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Hall Current Effects on Unsteady Mhd Fluid Flow with Radiative Heatflux and Heat Source over a Porous Medium

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Abstract

In this paper the study of unsteady hydromagnectic free flow of viscoelastic fluid (Walter's B) past an infinite vertical plate through porous medium was conducted. The temperature is assumed to be oscillating with time, also the effects of hall-current is taken in to account. The solution of velocity, temperature and concentration profiles have been obtained. The effects of various parameters on temperature, concentration primary and secondary velocity profiles were presented graphically.

Keywords: Hall current, Mhd fluid, radiative heatflux and porous medium.

1 Introduction

The effects of hall current in hydromagnetic fluid have attracted the attention of large number of scholars, particularly in the field of fluid dynamics. This may be as a result of wider range of its applications in this era of modern science, technology and vast industrialisation. The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal

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energy extraction, electromagnetic propulsion and the boundary layer control in the field of aerodynamics. Magneto hydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter [1].

In addition to this, the relevance of fluid flow in the health sector can never be overemphasised. As we all know, the cardiovascular system is sensitive to changes in the environment, and flow characteristics of blood are modified to satisfy changing demands of the organisms. In addition to transporting of oxygen, metabolites, and other dissolved substances to and from the tissues, blood flow alters heat transfer within the body [2].

Hydro magnetic flow and heat transfer between two horizontal, the lower plate being a stretching sheet had been studied by [3]. [4] made similarity analysis in magneto hydrodynamics, Hall effects on forced convective heat and mass transfer of non Newtonian power law fluids past semi infinite vertical flat plate. Heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with hall effects was examined by [5]. [6] investigated the effects of chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past semi infinite vertical moving plate embedded in a porous medium. [7] analysed the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. [8], described the effects of heat transfer to MHD oscillatory flow in a channel filled with porous medium. [9] investigated the thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity. Heat and mass transfer effects on unsteady MHD free convection flow near moving vertical plate in porous medium had been studied by [10]. The demonstration of effects of unsteady two dimensional hydro magnetic flow and heat transfer of a fluid [2]. This shows that an external magnetic field has the same effects on the flow as viscoelasticity. [7] analysed the radiation and mass transfer effects on MHD free convectional flow past an exponentially accelerated vertical plate with variable temperature. Kumar and Chand [11] illustrated the effects of slip condition and hall current on unsteady MHD flow of a visco-elastic fluid past an infinite porous vertical plate through porous medium. [11] further illustrated hall effects on heat and mass transfer in the flow of oscillating viscoelastic fluid through porous medium with slip condition.

This paper aimed to extend the study of heat and mass transfer for an electrically conducting incompressible fluid past a continuously moving plate to include hall current, radiative flux, heat source, mass flux and viscoelasticity through a porous medium in the presence of magnetic field.

2 Problem Formulation

We consider the unsteady flow of a viscous incompressible and electrically conducting viscoelastic fluid with oscillating temperature. The flow occurs over an infinite vertical porous plate. The x^* axis is assumed to be oriented vertically upward along the plate and y^*axis taken normal to the plane of the plate. It is assumed that the plate is electrically none conducting and a uniform magnetic field of strength B_0 is applied normal to the plate. The induced magnetic field is assumed to be negligible so that $\vec{B}\left(0, B_{0,} 0\right)$. The plate is subjected to a constant suction velocity V_0.

Since the plate is infinite in extend all physical quantities are functions of y^* and t^* only. Thus the governing equations of the flow under the usual Boussinesq approximation are:

The Momentum Equations:

$$\frac{\partial u'}{\partial t} - \upsilon_0 \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y^2} - \upsilon \frac{u'}{K_l} - g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma\beta_0^2}{\rho(1+m^2)}(u + mw) - K_0\left\{\frac{\partial^3 u'}{\partial t \partial y'^2} - \upsilon_0 \frac{\partial^3 u'}{\partial y'^2}\right\} \tag{1}$$

$$\frac{\partial w'}{\partial t'} - \upsilon_0 \frac{\partial w'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \upsilon \frac{w'}{K_l} + \frac{\sigma\beta_0^2}{\rho(1+m^2)}(mu - w) - K_0\left\{\frac{\partial^3 w'}{\partial t \partial y'^2} - \upsilon_0 \frac{\partial^3 w'}{\partial y'^3}\right\} \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} - \upsilon_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho cp}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho cp}(T' - T'_\infty) - \frac{1}{\rho cp}\frac{\partial q_r}{\partial y} \tag{3}$$

$$\sigma^* k^*$$

Concentration Equation:

$$\frac{\partial C'}{\partial t'} - \upsilon_0 \frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K_c(C' - C'_\infty) \tag{4}$$

The initial boundary conditions are :

$$\begin{array}{lllll} u' = 0, & w' = 0, & \theta = 1 + \varepsilon e^{i\Omega t} & C' = 1 + \varepsilon e^{\Omega t i}, & y=0 \\ u' \to 0, & w' \to 0, & T' \to T'_\infty, & C' \to C'_\infty & y \to \infty \end{array} \tag{5}$$

Introducing the following non dimensional quantities

$$\begin{array}{lllll} y = \frac{\upsilon_0 y'}{\upsilon} & u' = Uu_0 & u'_1 = \frac{U}{e}u_1, & w'_0 = Uw_0, & w'_1 = \frac{U}{e}w_1, \\ y = \frac{\upsilon_0 y'}{\upsilon} & \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty} & C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty} & M = \frac{\sigma\beta_0^2\upsilon}{\rho\upsilon_0^2} & \\ Gr = \frac{\upsilon\beta(T'_\omega - T'_\infty)}{U\upsilon_0^2} & S = \frac{\upsilon^2 Q_0}{\upsilon_0^2} & Gc = \frac{\upsilon\beta^*\upsilon(C'_\omega - C'_\infty)}{U\upsilon_0^2} & Km = \frac{K_0\upsilon_0^2}{\upsilon^2} & Sc = \frac{\upsilon}{D} \\ \Pr = \frac{\mu Cp}{k} & K = \frac{K_l \upsilon}{\upsilon_0^2} & & & \end{array} \tag{6}$$

Where, β volumetric coefficient of the thermal expansion, v the kinematic viscosity, ρ is density, μ the coefficient of viscosity, β^* is volumetric coefficient of expansion with concentration, u_0 the velocity of the plate, y the coordinate axis normal to the plate, g acceleration due to gravity, q_r the radiation heat flux in the y direction, Cp is specific heat at constant pressure, C' is specific concentration in the fluid, C'_∞ the concentration in the fluid far away from the plate, C'_W the concentration on the plate, D the mass diffusion coefficient, t' is time, T the temperature of the fluid near the plate, T_W the temperature of the plate, T_∞ the temperature of the fluid far away from the plate. M the Hartmann number, Km viscoelastic parameter, Ω, is the frequency of oscillation.

Let the assumed solutions be,

$$\begin{aligned} u' &= u'_0 + \varepsilon u'_1 e^{i\Omega t} \\ \theta' &= \theta'_0 + \varepsilon \theta'_1 e^{\Omega t i} \\ w' &= w'_0 + \varepsilon w'_1 e^{\Omega t i} \\ C' &= C'_0 + \varepsilon C'_1 e^{\Omega t i} \end{aligned} \tag{7}$$

Substituting equation (7) in (1) above

Equations (1) to (4) using (6) and (7) reduced to

$$\frac{Km\,\partial^3 u_0}{\partial y^3} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - L\,u_0 - Jw_0 + Gr\theta_0 + GcC_0 = 0 \tag{8}$$

$$Km\frac{d^3 w_0}{dy^3} + \frac{d^2 w_0}{dy^2} + \frac{dw_0}{dy} + Lw_0 - Ju_0 = 0 \tag{9}$$

$$\Rightarrow Km\,\frac{d^3 u_1}{dy^3} + (1 - i\,\Omega Km)\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - Lnu_1 + Jw_1 = -Gr\theta_1' - GcC'_1 \tag{10}$$

$$\Rightarrow Km\frac{d^3 w_1}{dy^3} + (1 - i\Omega Km)\frac{d^2 w_1}{dy^2} + \frac{dw_1}{dy} - (Ln)w_1 + Ju_1 = 0 \tag{11}$$

$$Z\frac{\partial^2 \theta_0^1}{\partial y^2} + \Pr\frac{\partial \theta'_0}{\partial y} - S\theta'_0 = 0 \tag{12}$$

$$Z\frac{\partial^2 \theta'_1}{\partial y^2} + \Pr\frac{\partial \theta'_1}{\partial y} - q\theta'_1 = 0 \tag{13}$$

$$\frac{1}{SC}\frac{\partial^2 C'_0}{\partial y^2} + \frac{\partial C'_0}{\partial y} - KC'_0 = 0 \tag{14}$$

$$\frac{\partial^2 C'_1}{\partial y^2} + SC\frac{\partial C'_1}{\partial y} - SC(K + i\Omega)C'_1 = 0 \tag{15}$$

The corresponding boundary conditions are:

$$U'_0 = U'_1 = 0\ ,\ \ w'_0 = w'_1 = 0 \quad ,\theta'_0 = \theta'_1 = 1\ ,\quad C'_0 = C'_1 = 0 \qquad at\ y = 0$$
$$U'_1 \to 0\ ,\qquad w'_0 \to 0\ ,\qquad U'_0 \to 0 \qquad w'_1 \to 0 \qquad \theta'_0 \to 0\ ,\quad \theta'_1 \to 0\ ,$$
$$C'_0 \to 0\ ,\ \ C'_1 \to 0 \qquad\qquad as\ \ y \to \infty \tag{16}$$

3 Method of Solution

Introducing $F = (u_0 + iw_0),$ and $i = \sqrt{-1}$ also $H = (u_1 + iw_1)$ equation (8) to (11) transformed to

$$Km\frac{d^3F}{dy^3} + \frac{d^2F}{dy^2} + \frac{dF}{dy} - FL - FJ = -Gr\theta_0 - GcC_0 \tag{17}$$

$$Km\frac{d^3H}{dy^3} + E\frac{d^2H}{dy^2} + \frac{dH}{dy} - NnH = -Gr\theta_1 - GcC_1 \tag{18}$$

Equations (17) and (18) are third order differential equations due to the presence of viscoelasticity. Therefore, F_0 and H_0 are expanded using [12] in terms of K_m

$F_0 = F_{00} + KmF_{01}$ and $H_1 = H_{11} + KmH_{12}$

Zeroth –order

$$F_{00}''' + F_{01}'' + F_{01}' - NF_{01} = 0 \tag{19}$$

$$F_{01}'' + F_{01}' - NF_{01} = -\ F_{00}''' \tag{20}$$

$$H_{11}''' + EH_{12}'' + H_{11}' - NnH_{12} = 0 \tag{21}$$

$$EH_{12}'' + H_{12}' - NnH_{12} = -H_{11}''' \tag{22}$$

The corresponding boundary condition transformed to

$$Y = 0 : F_{00} = F_{01} = 0$$
$$Y \to \infty : F_{00} = F_{01} \to 0$$

Similarly,

$$Y = 0: \qquad H_{00} = H_{01} = 0$$
$$Y \to \infty : \quad H_{00} = H_{01} \to 0 \tag{23}$$

Solving equations (19) to (22) under the boundary condition (23) to obtained

The primary velocity is given by,

$$U=(A_{10}+KmA_{15})e^{-n_5y}-(A_{11}+KmA_{16})e^{-n_5y}-$$
$$(A_{12}+KmA_{17})e^{-n_3y}+KmA_{14}e^{-n_6y}+$$
$$\varepsilon e^{\Omega it}\begin{bmatrix}(B_2+KmB_7)e^{-f1y}-(B_3+KmB_8)e^{-n2y}\\ -(B_4+KmB_9)e^{-n4y}+KmB_6e^{-f2y}\end{bmatrix}$$

The secondary velocity is also given by

$$W=(A_{11}+KmA_{16})e^{-n_1y}+(A_{12}+KmA_{17})e^{-n_3y}-$$
$$KmA_{14}e^{-n_6y}-(A_{10}+KmA_{15})e^{-n_5y}+$$
$$i\,\varepsilon e^{\Omega it}\begin{bmatrix}(B_3+KmB_8)e^{-n2y}+(B_4+KmB_9)e^{-n4y}\\ -KmB_6e^{-f2y}-(B_2+KmB_7)e^{-f1y}\end{bmatrix}$$

The temperature expression

$$\theta'(y)=e^{-n_1y}+\varepsilon e^{(i\pi t-n_2y)}$$

The concentration expression

$$\therefore C'(y)=e^{-n_3y}+\varepsilon e^{(i\pi t-n_4y)}$$

4. Results and Discussion

In order to illustrate the influence of various parameters which include Grashof number Gr, mass Grashof number Gc, magnetic number M, chemical reaction parameter R, Schmits number Sc, Prandtl number Pr, radiation parameter R, heat source s, hall current parameter m and viscoelastic parameter Km on velocity, temperature and concentration profiles. Computations were carried out using Gr=2, Gc=2, Pr=0.71, Sc=0.6, R=0.4, M=5, s=0.2, m=0.2Km=0.05 various values based on physical quantities are computed and presented in Figs. 1-16. The appendix in the paper contained the letters represented by equations to ease the computations.

5. Conclusions

The hall current effects on unsteady MHD fluid flow with radiative heat flux and heat source over a porous medium is investigated by transforming the governing partial differential equations into ordinary differential equations which are then solved using perturbation techniques. The result of the flow variables indicates that the fluid temperature is reduced by increasing Prandtl number (Pr) and radiation parameter (R). Concentration is reduced with increase in Schmidt number (Sc) and chemical reaction parameter (K). The primary velocity decrease with increasing prandtl (Pr), radiation parameter and hall-current while the opposite trend is observed in secondary velocity.

The primary velocity increases with increase in mass Grashoof number (Gr) and thermal Grasshoof number (Gc) also the reverse is the case in secondary velocity. The primary velocity decreases with increase in M, s and Sc.

5.1 Temperature Profiles

Fig.1 illustrates the effects of Prandt number on temperature field. It is noticed that as the Prandt number increases the temperature decreases. The effects of Radiation parameter have been illustrated in Fig. 2 Increase in radiation parameter R is accompanied with increase in temperature profile.

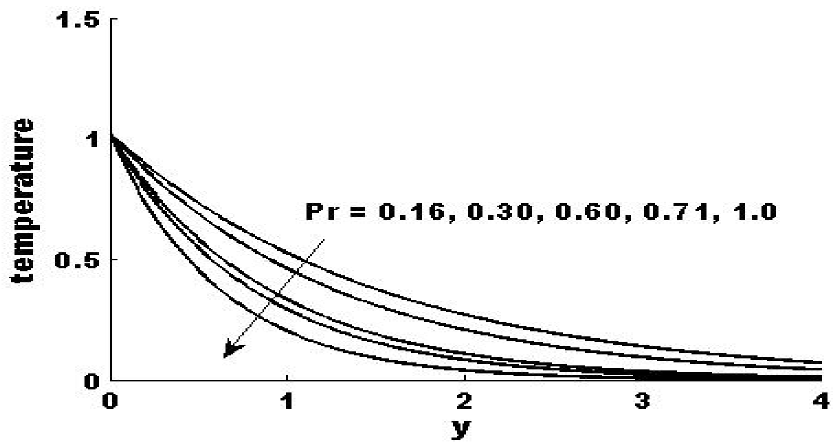

Fig. 1. Effects of Pr on temperature profile

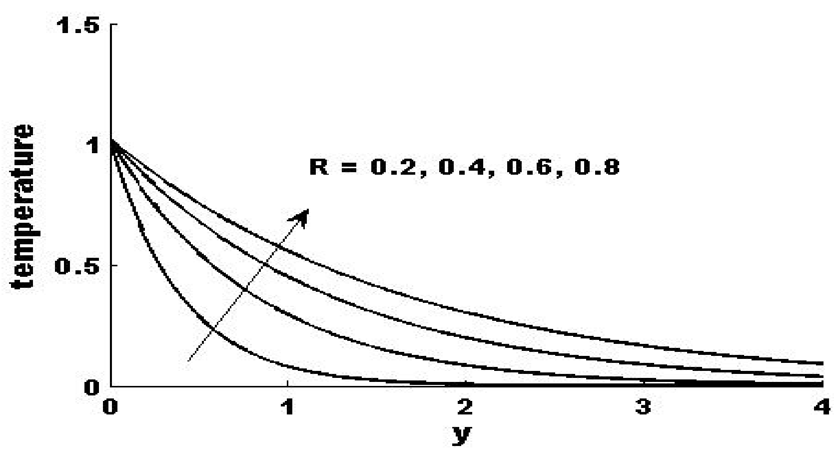

Fig. 2. Effects of R radiation parameter on temperature

5.2 Concentration Profiles

Fig. 3 Shows the effects of Schmidt number on concentration profile with fixed values of t=0.2, K= 0.22 . The graph shows that, increase in the values of Schmidt number Sc results in increase in the concentration profile. Fig. 4 demonstrates the effects of chemical reaction parameter K, on concentration field. It also exhibits the same behaviour as Schmidt number.

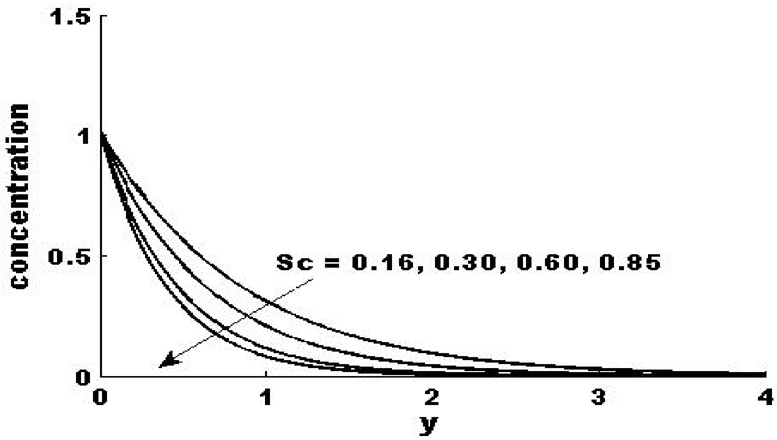

Fig. 3. Effects of Sc on concentration profile

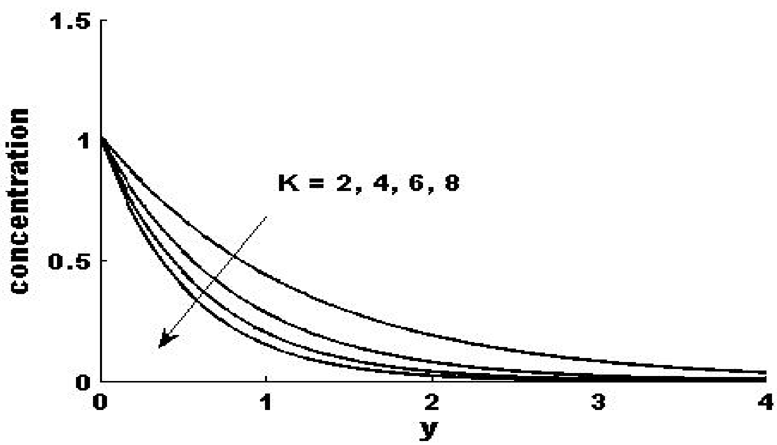

Fig. 4. Effects of K on concentration profile

5.3 Velocity Profiles

Fig. 5 and Fig. 6, demonstrate the effects of primary velocity on Gr and Gc, while Fig. 7 and Fig. 8 show the effects of secondary velocity on Gr and Gc, in the former two cases the velocity decrease with increase in the values of both Gr and Gc. In the later two cases the velocity is directly proportional to the values of both Gr and Gc.

Fig. 9 and 10 described the effects of Hartmann number (M), on primary and secondary velocity respectively. Increase in the value of M increases the primary velocity profile. Likewise decrease in M results increase in profile of W. Fig. 11 and 12 illustrate the effects of heat source (s) on both primary and secondary velocity. Increase in heat source increases the primary velocity, while decrease in s accompanied with increases secondary velocity. Fig. 13 and 14 demonstrates the effects of Km on both primary and secondary velocities. Km exhibit the same behaviour in both primary and secondary velocity as Gr and Gc. Increase in the values of Km accompanies with decrease in primary velocity and increase secondary velocity. Fig. 15 and 16 illustrated the effects of hall current on both primary and secondary velocity. Increase in hall current parameter results to decrease in primary velocity and vice versa in secondary velocity as it is expected.

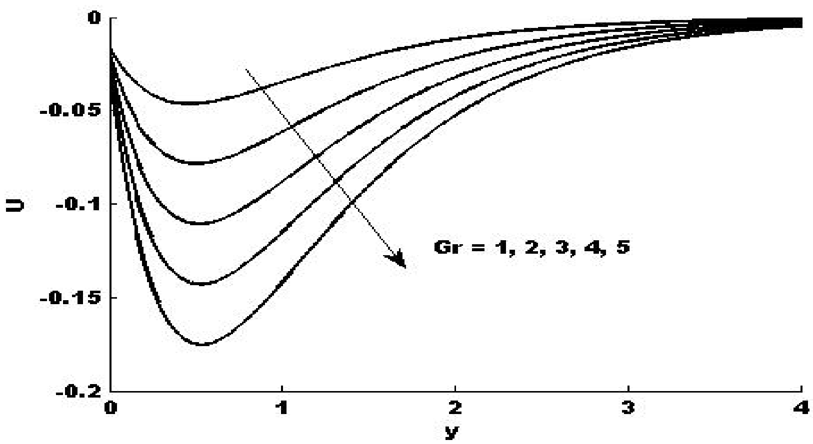

Fig. 5. Effects of Thermal Grashof number Gr on Primary velocity

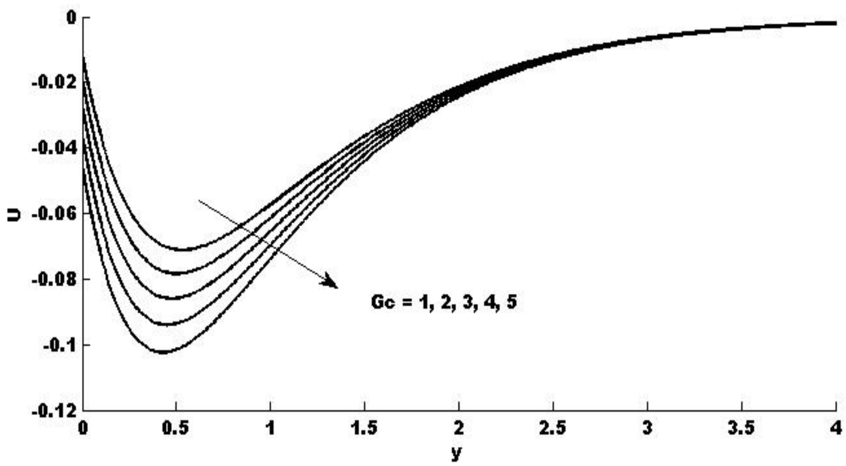

Fig. 6. Effects of Mass Grashof numberGc on Primary velocity

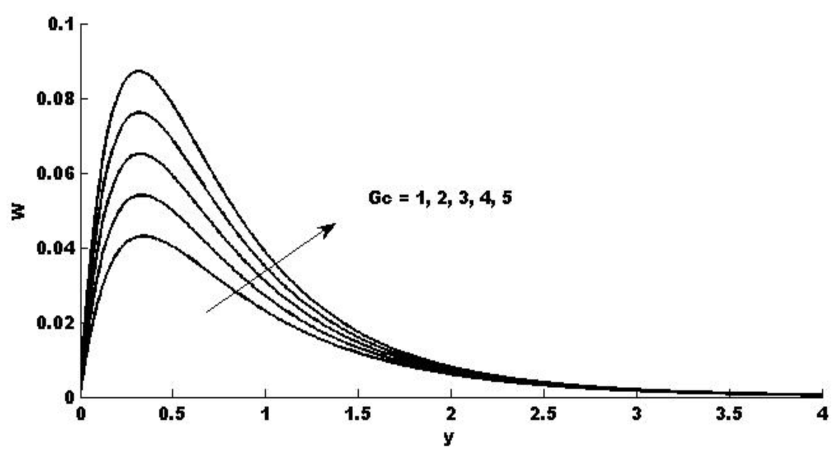

Fig. 7. Effects of Thermal Grashof number Gr on secondary velocity

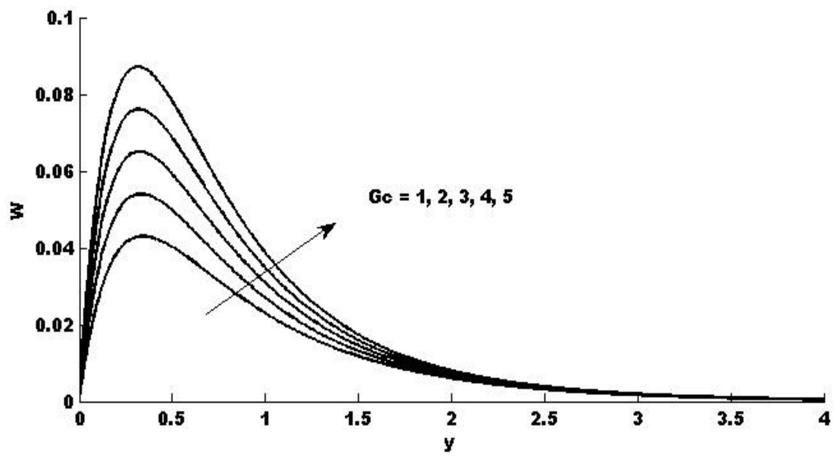

Fig. 8. Effects of Gc on secondary velocity

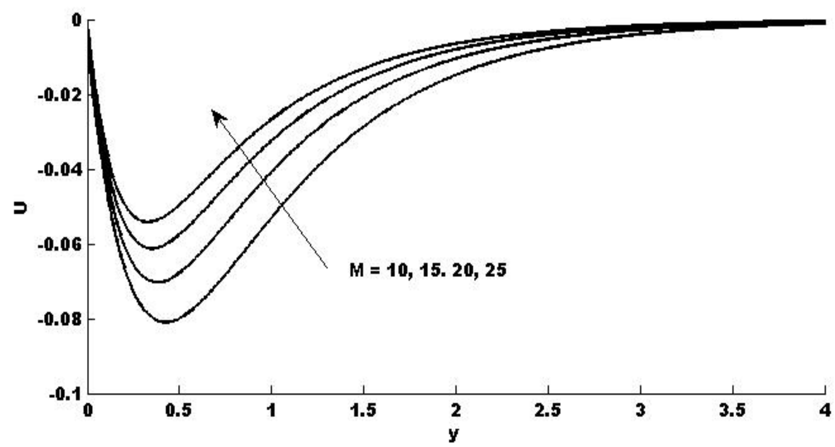

Fig. 9. Effects of Magnetic field M on primary velocity

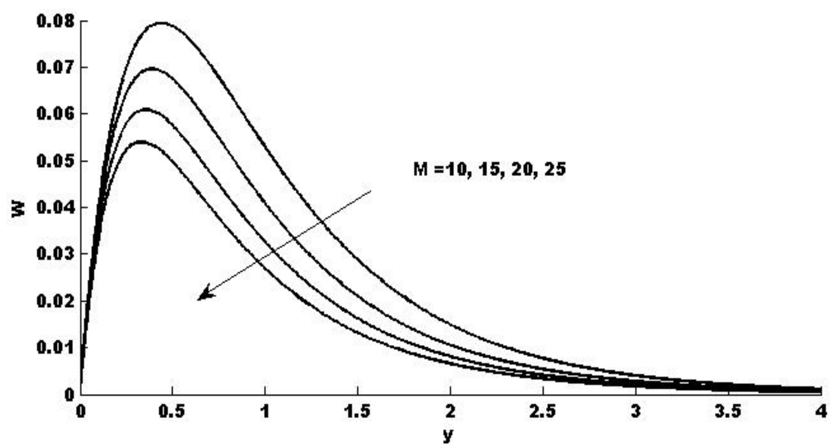

Fig. 10. Effects of Magnetic field M on secondary velocity

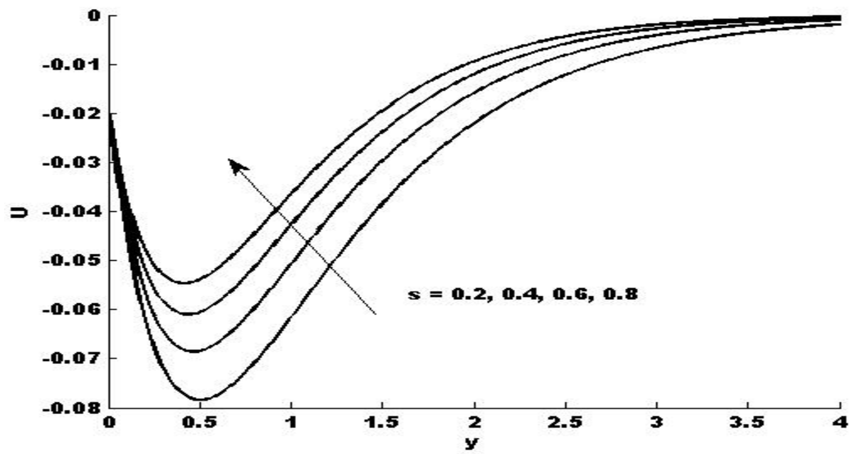

Fig. 11. Effects of heat source s on primary velocity

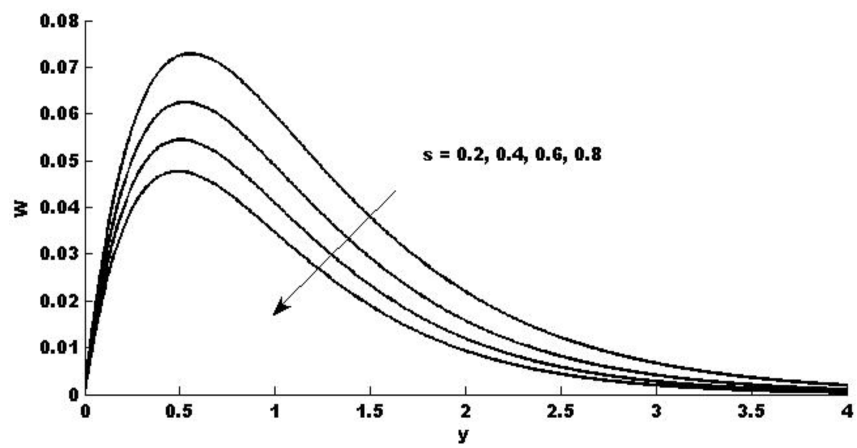

Fig. 12. Effects of heat source s on secondary velocity

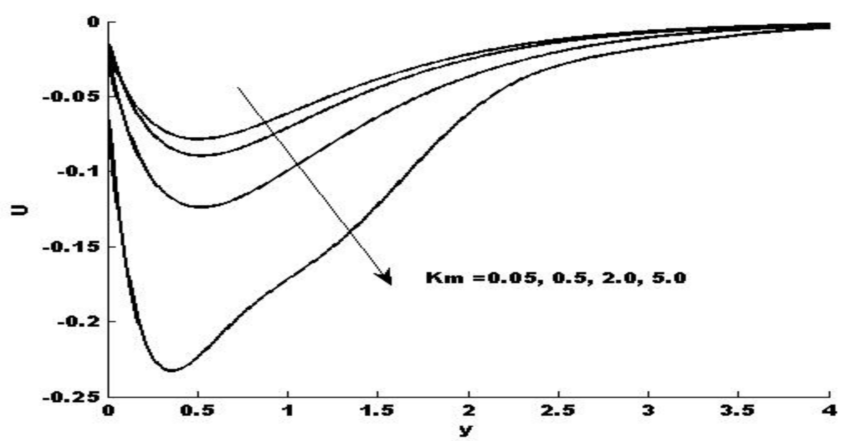

Fig. 13. Effects of Km on primary velocity

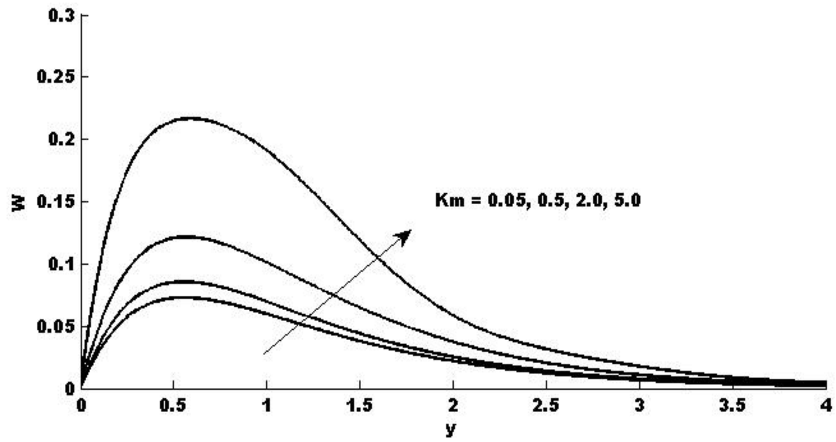

Fig. 14. Effects of Km on secondary velocity

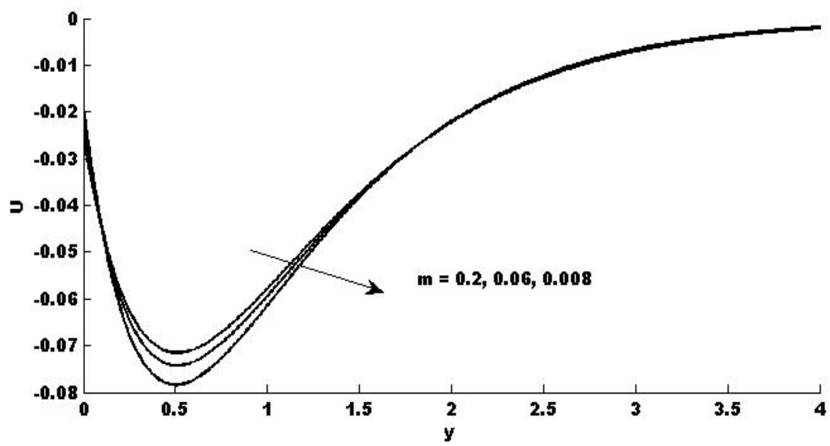

Fig. 15. Effects of hall current m on primary velocity

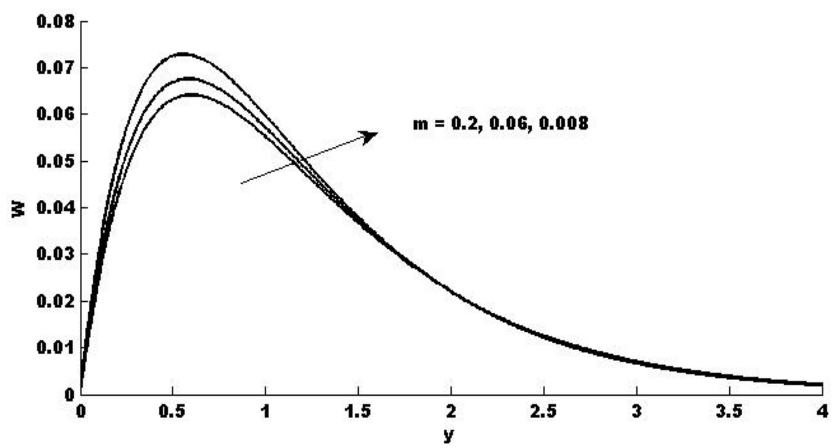

Fig. 16. Effects of hall current (m) on secondary velocity

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

$$L=\left(\frac{1}{K_s}+\frac{M}{(1+m^2)}\right)\quad J=\frac{Mm}{(1+m^2)}\quad Z=\left(\frac{3+4R}{3\Pr}\right)\quad n_1=\frac{-\frac{\Pr}{Z}\pm\sqrt{\left(\frac{\Pr}{Z}\right)^2+4\frac{S}{Z}}}{2}\quad n_3=\frac{-SC\pm\sqrt{(SC)^2+4SCK}}{2}$$

$$n_5=\frac{-1\pm\sqrt{1+4N}}{2}\quad A_{11}=\frac{-Gr}{\left(n_1^{\,2}-n_1-N\right)}\quad \therefore A_{12}=\frac{-Gc}{\left(n_3^{\,2}-n_3-N\right)}\quad A_{10}=A_{11}+A_{12}$$

$$\therefore A_{15}=\frac{n_5^{\,3}A_{10}}{\left(n_5^{\,2}-n_5-N\right)}\quad \therefore A_{17}=\frac{-n_3^{\,3}A_{12}}{\left(n_3^{\,2}-n_3-N\right)},\quad A_{14}=-A_{15}+A_{16}+A_{17}\quad n_2=\frac{-\frac{1}{2}\pm\sqrt{\left(\frac{1}{2}\right)^2+\frac{4q}{Z}}}{2},$$

$$q=(K+i\pi)\quad n_4=\frac{-SC\pm\sqrt{(SC)^2+4SCq}}{2}\quad f_1=\frac{-1\pm\sqrt{1+4ENn}}{2E}\quad B_3=-\frac{Gr}{En_2^2-n-Nn}$$

$$B_4=-\frac{Gc}{En_4^2-n_4-Nn}\quad B_2=B_3+B_4\quad f_2=\frac{-1\pm\sqrt{1+4ENn}}{2E}\quad B_7=\frac{f_1^3B_2}{Ef_1^2-f_1-Nn},$$

$$B_8=\frac{-n_2^3B_3}{En_2^2-n-Nn},\quad B_9=\frac{-n_4^3B_4}{En_4^2-n_4-Nn},\quad B_6=-B_7+B_8+B_9\quad Nn=Ln+J\quad Ln=L+i\Omega$$

$$N=(L+J)\quad n_5=n_6$$

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www.sciencedomain.org/review-history.php?iid=733&id=6&aid=7610