

British Journal of Mathematics & Computer Science 4(8): 1027-1034, 2014

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Traveling Wave Solutions for a Coupled KdV Equations

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Authors' contribution

This work was carried out in collaboration between all authors. Author ST designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author YZ managed the analyses of the study. Author YZ managed the literature searches. All authors read and approved the final manuscrip

Original Research Article

Received: 22 June 2013 Accepted: 21 August 2013 Published: 17 February 2014

Abstract

In this paper, we use a method in order to find exact explicit traveling solutions in the subspace of the phase space for coupled KdV equations. The key idea is removing a coupled relation for the given system so that the new systems can be solved. The existence of solitary wave solutions is obtained. It is shown that bifurcation theory of dynamical systems provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Keywords: Solitary wave solution, bifurcation theory, dynamical systems, coupled KdV equations.

1 Introduction

The study on the various physical structures of nonlinear dispersive equations has attracted much attention in connection with the important problems that arise in scientific applications. Mathematically, these physical structures have been studied by using various analytical methods, such as inverse scattering method [1], Darboux transformation method [2,3], Hirota bilinear method [4], Lie group method [5,6], sine-cosine method [7,8], tanh function method [9,10], Fan expansion method [11,12] and so on. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations.

In 1981, Hirota [4] presented the following coupled KdV equations,

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\n
$$
\begin{cases}\n u_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xxx} = 0, \\
 v_t + 3\beta u v_x + \beta v_{xxx} = 0,\n\end{cases}
$$
\n(1.1)

\nch were derived to model the interaction of water waves. Here α, β, b are constants. Liu et al.

\n13] based on a subtle balance method, a given function expansion which was applied to Eq.

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 $6 \alpha u u_x - 2b v v_x + c u_{xxx} = 0,$
 $3 \beta u v_x + \beta v_{xxx} = 0,$ (1.1)

e derived to model the interaction of water waves. Here α , β , b are constants. Liu e British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014

6 $\alpha u u_x - 2bv v_x + \alpha u_{xx} = 0$,
 $3\beta u v_x + \beta v_{xx} = 0$,

(1.1)

re derived to model the interaction of water waves. Here α , β , b are constants. Liu e *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $u_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xxx} = 0,$
 $v_t + 3\beta uv_x + \beta v_{xxx} = 0,$ (1.1)

a were derived to model the interaction of water waves. Here α, β, b are constants. *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $u_t + 6\alpha u u_x - 2b v v_x + \alpha u_{xxx} = 0,$
 $v_t + 3\beta u v_x + \beta v_{xxx} = 0,$ (1.1)

in were derived to model the interaction of water waves. Here α, β, b are constan *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $\left\{ u_t + 6\alpha u u_x - 2bv v_x + \alpha u_{xx} = 0,$
 $\left\{ v_t + 3\beta uv_x + \beta v_{xx} = 0, \right.$

(1.1)

ch were derived to model the interaction of water waves. Here α, β, b a which were derived to model the interaction of water waves. Here α , β , b are constants. Liu et al. in [13] based on a subtle balance method, a given function expansion which was applied to Eq. (1.1), a series of periodic solutions, solitary wave solutions and singular solutions are obtained by aid of symbolic computation. *British Journal of Mathematics & Compater Science 4(8), 1027-1034, 2014*
 $6 \alpha u u_x - 2 b v v_x + \alpha u_{xxx} = 0,$
 $3 \beta h v_x + \beta v_{xxx} = 0,$ (1.1)
 c derived to model the interaction of water waves. Here α, β, b are constants. Liu et a *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 5*Gautt₄* - - $2h\nu_x + \alpha u_{xx} = 0$,
 $3\beta I\nu v_x + f\nu_{xx} = 0$,
 $\alpha u_x = 0$,
 $\alpha u_y = 0$,
 $t_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xxx} = 0,$
 $t + 3\beta u v_x + \beta v_{xxx} = 0,$

were derived to model the interaction of water waves. Here α , β , b are constant

based on a subtle balance method, a given function expansion which was apply
 Example 18 Initial Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014
 $u_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xx} = 0,$
 $v_t + 3\beta uv_x + \beta v_{xx} = 0,$ (1.1)

the were derived to model the interaction of water waves. Here α, β, b a *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $u_t + 6\alpha t u_t$, $-2b v v_x + \alpha u_{xx} = 0$,
 $v_t + 3\beta uv_x + \beta v_{xx} = 0$, (1.1)

were derived to model the interaction of water waves. Here α , β , b are co *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $\left(u_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xxx} = 0,$
 $v_t + 3\beta uv_x + \beta v_{xxx} = 0,$ (1.1)

ch were derived to model the interaction of water waves. Here α, β, b are consta **British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014**
 $\left\{ u_t + 6\alpha u u_s - 2bvv_x + \alpha u_{xx} = 0,$
 $v_t + 3\beta uv_x + \beta v_{xxx} = 0,$ (1.1)

ch were derived to model the interaction of water waves. Here α , β , b are c $u u_{k} - 2b v v_{x} + \alpha u_{xxx} = 0,$ (1.1)
 $u v_{x} + \beta v_{xx} = 0,$ (1.1)

erived to model the interaction of water waves. Here α , β , b are constants. Liu et al.

on a subtle balance method, a given function expansion which was $uv_x + \beta v_{\text{vac}} = 0,$ (1.1)

erived to model the interaction of water waves. Here α , β , b are constants. Liu et al.

on a subtle balance method, a given function expansion which was applied to Eq.

of periodic soluti + 6*Gtutt_s* - 2*bvv_s* + *GtU_{xxx}* = 0,

+ 3*Gtv_s* + *fV<sub>y_{xxx}* = 0,

ever derived to model the interaction of water waves. Here *α*, *β*, *b* are constants. Liu et al.

based on a subtle balance method, a given </sub> +3*βtv₃* + *βv_{zax}* = 0, (1.1)
 *cvere derived to model the interaction of water waves. Here α, β, b are constants. Lia et al.

based on a subtle balance method, a given function expansion which was applied to Eq.

s minor bonuta y and bonuta is a compare science view* $\pi/2$ *, <i>no.*² *x D*₂ *x A x y A x y A x y A x y A x y A* which were derived to model the interaction of water waves. Here α , β , b are constants. Liu et al.

In [13] based on a subtle balance method, a given function expansion which was applied to Eq.

1.1.), a seres of pe

For $b = 3, \alpha = \beta$, Eq. (1.1) becomes to

$$
\begin{cases} u_t + 6\alpha u u_x - 6\nu v_x + \alpha u_{xxx} = 0, \\ v_t + 3\alpha u v_x + \alpha v_{xxx} = 0, \end{cases}
$$
\n(1.2)

Recently, Wang et al. [14] considered the solitary wave solutions of Eq. (1.2), by using a homogeneous balance method.

In this paper, we consider the dynamical bifurcation behavior for the traveling wave solutions of Eq. (1.2). Let $u(x,t) = u(\xi)$, $v(x,t) = v(\xi)$, $\xi = x - ct$, where *c* is the wave speed. Then Eq. (1.2) become to

$$
\begin{cases}\n-cu' + 6\alpha uu' - 6vv' + \alpha u''' = 0, \\
-cv' + 3\alpha uv' + \alpha v''' = 0,\n\end{cases}
$$
\n(1.3)

where "'" is the derivative with respect to ξ . To remove the coupled relationship of Eq. (1.3), let

3) based on a suble balance method, a given function expansion which was applied to Eq.
\n0. a series of periodic solutions, solving wave solutions and singular solutions are obtained by
\n
$$
b = 3
$$
, $\alpha = \beta$, Eq. (1.1) becomes to
\n
$$
\left[u_t + 6\alpha t u u_x - 6vv_x + \alpha u_{xxx} = 0, \qquad (1.2)
$$
\n $v_t + 3\alpha uv_x + \alpha v_{xxx} = 0,$ \n
$$
\left[u_t + 6\alpha t u u_x - 6vv_x + \alpha u_{xxx} = 0, \qquad (1.2)
$$
\n $v_t + 3\alpha uv_x + \alpha v_{xxx} = 0,$ \n
$$
\left[u_t + 6\alpha t u u_x + \alpha v_x - 0, \qquad (1.2)
$$
\n $v_t + 3\alpha t u_x + \alpha v_x = 0, \qquad (1.3)$ \n $v_t = u(\xi), v(x, t) = v(\xi), \xi = x - ct$, where c is the wave speed. Then Eq.
\n(1.3). Let $u_t = u(t, t) = u(t, t) + \alpha u_t = 0$,
\n
$$
\left[-cu' + 6\alpha t u - 6vv' + \alpha u_t = 0, \qquad (1.3)
$$
\n
$$
\left[-cv' + 3\alpha t v'' + \alpha v'' = 0, \qquad (1.3)
$$
\n
$$
\left[-cv' + 3\alpha t v'' + \alpha v'' + 0, \qquad (1.3)
$$
\n
$$
\left[-av + 3\alpha t^2 + (6\alpha a b - bc - 3) v^2 + 3\alpha b^2 v^4 + 2\alpha b \left(v v'' + (v')^2 \right) \right] = g_1,
$$
\n
$$
\left[-cv + \alpha v (3a + bv^2) + \alpha v'' = g_2,
$$
\n
$$
\left[-cv + \alpha v (3a + bv^2) + \alpha v'' = g_2,
$$
\n
$$
\left[-c v + \alpha v (3a + bv^2) + \alpha v'' = g_2,
$$
\n
$$
\left[-c v + \alpha v (3a + bv^2) + \alpha v'' = g_2, \qquad (1.4)
$$
\n
$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{g_1 + ac - 3\alpha a^2 + (bc - 6\alpha
$$

where g_1 and g_2 are integral constants.

Thus, we have two Hamiltonian systems

$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{g_1 + ac - 3\alpha a^2 + (bc - 6\alpha ab + 3)v^2 - 3\alpha b^2 v^4 - 2\alpha by^2}{2\alpha bv},
$$
(1.5)

$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{1}{\alpha} \left(g_2 + (c - 3\alpha a)v - \alpha bv^3 \right)
$$
\n(1.6)

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with the first integral

$$
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$$
\n
$$
t = \text{first integral}
$$
\n
$$
y^2 = \frac{2(g_1 + ac - 3\alpha a^2)v^2 + (bc - 6\alpha ab - 3)v^4 - 2\alpha b^2 v^6 + 4h_1}{4\alpha b v^2},
$$
\n
$$
y^2 = \frac{4g_2v + 2(c - 3\alpha a)v^2 - \alpha bv^4 + 2h_2}{2\alpha},
$$
\n
$$
H_1(v, y) = \alpha bv^2 y^2 - \left[\frac{1}{2}(g_1 + ac - 3\alpha a^2)v^2 + \frac{1}{4}(bc - 6\alpha ab - 3)v^4 - \frac{1}{2}\alpha b^2 v^6\right] = h_1, (1.9)
$$

$$
y^{2} = \frac{4g_{2}v + 2(c - 3\alpha a)v^{2} - abv^{4} + 2h_{2}}{2\alpha},
$$
\n(1.8)

and

$$
British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014
$$
\nthe first integral

\n
$$
y^{2} = \frac{2(g_{1} + ac - 3\alpha a^{2})v^{2} + (bc - 6\alpha ab - 3)v^{4} - 2\alpha b^{2}v^{6} + 4h_{1}}{4\alpha b^{3}},
$$
\n
$$
y^{2} = \frac{4g_{2}v + 2(c - 3\alpha a)v^{2} - \alpha bv^{4} + 2h_{2}}{2\alpha},
$$
\n
$$
H_{1}(v, y) = \alpha bv^{2}y^{2} - \left[\frac{1}{2}(g_{1} + ac - 3\alpha a^{2})v^{2} + \frac{1}{4}(bc - 6\alpha ab - 3)v^{4} - \frac{1}{2}\alpha b^{2}v^{6}\right] = h_{1}, (1.9)
$$
\n
$$
H_{2}(v, y) = \alpha y^{2} - \left[2g_{2}v + (c - 3\alpha a)v^{2} - \frac{\alpha b}{2}v^{4}\right] = h_{2},
$$
\n
$$
h_{1} = h_{2} = g_{2} = 0, a = \frac{c \pm \sqrt{c^{2} + 12\alpha g_{1}}}{6\alpha}, b = \frac{1}{c - 2\alpha a}, c^{2} + 12\alpha g_{1} \ge 0,
$$
\n
$$
u(1.11)
$$
\nthen (1.5) and system (1.6) are equivalent to each other. In this paper, we always assume that (1.2) satisfy (1.11). Therefore, under condition (1.11), we only consider the system (1.6) and we have two Hamiltonians systems

\n
$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{v}{2\alpha} \left(c - \sqrt{c^{2} + 12\alpha g_{1}} - \frac{6\alpha v^{2}}{2c - \sqrt{c^{2} + 12\alpha g_{1}}}\right),
$$
\n
$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{v}{2\alpha} \left(c + \sqrt{c^{2} + 12\alpha g_{1}} - \frac{3v^{4}}{2c + \sqrt{c^{2} + 12\alpha g_{1}}}\right),
$$
\nthe first integral

\n
$$
y^{2} = \frac{1}{2\alpha} \left(c
$$

For

$$
h_1 = h_2 = g_2 = 0, a = \frac{c \pm \sqrt{c^2 + 12\alpha g_1}}{6\alpha}, b = \frac{1}{c - 2\alpha a}, c^2 + 12\alpha g_1 \ge 0,
$$
 (1.11)

system (1.5) and system (1.6) are equivalent to each other. In this paper, we always assume that Eq. (1.2) satisfy (1.11). Therefore, under condition (1.11), we only consider the system (1.6) and then we have two Hamiltonian systems

$$
H_2(v, y) = \alpha y^2 - \left[2g_2 v + (c - 3\alpha a) v^2 - \frac{\alpha b}{2} v^4 \right] = h_2,
$$
\n
$$
h_1 = h_2 = g_2 = 0, a = \frac{c \pm \sqrt{c^2 + 12\alpha g_1}}{6\alpha}, b = \frac{1}{c - 2\alpha a}, c^2 + 12\alpha g_1 \ge 0,
$$
\n
$$
(1.11)
$$
\n
$$
\text{Herm} (1.5) \text{ and system (1.6) are equivalent to each other. In this paper, we always assume that}
$$
\n
$$
H_2(1.2) \text{ satisfy (1.11). Therefore, under condition (1.11), we only consider the system (1.6) and}
$$
\n
$$
u = \text{ have two Hamiltonian systems}
$$
\n
$$
\frac{dv}{d\zeta} = y, \frac{dy}{d\zeta} = \frac{v}{2\alpha} \left(c - \sqrt{c^2 + 12\alpha g_1} - \frac{6\alpha v^2}{2c - \sqrt{c^2 + 12\alpha g_1}} \right),
$$
\n
$$
\frac{dv}{d\zeta} = y, \frac{dy}{d\zeta} = \frac{v}{2\alpha} \left(c + \sqrt{c^2 + 12\alpha g_1} - \frac{6\alpha v^2}{2c + \sqrt{c^2 + 12\alpha g_1}} \right),
$$
\n
$$
u = 1.13
$$
\n
$$
u = \frac{dv}{2\alpha} \left(c - \sqrt{c^2 + 12\alpha g_1} \right) v^2 - \frac{3v^4}{4 \left(2c - \sqrt{c^2 + 12\alpha g_1} \right)},
$$
\n
$$
u = 1.14
$$

$$
\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{v}{2\alpha} \left(c + \sqrt{c^2 + 12\alpha g_1} - \frac{6\alpha v^2}{2c + \sqrt{c^2 + 12\alpha g_1}} \right),\tag{1.13}
$$

with the first integral

$$
y^{2} = \frac{1}{2\alpha} \left(c - \sqrt{c^{2} + 12\alpha g_{1}} \right) v^{2} - \frac{3v^{4}}{4 \left(2c - \sqrt{c^{2} + 12\alpha g_{1}} \right)},
$$
\n(1.14)

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$$
Brtish Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014
$$
\n
$$
y^{2} = \frac{1}{2\alpha} \Big(c + \sqrt{c^{2} + 12\alpha g_{1}} \Big) v^{2} - \frac{3v^{4}}{4 \Big(2c + \sqrt{c^{2} + 12\alpha g_{1}} \Big)},
$$
\n(d)
\n
$$
H_{21}(v, y) = y^{2} - \frac{1}{2\alpha} \Big(c - \sqrt{c^{2} + 12\alpha g_{1}} \Big) v^{2} + \frac{3v^{4}}{4 \Big(2c - \sqrt{c^{2} + 12\alpha g_{1}} \Big)} = 0.
$$
\n
$$
(1.16)
$$
\n
$$
H_{22}(v, y) = y^{2} - \frac{1}{2\alpha} \Big(c + \sqrt{c^{2} + 12\alpha g_{1}} \Big) v^{2} + \frac{3v^{4}}{4 \Big(2c + \sqrt{c^{2} + 12\alpha g_{1}} \Big)} = 0.
$$
\n
$$
(1.17)
$$
\n**Exact Explicit Traveling Solutions of System (1.2)**\n
$$
1. a = \frac{c - \sqrt{c^{2} + 12\alpha g_{1}}}{6\alpha}
$$

and

$$
H_{21}(v, y) = y^2 - \frac{1}{2\alpha} \left(c - \sqrt{c^2 + 12\alpha g_1} \right) v^2 + \frac{3v^4}{4 \left(2c - \sqrt{c^2 + 12\alpha g_1} \right)} = 0. \tag{1.16}
$$

$$
H_{22}(v, y) = y^2 - \frac{1}{2\alpha} \left(c + \sqrt{c^2 + 12\alpha g_1} \right) v^2 + \frac{3v^4}{4 \left(2c + \sqrt{c^2 + 12\alpha g_1} \right)} = 0. \tag{1.17}
$$

2 Exact Explicit Traveling Solutions of System (1.2)

$$
2.1 \t a = \frac{c - \sqrt{c^2 + 12\alpha g_1}}{6\alpha}
$$

Suppose that $g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$. Then, the origin of *latitish Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $\frac{1}{2\alpha} (c + \sqrt{c^2 + 12\alpha g_1}) v^2 - \frac{3v^4}{4(2c + \sqrt{c^2 + 12\alpha g_1})}$, (1.15)
 $v, y) = y^2 - \frac{1}{2\alpha} (c - \sqrt{c^2 + 12\alpha g_1}) v^2 + \frac{3v^4}{4(2c - \sqrt{c^2 + 12\alpha g_1})$ *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $\sqrt{(c+\sqrt{c^2+12\alpha g_1})v^2-\frac{3v^4}{4(2c+\sqrt{c^2+12\alpha g_1})}},$ (1.15)
 $y) = y^2 - \frac{1}{2\alpha}(c-\sqrt{c^2+12\alpha g_1})v^2 + \frac{3v^4}{4(2c-\sqrt{c^2+12\alpha g_1})} = 0.$ (1.16)
 $y) = y^$ *British Journal of Mathematics & Computer Science 4(8), 1027-1034, 2014*
 $y^2 = \frac{1}{2\alpha} \Big(c + \sqrt{c^2 + 12\alpha g_1} \Big) y^2 - \frac{3y^4}{4 \Big(2c + \sqrt{c^2 + 12\alpha g_1} \Big)},$ (1.15)

and
 H₂₁(v, y) = $y^2 - \frac{1}{2\alpha} \Big(c - \sqrt{c^2 + 12\alpha g_1} \Big)$ $y^2 = \frac{1}{2\alpha} \Big(c + \sqrt{c^2 + 12\alpha g_1} \Big) v^2 - \frac{3v^4}{4 \Big(2c + \sqrt{c^2 + 12\alpha g_1} \Big)},$ (1.15)

and

and
 $H_{21}(y, y) = y^2 - \frac{1}{2\alpha} \Big(c - \sqrt{c^2 + 12\alpha g_1} \Big) v^2 + \frac{3v^4}{4 \Big(2c - \sqrt{c^2 + 12\alpha g_1} \Big)} = 0.$ (1.16)
 $H_{22}(y, y) = y^2 - \frac{1}{2\alpha$ homoclinic orbits of Eq. (1.12) have the following parametric representation (Fig. 1) it Traveling Solutions of System (1.2)
 $\frac{2\alpha g_1}{\alpha > 0, c > 2\sqrt{-3\alpha g_1}}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$. Then, the origin of

dle point. Corresponding to the level curves defined by $H_{21}(v, y) = 0$; two

dq. (1.12) have th eling Solutions of System (1.2)
 $> 2\sqrt{-3\alpha g_1}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$. Then, the origin of

Corresponding to the level curves defined by $H_{21}(v, y) = 0$; two

nave the following parametric representation (Fig. 1) $\sqrt{c^2 + 12\alpha g_1}$ $y^2 + \frac{3y^4}{4(2c - \sqrt{c^2 + 12\alpha g_1})} = 0.$ (1.16)
 $\sqrt{c^2 + 12\alpha g_1}$ $y^2 + \frac{3y^4}{4(2c + \sqrt{c^2 + 12\alpha g_1})} = 0.$ (1.17)
 Fraveling Solutions of System (1.2)
 $\frac{g_1}{g_2}$
 $\frac{g_2}{g_1}$
 $\frac{g_2}{g_2}$
 $\frac{3v^4}{2c - \sqrt{c^2 + 12\alpha g_1}} = 0.$ (1.16)
 $\frac{3v^4}{2c + \sqrt{c^2 + 12\alpha g_1}} = 0.$ (1.17)
 System (1.2)
 System (1.2)
 $\iota < 0, c > 2\sqrt{\alpha g_1}$. Then, the origin of curves defined by $H_{21}(v, y) = 0$; two

tric representation (Fi $\sqrt{c^2 + 12\alpha g_1}$) $v^2 + \frac{3v^4}{4(2c - \sqrt{c^2 + 12\alpha g_1})} = 0.$ (1.16)
 $\sqrt{c^2 + 12\alpha g_1}$) $v^2 + \frac{3v^4}{4(2c + \sqrt{c^2 + 12\alpha g_1})} = 0.$ (1.17)
 ling Solutions of System (1.2)
 $2\sqrt{-3\alpha g_1}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$. Then, $y^2 - \frac{1}{2\alpha} \left(c - \sqrt{c^2 + 12\alpha g_1} \right) y^2 + \frac{3y^4}{4 \left(2c - \sqrt{c^2 + 12\alpha g_1} \right)} = 0.$ (1.16)
 $y = y^2 - \frac{1}{2\alpha} \left(c + \sqrt{c^2 + 12\alpha g_1} \right) y^2 + \frac{3y^4}{4 \left(2c + \sqrt{c^2 + 12\alpha g_1} \right)} = 0.$ (1.17)
 xplicit Traveling Solutions of Syste $I_{21}(v, y) = y^2 - \frac{1}{2\alpha} \Big(c - \sqrt{c^2 + 12\alpha g_1} \Big) v^2 + \frac{3v^4}{4 \Big(2c - \sqrt{c^2 + 12\alpha g_1} \Big)} = 0.$ (1.16)
 $I_{22}(v, y) = y^2 - \frac{1}{2\alpha} \Big(c + \sqrt{c^2 + 12\alpha g_1} \Big) v^2 + \frac{3v^4}{4 \Big(2c + \sqrt{c^2 + 12\alpha g_1} \Big)} = 0.$ (1.17)
 xact Explicit Trav 4 $(2c+\sqrt{c^2+12\alpha g_1})$
 $\frac{3y^4}{4(2c-\sqrt{c^2+12\alpha g_1})} = 0.$ (1.16)
 $\frac{y^2-\frac{1}{2\alpha}(c+\sqrt{c^2+12\alpha g_1})y^2+\frac{3y^4}{4(2c+\sqrt{c^2+12\alpha g_1})} = 0.$ (1.17)
 plicit Traveling Solutions of System (1.2)
 plicit Traveling Solutions of $(0, y) = y^2 - \frac{1}{2\alpha} \left(c - \sqrt{c^2 + 12\alpha g_1} \right) y^2 + \frac{3y^4}{4 \left(2c - \sqrt{c^2 + 12\alpha g_1} \right)} = 0.$ (1.16)
 $2 \left(y, y \right) = y^2 - \frac{1}{2\alpha} \left(c + \sqrt{c^2 + 12\alpha g_1} \right) y^2 + \frac{3y^4}{4 \left(2c + \sqrt{c^2 + 12\alpha g_1} \right)} = 0.$ (1.17)
 act Explicit Travelin

$$
v = \pm \sqrt{\frac{\left(c - \sqrt{c^2 + 12\alpha g_1}\right)\left(2c - \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h \sqrt{\frac{c - \sqrt{c^2 + 12\alpha g_1}}{2\alpha}} \xi.
$$
 (2.1)

Thus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as follows:

$$
H_{22}(v, y) = y^2 - \frac{1}{2\alpha} \Big(c + \sqrt{c^2 + 12\alpha g_1} \Big) v^2 + \frac{3v^4}{4 \Big(2c + \sqrt{c^2 + 12\alpha g_1} \Big)} = 0.
$$
 (1.17)
2 Exact Explicit Travelling Solutions of System (1.2)
2.1 $a = \frac{c - \sqrt{c^2 + 12\alpha g_1}}{6\alpha}$
Suppose that $g_1 < 0$, $\alpha > 0$, $c > 2\sqrt{-3\alpha g_1}$ or $g_1 < 0$, $\alpha < 0$, $c > 2\sqrt{\alpha g_1}$. Then, the origin of
system (1.12) is a saddle point. Corresponding to the level curves defined by $H_{21}(v, y) = 0$, two
nomoclinic orbits of Eq. (1.12) have the following parametric representation (Fig. 1)
 $v = \pm \sqrt{\frac{\Big(c - \sqrt{c^2 + 12\alpha g_1} \Big) \Big(2c - \sqrt{c^2 + 12\alpha g_1} \Big)}}{3\alpha} \Bigg| 3c \cos h \sqrt{\frac{c - \sqrt{c^2 + 12\alpha g_1}}{2\alpha} \Bigg| 5c}.$ (2.1)
Thus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as
follows:

$$
\int u(x,t) = \frac{c - \sqrt{c^2 + 12\alpha g_1}}{6\alpha} + \left(\frac{6\alpha}{2c - \sqrt{c^2 + 12\alpha g_1}} \right) v^2,
$$
 (2.2)

$$
v(x,t) = \pm \sqrt{\frac{\Big(c - \sqrt{c^2 + 12\alpha g_1} \Big) \Big(2c - \sqrt{c^2 + 12\alpha g_1} \Big)}{3\alpha}} \sec h \sqrt{\frac{c - \sqrt{c^2 + 12\alpha g_1}}{2\alpha}} (x - ct).
$$

$$
g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1} \text{ or } g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}
$$

Fig. 1. The phase portrais of (2.12) for $H_{21}(v, v) = 0$.

$$
2.2 \t a = \frac{c + \sqrt{c^2 + 12\alpha g_1}}{6\alpha}
$$

Suppose that $g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ or $g_1 > 0, \alpha > 0, c > 0$. Then, the origin of system homoclinic orbits of Eq. (1.13) have the following parametric representation (Fig. 2) 2 2 ² $c > 2\sqrt{-3\alpha g_1}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$.

phase portraits of (2.12) for $H_{21}(v, y) = 0$.
 $> 2\sqrt{-3\alpha g_1}$ or $g_1 > 0, \alpha > 0, c > 0$. Then, the origin of system

esponding to the level curves defined by $H_{22}(v, y) = 0$; $\left\{\n\begin{array}{l}\n\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}\n\right\}$
 $\left\{\n\begin{array}{l}\n0, \alpha < 0, c > 2\sqrt{\alpha g_1} \\
\bullet \\
\end{array}\n\right\}$.
 $\left\{\n\begin{array}{l}\n\bullet \\
\bullet \\
\end{array}\n\right\} = 0.$
 $\left\{\n\begin{array}{l}\n\bullet \\
\bullet \\
\end{array}\n\right\} = 0.$ Then, the origin of system

surves defined by $H_{22}(v$

$$
v = \pm \sqrt{\frac{\left(c + \sqrt{c^2 + 12\alpha g_1}\right)\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h \sqrt{\frac{c + \sqrt{c^2 + 12\alpha g_1}}{2\alpha}} \xi.
$$
 (2.3)

Thus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as follows:

$$
g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1} \text{ or } g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}.
$$

\nFig. 1. The phase portrait of (2.12) for $H_{21}(v, y) = 0$.
\n
$$
a = \frac{c + \sqrt{c^2 + 12\alpha g_1}}{6\alpha}
$$

\n
$$
ppose that g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1} \text{ or } g_1 > 0, \alpha > 0, c > 0
$$
. Then, the origin of system
\n13) is a saddle point. Corresponding to the level curves defined by $H_{22}(v, y) = 0$; two
\nmodinic orbits of Eq. (1.13) have the following parametric representation (Fig. 2)
\n
$$
v = \pm \sqrt{\frac{\left(c + \sqrt{c^2 + 12\alpha g_1}\right)\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h \sqrt{\frac{c + \sqrt{c^2 + 12\alpha g_1}}{2\alpha}} g.
$$
 (2.3)
\nus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as
\n
$$
\begin{cases}\nu(x,t) = \frac{c + \sqrt{c^2 + 12\alpha g_1}}{6\alpha} + \left(\frac{6\alpha}{2c + \sqrt{c^2 + 12\alpha g_1}}\right)v^2, \\
v(x,t) = \pm \sqrt{\frac{\left(c + \sqrt{c^2 + 12\alpha g_1}\right)\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h \sqrt{\frac{c + \sqrt{c^2 + 12\alpha g_1}}{2\alpha}} (x - ct).\n\end{cases}
$$
 (2.4)

Remark. Obviously, solutions (2.2) and (2.4) are different from solutions (3.22) and (3.23) of [13], respectively.

3 Conclusion

In this paper, we have considered traveling wave solutions for the coupled KdV equations of Eq. (1.2) in its subspace of parameter space by using the method of dynamical systems. We obtain parametric representations for the solitary wave solutions of Eq. (1.1) in different parameter regions of the parameter space.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Nos. 11361017, 11061010 and 11161013) and the Natural Science Foundation of Guangxi (Nos. 2012GXNSFAA053003 and 2013GXNSFAA019010). The authors thank the referees and the editors for their valuable comments and suggestions on the improvement of this paper.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Ablowitz MJ, Clarkson PA.Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, London; 1991.

- [2] Matveev VB, Salle MA. Darboux Transformation and Solitons, Springer-Verlag, Berlin; 1991.
- [3] Gu CH, Hu HS, Zhou ZX. Darboux Transformations in Soliton Theory and its Geometric Applications, Shanghai Sci. Tech. Publ., Shanghai; 1999.
- [4] Hirota R, Satsuma J. Soliton solutions of a coupled KdV equation, Phys. Lett. A,. 1981;85:407-408.
- [5] Olver PJ. Applications of Lie Groups to Differential Equations, Springer-Verlag, New York; 1993.
- [6] Bluman GW, Kumei S. Symmetries and Differential Equations, Springer-Verlag, Berlin; 1989.
- [7] Tian L, Yin J. New compacton solutions and solitary wave solutions of fully nonlinear generalized Camassa-Holm equations, Chaos Soliton. Fract. 2004;20:289-299.
- [8] Wazwaz AM. Analytic study on nonlinear variant of the RLW and the PHI-four equations, Commun. Nonlinear Sci. Numer. Simul. 2007;12:314-327.
- [9] Wazwaz AM. A reliable treatment of the physical structure for the nonlinear equation K(m,n), Appl. Math.Comput. 2005;163:1081-1095.
- [10] Senthamarai R, Rajendran L. Traveling wave solution of non-linear coupled reaction diffusion equation arising in mathematical chemistry, J. Math. Chem. 2009;46:550-561.
- [11] Fan EG. Uniformly constructing a series of explicit exact solutions to nonlinear equations in mathematical physics, Chaos. Soliton. Fract. 2003;16:819-839.
- [12] Feng D. Luo G. The improved Fan sub-equation method and its application to the SK equation, Appl. Math. Comput. 2009;215:1949-1967.
- [13] Liu J. Yang L, Yang K. Jacobi elliptic function solutions of some nonlinear PDEs, Phys. Lett. A,. 2004;325:268-275.

[14] Wang ML, Zhou Y, Li Z. Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Phys. Lett. 1996;A(216):67-75.

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