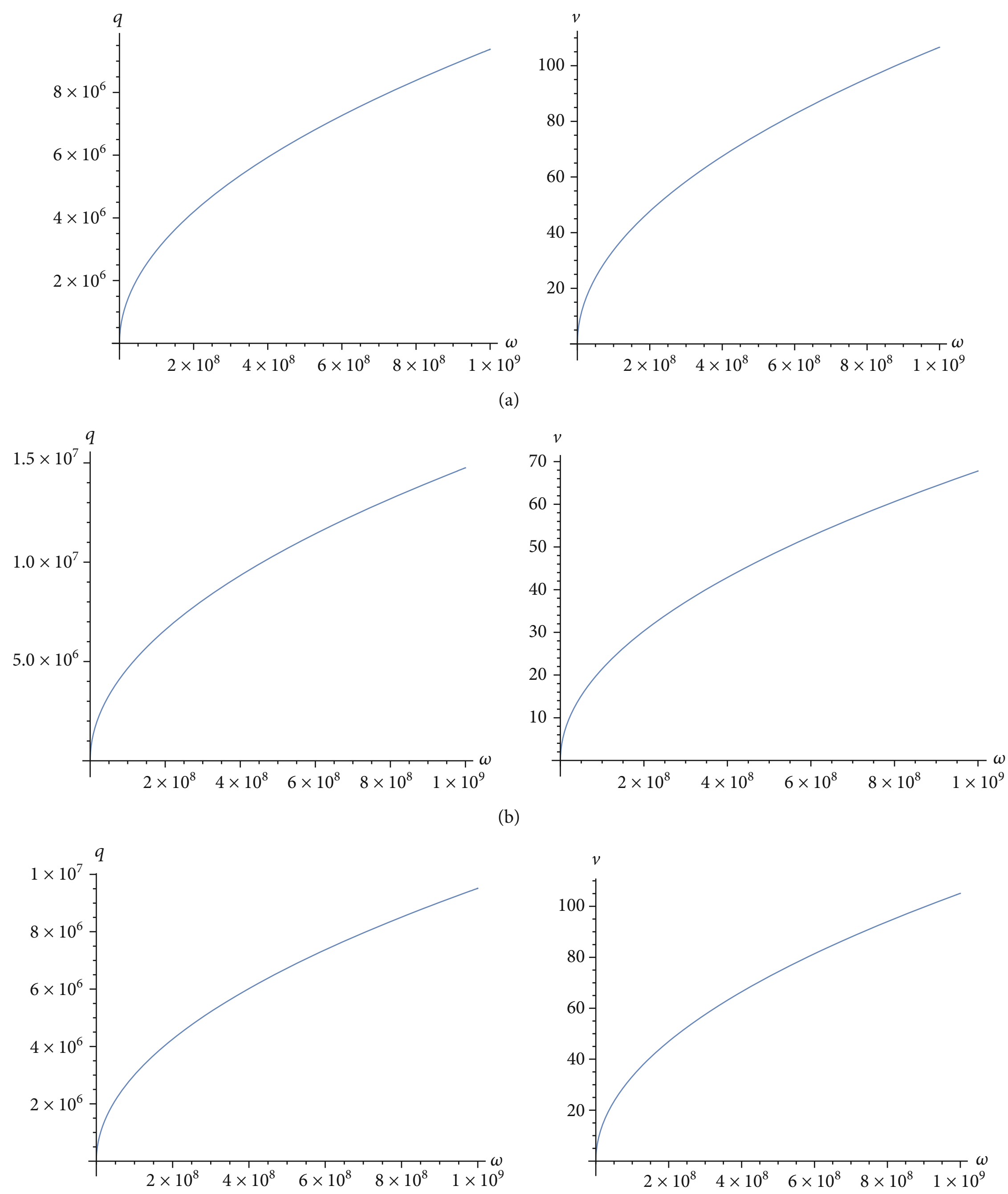

Figure 1: Continued.

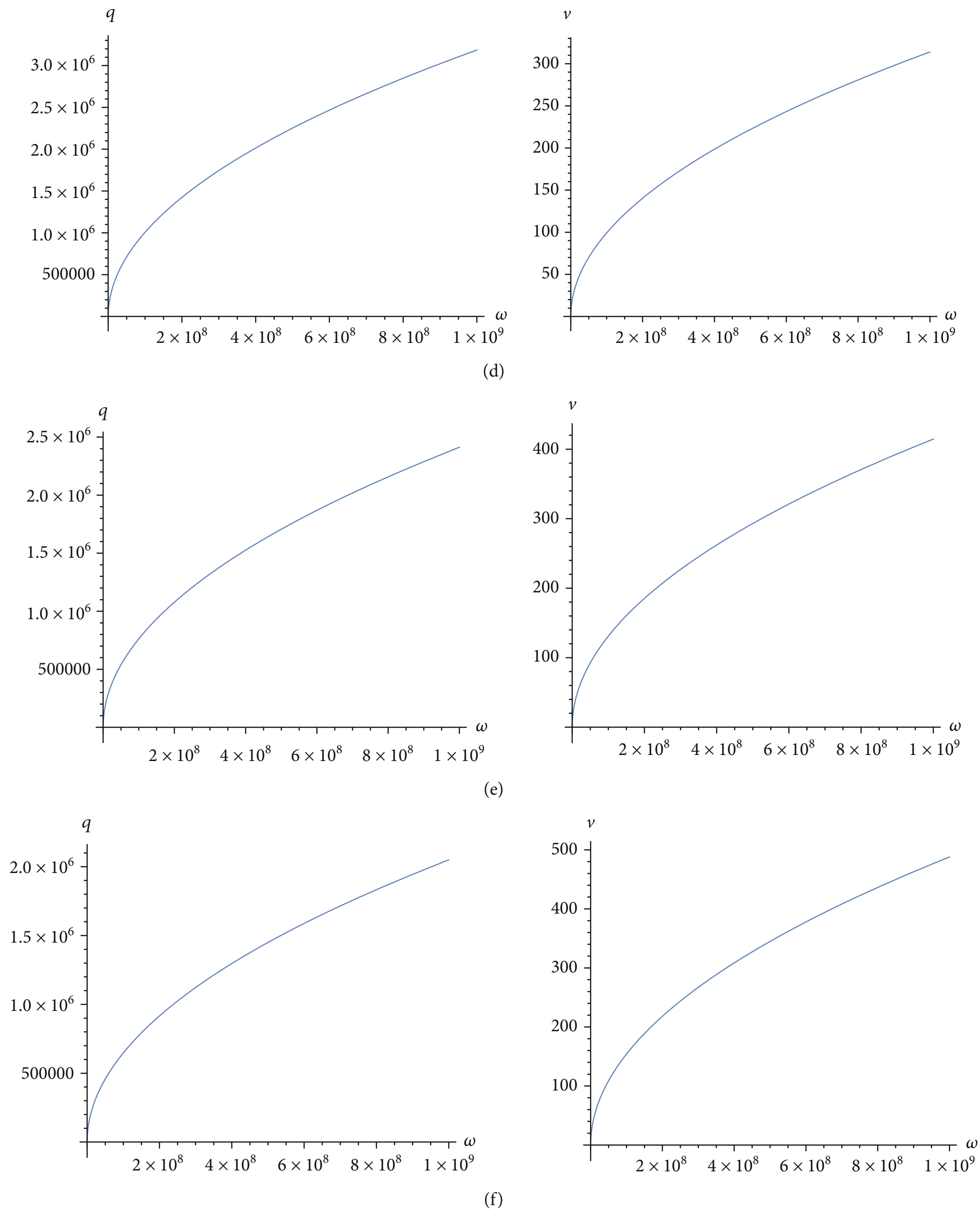

Figure 1: The dependence of the attenuation coefficient and the phase velocity of the heat wave on the angular frequency in different mediums. (a) Quartz. (b) Calcite. (c) Bismuth. (d) Graphite. (e) Aluminum. (f) Copper.

5. Conclusion

In this paper, the propagation of elastic longitudinal and thermal waves in anisotropic medium of monoclinic, trigonal, hexagonal, and cubic crystal systems is considered on the basis of the matrix method. In particular, the problem of heat wave propagation in the one-dimensional case is solved, the solution of which coincides with the known classical solution.

Moreover, by using the matricant method the solutions of equations of wave propagation in elastic medium are obtained. From these solutions, it is possible to determine the attenuation coefficient and phase velocity of the thermal waves. Finally, the results obtained by the matricant method are in consistent with the models of poroelastic equations obtained by using another analytical solution [5, 6]. It is expected that the results obtained will be helpful for better understanding the thermoelastic wave propagation in various mediums. In this paper, we got the dependence of the attenuation coefficient and the phase velocity of the heat wave on the angular frequency in different mediums. We analyzed these dependencies.

Data Availability

Data is available on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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