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Numerical Solution of Some COVID-19 Models

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper presents a two-step Bernstein induced scheme for the numerical solution of some COVID-19 models. The scheme is developed via collocation and interpolation techniques invoked on Bernstein polynomials; the proposed scheme is consistent, between orders four and three. This method can estimate the approximate solution at step points simultaneously by using variable step size. A numerical implementation of the scheme was used on the COVID-19 model, and this showed that the scheme can be conveniently applied to some mathematical models of COVID-19.

Keywords: Mathematical model; COVID-19; bernstein; collocation; interpolation.

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1 Introduction

In the sciences and engineering, mathematical models are formulated to aid in the understanding of physical phenomena. The formulated model often yields an equation that contains the derivatives of the unknown function; such an equation is referred to as a "differential equation [1,2]. Interestingly, differential equations arising from the modeling of physical phenomena often do not have exact solutions. “Hence, the development of numerical methods to obtain approximate solutions becomes necessary. To that extent, several numerical methods, such as finite difference methods, finite element methods, and finite volume methods, among others, have been developed based on the nature and type of the differential equation to be solved” [3].

“The knowledge of mathematical models is very useful in understanding the behavior of an infection when it enters a community and investigating the conditions it will take to be wiped out. Recently, COVID-19 has been of great interest to everybody—researchers and governments—because of the high rate of infection and the significant number of deaths that occur as a result of the disease. In 2019, coronavirus was first reported in Wuhan, China, as an infectious disease caused by the deadly virus coronavirus, which is mainly transmitted through droplets generated when an infected person coughs or sneezes” [4]. Researchers have been tracking assiduously the spread of the virus, have mobilized to speed innovative diagnostics, and are working on a number of vaccines to protect against COVID-19 [5,6].

Adeniran and Longe [7], in their paper, construct “a one-step hybrid numerical scheme with one off-grid point for solving directly the general second-order initial value problems. The scheme is developed using collocation and interpolation techniques invoked on the Lucas polynomial. The numerical results of the scheme show some efficiency over some existing schemes of the same or higher order”. Ojo and Okoro [8] used a “Bernstein polynomial to develop a one-step hybrid scheme with one off-grid point via collocation and interpolation techniques for the direct solution of general second-order ordinary differential equations, and the method was found to be reliable”. Farouki, & Goodman [9] and Yasser (2023) showed that Bernstein polynomial recurrence has been proven to be conditionally stable and has an exponential rate of convergence.

1.1 Background of the Study

Bernstein Polynomial of degree m on interval [0, 1] is defined mathematically as

$$B_{i,m}(x) = \binom{m}{i} x^i (1-x)^{m-i} \tag{1}$$

Where the binomial coefficient is

$$\binom{m}{i} = \frac{m!}{i!\,(m-i)!}$$

See Ojo and Okoro [8] for details.

1.2 Development of the Method

We consider a first order initial value problem of the form

$$y'(x) = f\big(x, y(x)\big), \qquad y(0) = y_0 \tag{2}$$

We seek numerical approximation of the analytical solution y(x) by assuming an approximate solution of the form

$$y(x) = \sum_{k=0}^{c+i-1} a_k B_{k,n}(x) \tag{3}$$

where c and i are number of distinct collocation and interpolation points respectively and $B_{k,n}(x)$ is the Bernstein Polynomial derived from the recursive relation,

$$B_{k,n} = (1-x)B_{k,\ n-1}(x) + xB_{k-1,\ n-1}(x) \tag{4}$$

Differentiating equation (2.2) once and substituting into equation (2) gives:

$$f\big(x, y(x)\big) = \sum_{k=0}^{c+i-1} \quad a_k B'_{k,n}(x) \tag{5}$$

We consider a grid point of step length one (1) Collocating (5) at $x = x_n,\ x = x_{n+1}$,

$x = x_{n+2}$ and interpolating (2.2) at $x = x_n$ give a system of equations below

$$a_0(1-x_n)^3 + 3a_1x_n(1-x_n)^2 + 3a_2{x_n}^2(1-x_n) + a_3{x_n}^3 = y_n$$

$$-3a_0(1-x_n)^3 + 3a_1(1-{x_n}^2)^3 - 6a_1x_n(1-x_n) + 6a_2x_n(1-x_n) - 3a_2{x_n}^2 + 3a_3{x_n}^2 = f_n$$

$$\therefore\ \ -3a_0(1-x_{n+1})^3 + 3a_1(1-{x_{n+1}}^2)^3 - 6a_1x_{n+1}(1-x_{n+1}) + 6a_2x_{n+1}(1-x_{n+1}) - 3a_2{x_{n+1}}^2 + 3a_3{x_{n+1}}^2 = f_{n+1}$$

$$hence\ \ -3a_0(1-x_{n+2})^3 + 3a_1(1-{x_{n+2}}^2)^3 - 6a_1x_{n+2}(1-x_{n+2}) + 6a_2x_{n+2}(1-x_{n+2}) - 3a_2{x_{n+2}}^2 + 3a_3{x_{n+2}}^2 = f_{n+2}$$

The above are solved using Gaussian elimination method to obtained the parameters $a_j's$, $j = 0,1,2,3$. The parameter $a_j's$ is obtained are then substituted back into equation (3) to give a continuous one step method of the form

$$y(x) = \alpha_0 y_n + h[\beta_0 f_n + \beta_1 f_{n+1} + \beta_2 f_{n+2}] \tag{6}$$

where α and β are continuous coefficients. The continuous method (6) is used to generate the main methods. That is, we evaluate at $x = x_{n+1}$ and $x = x_{n+2}$

$$y_{n+1} = y_n + \frac{h}{3}[f_n + 4f_{n+1} + f_{n+2}] \tag{7}$$

$$y_{n+2} = y_n + \frac{h}{12}[5\,f_n + 8f_{n+1} - f_{n+2}] \tag{8}$$

2 Analysis of the Method

Order and Error Constant of the main method (7 & 8)

Writing (7) in the form

$$y_{n+1} - y_n - \frac{h}{3}[f_n + 4f_{n+1} + f_{n+2}] = 0$$

Expanding $y(x_n + jh)$ and $y'(x_n + jh)$, collecting the terms in power in terms of h and y_n gives the order and error constant.

Order $\boldsymbol{p}$ = 4 with Error Constant $C_{p+1} = -\frac{1}{90}$. Thus equation (7) can be writing in the form

$$y_{n+1} = y_n + \frac{h}{3}[f_n + 4f_{n+1} + f_{n+2}] - \frac{h^5}{90}f^{iv}(\epsilon)$$

Writing (8) in the form

$$y_{n+2} - y_n - \frac{h}{12}[5\,f_n + 8f_{n+1} - f_{n+2}] = 0$$

Order $\boldsymbol{p}$ = *3* with Error Constant $C_{p+1} = \frac{1}{24}$, thus equation (8) can be writing in the form

$$y_{n+2} = y_n + \frac{h}{12}[5\, f_n + 8f_{n+1} - f_{n+2}] + \frac{h^4}{24} f^{i''}(\epsilon)$$

Consistency: According to Gurjinder, Kanwar, and Saurabh (2013), A linear multistep method is said to be Consistent, if it has an order of convergence, say $p \geq 1$, thus, our derived methods in equation (7 & 8) are consistent, since they are of order three and four respectively.

3 Numerical Implementation

The effectiveness and validity of our newly derived method was tested on the model of [7]. All plotting of graphs, calculations and programs are carried out with the python software.

Model 1: Ud Din et al. [1].

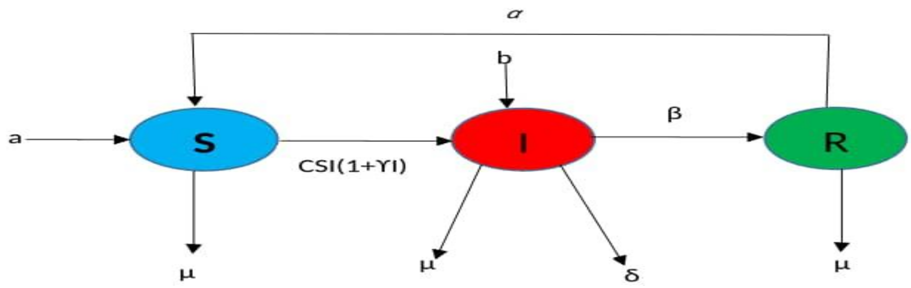

Fig. 1. Showing the COVID-19 model of Ud Din et al. [1]

Table 1. Showing the model Parameters of Ud Din et al. [1]

Parameters		Physical description	Numerical value
S(t)		Susceptible population	12.6 million
I(t)		Infected population	0.084 million
R(t)		Recovered population	0 million
a		The birth rate of infection	0.1243
μ		Natural death	0.002
	δ	Death due to corona	0.05
B		The immigration rate	0.0205
	β	Corona infection recovery rate	0.09871
C		Infection rate	0.580
	γ	Rate at which recovered individuals lose immunity	0.0003
	α	Rate of recovery	0.854302

The model equation are

$$\frac{dS(t)}{dt} = a - CI(t)S(t)(1 + \gamma I(t)) - \mu S(t) + \alpha R(t)$$
$$\frac{dI(t)}{dt} = CI(t)S(t)(1 + \gamma I(t)) - (\beta + \mu + \delta - b)I(t)$$
$$\frac{dR(t)}{dt} = \beta I(t) - (\alpha + \mu)R(t)$$

With initial conditions

$$S(0) = S_0 \geq 0,\ I(0) = I_0 \geq 0,\ R(0) = R_0 \geq 0$$

Using the numerical method derived in equation (7 & 8), the solution of the above model is plotted in the graph below.

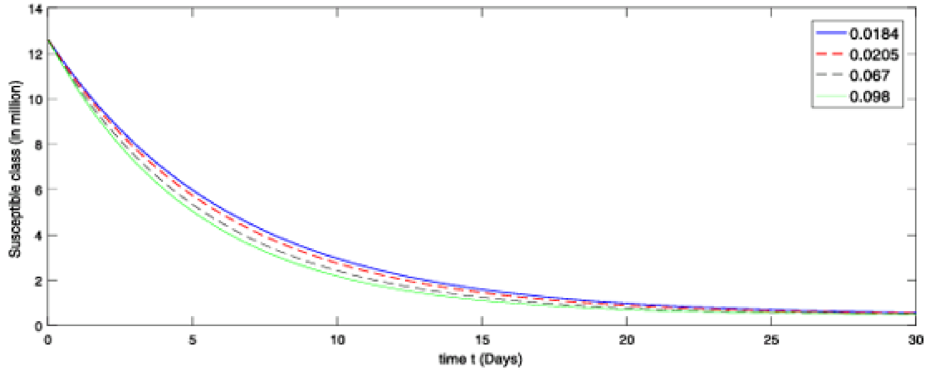

Fig. 2. Showing the graph of *S(t)* against t with varying emigration rate

In Fig. 2 above, it is observed that in the first 30 days, the susceptible population is decreasing with respect to the excessive rate of immigration. Whenever the immigration rate is high, a decline in the population of susceptible people is observed because they are exposed to infection. Hence, the higher the immigration rate, the faster the growth rate of the infected population, and vice-versa.

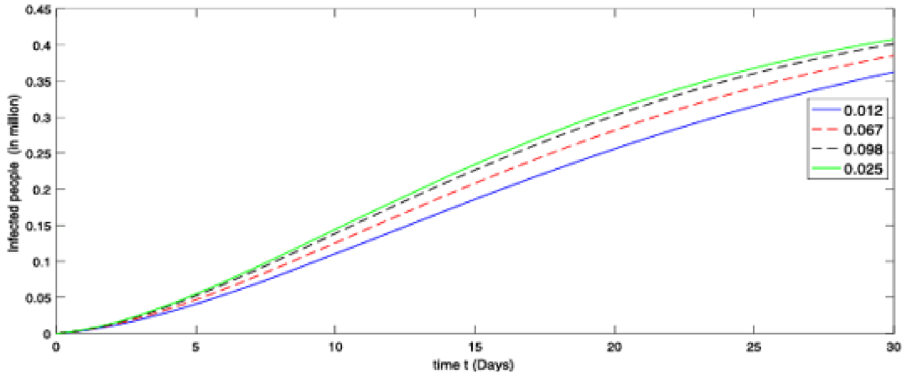

Fig. 3. Showing the graph of I(t) against t with varying infection rate

Fig. 3 shows that the infection rate is increasing in the first days even with a varying infection rate, which signifies that as the susceptible population is exposed to the disease, the number of Infected will increase gradually.

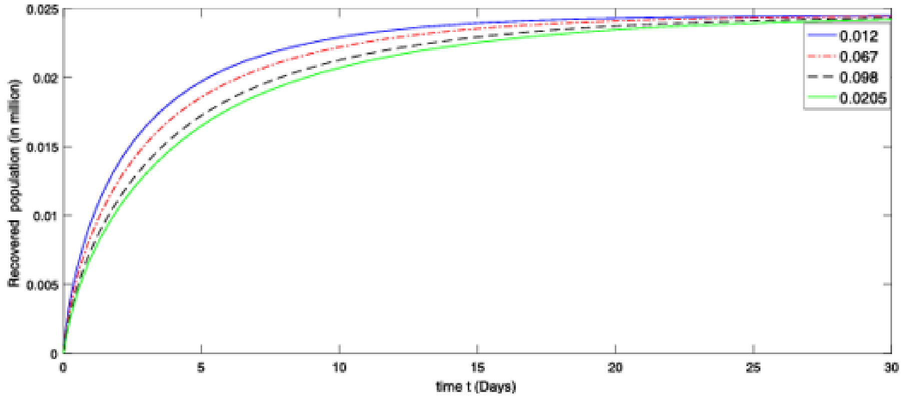

Fig. 4. Showing the graph of *R(t)* against *t* with varying infection rate

Fig. 4 shows that the recovery rate will be increasing gradually with varying infection rates during the first 10 days, after which it will remain constant for the number of days considered. After day 10, varying infection rates have little or no effect on the rate of recovery.

Model 2: The model equation of Zeb et al. [4] is stated below

$$\frac{ds}{dt} = \mu - \mu s + \beta N s(e + i)$$
$$\frac{de}{dt} = \beta N s(e + i) - \pi e - (\mu + \gamma)e$$
$$\frac{di}{dt} = \pi e - \sigma i - \mu i$$
$$\frac{dq}{dt} = \gamma e + \sigma i - \Theta q - \mu q$$

With initial conditions

$$s(0) = s_0 \geq 0,$$
$$e(0) = e_0 \geq 0,$$
$$i(0) = i_0 \geq 0,$$
$$q(0) = q_0 \geq 0$$

Table 2. Showing the model Parameters of Zeb et al. [4]

Parameters	Physical description
s(t)	Susceptible population
e(t)	Exposed population
i(t)	Infected population
B	Rate at which susceptible population moves to infected and exposed class
$\boldsymbol{\pi}$	Rate at which exposed population moves to infected one
$\boldsymbol{\gamma}$	Presents the rate at which exposed
Θ	Rate at which isolated persons recovered
$\boldsymbol{\mu}$	Natural death rate plus disease-related death rate

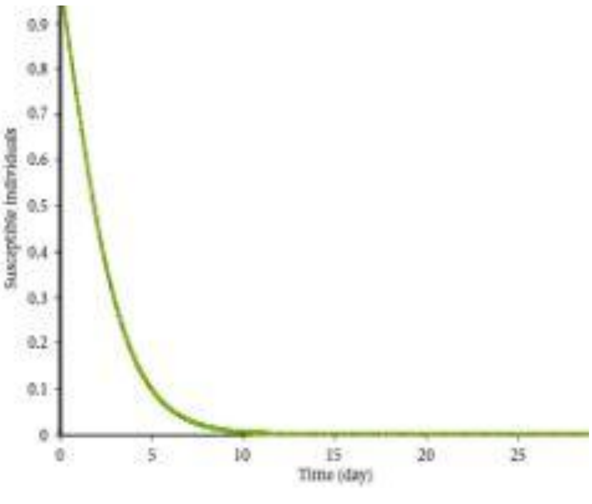

Fig. 5. Showing the graph of susceptible *s(t)* population against time *t*

Fig. 5 above, show that the *e* susceptible population is decreasing with respect to time until day 10, after which it will remain constant.

In Fig. 6, it is shown that the exposed population will be increasing until it begins to decline.

In Fig. 7, the infected population will be increasing with respect. Thus the disease will persist.

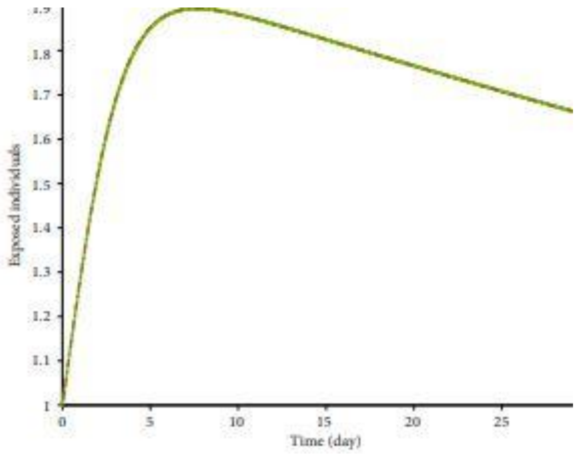

Fig. 6. Showing the graph of exposed population e(t) against time

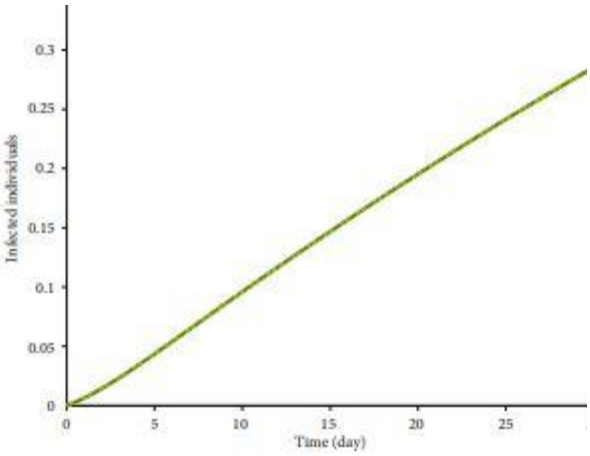

Fig. 7. Showing the graph of exposed population e(t) against time

4 Conclusion

The two-step method is derived via the multistep collocation technique using the Bernstein polynomial as the basis function and implemented in block form. The methods are of order 3 and 4 and consistent. The method is reliable and efficient and can conveniently solve the mathematical models addressing the current novel COVID-19 pandemic.

Competing Interests

Authors have declared that no competing interests exist.

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