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Statistical Modeling of Head Loss through Duct Fittings in Conditioned Air Distribution Systems

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Fractions of the total head loss which constitute the loss through duct fittings, obtained in an earlier study, for varying numbers of supply outlets and lengths of index duct run were used in a regression analysis to obtain second order equations which gave a general increase of the fraction from 0.598 to 0.713 for an increase in number of outlets from 3 to 19 and for a corresponding increase of 11.2 m to 43.2 m in index duct length. The correlation coefficient between the fraction of loss through duct fittings and each of the variables of number of supply outlets and length of index duct run was 0.823, which was found to be acceptable for a 95% confidence level. The ratios are useful for quick estimate of total pressure losses in conditioned air distribution systems which are needed in determining the fan pressure requirements.

Keywords: Loss ratio; duct fittings; air distribution; regression analysis.

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1. INTRODUCTION

The determination of the total pressure loss is an important exercise in system design as the air conditioning fan selection is facilitated thereby; this total pressure loss being the sum of the frictional loss and the loss through duct fittings such as elbows, tees, tap-ins, and reducers.

In an earlier study, the variation of the percentage of the total head loss which is due to duct fittings with varying number of air supply outlets and length of index run of ductwork was studied for a typical air distribution system [1]. In that study, equal air quantities are distribution to 19 rooms of a hotel building through wall-mounted outlets Fig. 1. Fig. 2 shows an isometric sketch of the duct system of Fig. 1. In order to obtain variations of number of supply outlets and length of index run of ductwork, the index run ABC - - - -L is analyzed in the independent successive runs ABC, ABCD, ABCDE, and so on. The isometric sketches of these successive runs are shown in Figs. 3 to 10; in addition to Fig. 2. In Figs. 2 to 10 the duct sections in the index runs are labeled using boxes which touch the sections. In each box, the number on the left is the duct section number, that on the top right is the length of the section (in m) while the number on the bottom right is the air quantity (in m^3/s) flowing through the duct section.

Utilizing standard system parameters for these duct runs and using commonly applied methods, the total frictional head loss and the loss through fittings were computed for each of the duct systems of Fig. 2 to 10. Second order variations of the fraction of pressure loss due to fittings with varying numbers of supply outlets and lengths of index duct run were observed within the limits of duct parameters utilized in the study.

In this paper, regression model equations are derived to describe the variations obtained in the earlier study. The equations are useful for quick estimate of pressure losses in air distribution systems, needed in determining the fan pressure requirements.

2. ESTIMATION OF PRESSURE LOSS COMPONENTS

For a composite index duct run, the head loss due to friction (based on the D'Arcy-Weisbach formula) was applied as [2]

$$h_{friction} = 0.3304\sum_{i=1}^{n}\frac{f_i l_i q_i^2}{d_i^5} \qquad ---(1)$$

where i denotes the i^{th} duct section, n is the number of sections in the composite run and

f = duct section friction factor
l = duct section length in m
q = air flow rate through the duct section in m^3/s
d = diameter of the duct section

f is a function of the flow Reynolds number given as

$$Re = \frac{\rho v d}{\mu} \qquad ----(2)$$

where

ρ = air density (taken as $1.204 kg/m^3$)
v = flow velocity in m/s
μ = air dynamic viscosity (taken as $1.8 \times 10^{-5} kg/ms$)

Putting values of ρ and μ in Eqn. 2 and noting that

$$v = \frac{4q}{\pi d^2}, \text{ yields}$$

$$Re = 8.515 \times 10^4 \frac{q}{d} \qquad ----(3)$$

Furthermore, the Nikuradse equation [3]

$$f = 0.0008 + 0.0053\ Re^{-0.237} \qquad ---(4)$$

was utilized in estimating f.

Thus, with knowledge of q from the supply air requirements, d from the 'equal friction' method of duct sizing [4,5] and l from length measurements, the friction head loss was obtained.

For a given composite duct run, the head loss through fittings is given as [2]

$$h_{fittings} = 0.08256\sum_{j=1}^{m}k_j q_j d_j^{-4} \qquad ---(5)$$

where j denotes the j^{th} duct fitting, m is the number of fittings in the duct run and k is the head loss coefficient of the particular type of fitting obtained from the literature [6,7,8].

Combining Eqns. 1 and 5, the equation of total head loss h for the composite duct run is given as

$$h = h_{friction} + h_{friction} = 0.3304\sum_{i=1}^{n}\frac{f_i l_i q_i^2}{d_i^5} + 0.08256\sum_{j=1}^{m}k_j q_j d_j^{-4} \quad ------- (6)$$

Tables 1 and 2, respectively, show the results of the calculation of the head loss components for the shortest and longest index runs (shown in the isometric sketches of Figs. 3 and 2, respectively). The summary of the head loss estimates for all the index run configurations of Figs. 2 to 10 is given in Table 3.

Furthermore, the 'Excel' plots of Figs. 11 and 12 give the variation of the fraction of head loss due to duct fittings with varying numbers of air supply outlets and lengths of index duct run.

3. REGRESSION ANALYSIS

In the analysis, the ratio of loss through fittings to total loss (denoted as y), is regressed on the measures of complexity of ductwork, namely number of air supply outlets (denoted as x_1) and length of index duct run (denoted as x_2). As earlier mentioned, the 'Excel' plots of Figs. 11 and 12 show second order variations of y with x, where $x = x_1$ or x_2, namely

$$y = a_0 + a_1 x + a_2 x^2 \quad ---- (7)$$

Then, the regression parameters a_0, a_1 and a_2 should be obtained for each of the variables x_1 and x_2 by the solution of the simultaneous equations [9]

$$\left.\begin{aligned}\sum y &= na_0 + a_1\sum x + a_2\sum x^2\\ \sum xy &= a_0\sum x + a_1\sum x^2 + a_2\sum x^3\\ \sum yx^2 &= a_0\sum x^2 + a_1\sum x^3 + a_2\sum x^4\end{aligned}\right\} - (8)$$

where n is the number of data points, in this case, equal to 9.

Table 4 summarizes the computations of the variables and terms which appear in Eqn. 8. Substitution of the variables in Eqn. 8 and simultaneous solution yields the respective relationships between y and the variables x_1 and x_2 as

$$y = 0.5893 + 2.167\text{x}10^{-3}\,x_1 + 2.273\text{x}10^{-4}\,x_1^{\,2} \quad -(9)$$

and

$$y = 0.5852 + 4.923 \text{ x } 10^{-4}\,x_2 + 5.682 \text{ x } 10^{-5}\,x_2^2 \quad -(10)$$

For instance, to obtain Eqn. 9, substitution of values from Table 4 into Eqn. 8 is done to yield the simultaneous equations

$$5.82 = 9a_0 + 99a_1 + 1329a_2 \quad - (a)$$
$$65.74 = 99a_0 + 1329a_1 + 19899a_2 \quad -- (b)$$
$$898.38 = 1329a_0 + 19899a_1 + 317337a_2 \quad - (c)$$

Solving for a_0, a_1 and a_2 gives a_0 = 0.5893, a_1 = 2.167 x 10^{-3} and a_2 = 2.273 x 10^{-4} to yield Eqn. 9.

Now, the coefficient of correlation is given as [9]

$$r = \sqrt{1-\left(\frac{s_{y.x}}{s_y}\right)^2} \quad --- (11)$$

where

$$s_{y.x} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - y_{ic})^2}{n-3}} \quad --- (12)$$

= standard error of estimate, y_i being the actual values of y obtained from the results of section 2 above, y_{ic} being the values of y computed from the regression equation (Eqns.9 and 10) and n the number of points (equal to 9).

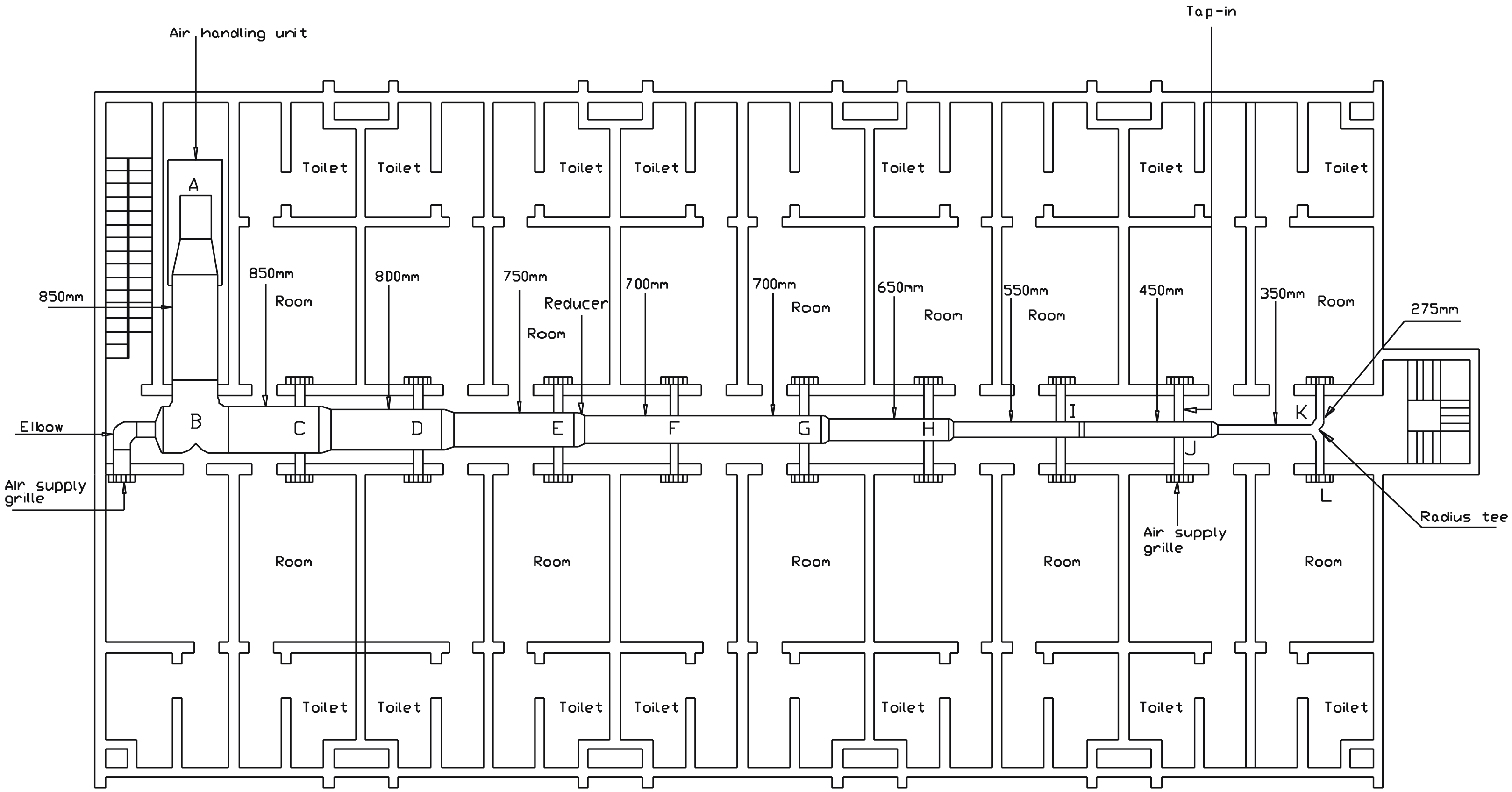

Fig. 1: Plan of air distribution ductwork

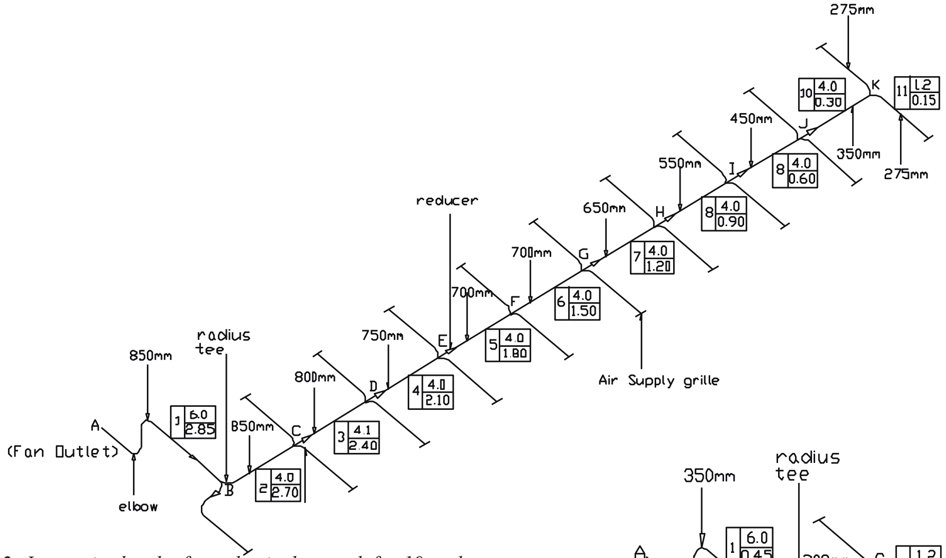

Fig. 2: Isometric sketch of supply air ductwork for 19 outlets

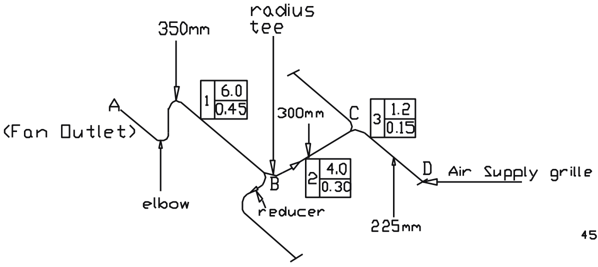

Fig. 3: Isometric sketch of supply air ductwork for 3 outlets

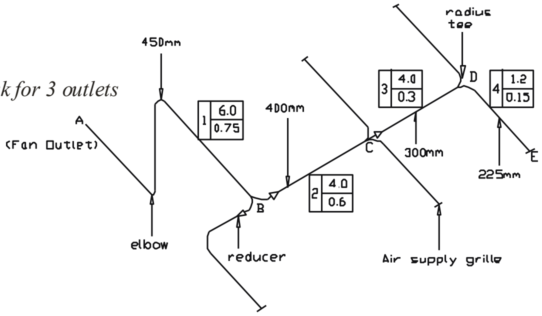

Fig. 4: Isometric sketch of supply air ductwork for 5 outlets

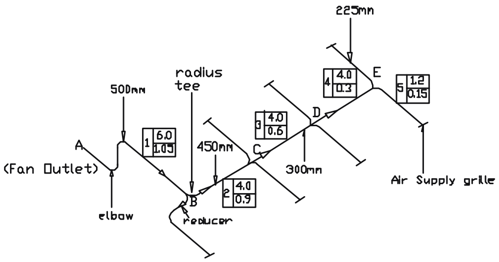

Fig. 5: Isometric sketch of supply air ductwork for 7 outlets

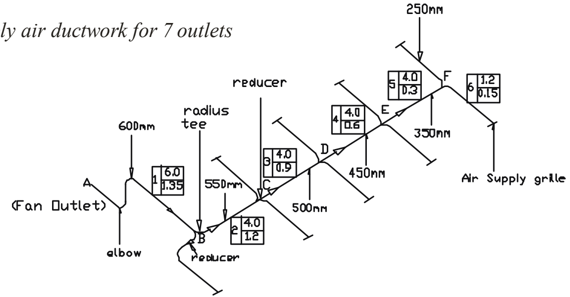

Fig. 6: Isometric sketch of supply air ductwork for 9 outlets

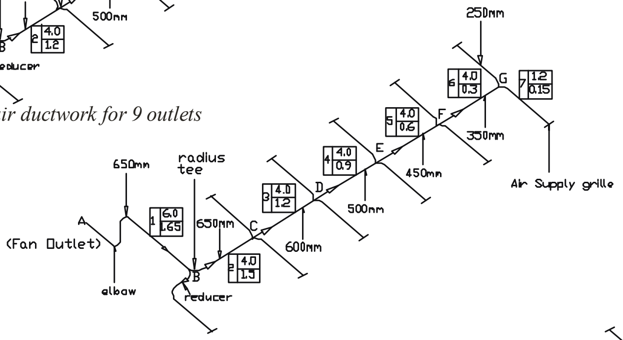

Fig. 7: Isometric sketch of supply air ductwork for 11 outlets

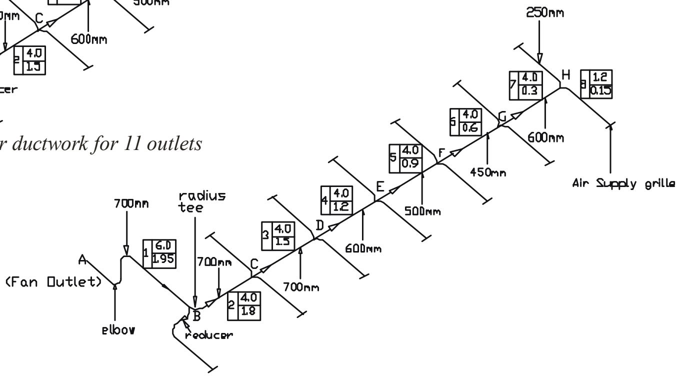

Fig. 8: Isometric sketch of supply air ductwork for 13 outlets

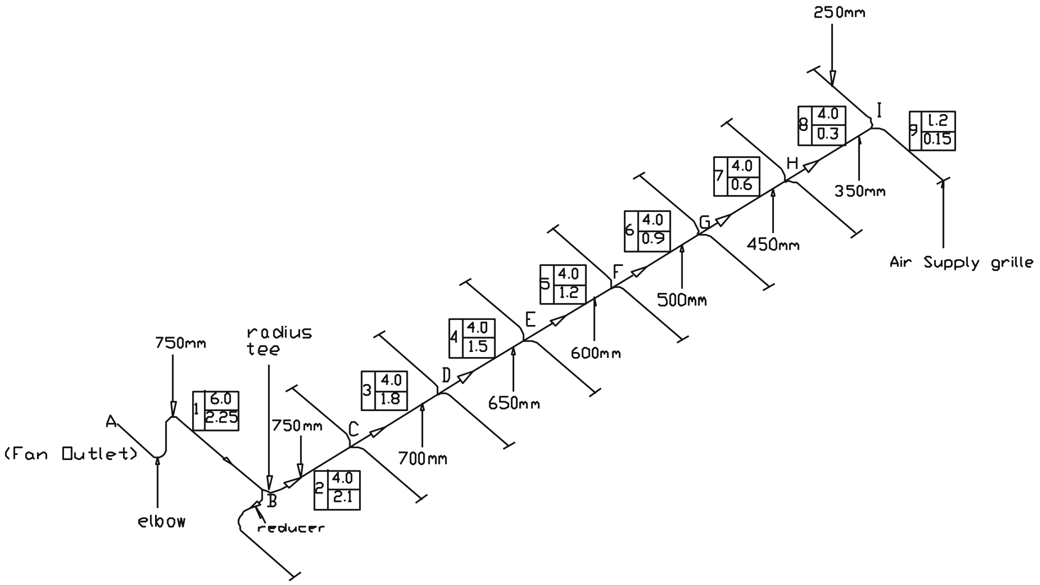

Fig. 9: Isometric sketch of supply air ductwork for 15 outlets

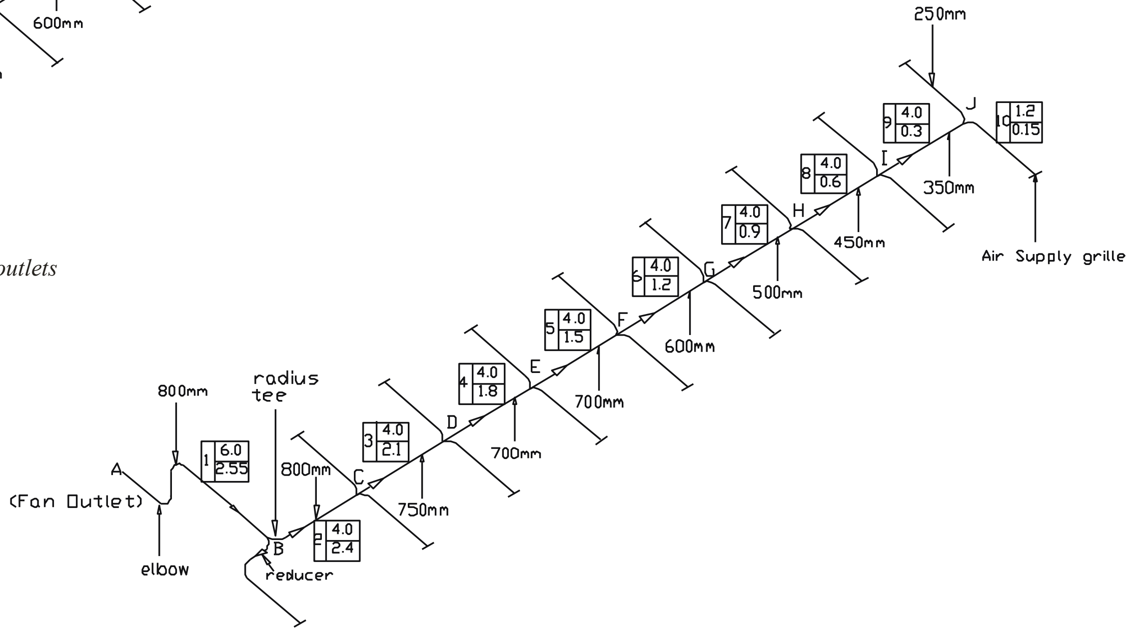

Fig. 10: Isometric sketch of supply air ductwork for 17 outlets

Table 1. Summary of head loss calculations for distribution system for 3 outlets (Fig. 3)

Duct section (in Fig. 3)	Flow rate, q (m^3/s)	Fractional flow with respect to total fan discharge Q	Length, 1 (m)	Diameter, d (mm)	Reynolds number, Re	Friction factor, f	Frictional head loss (m)	Fitting			
								Type	Number in duct section	Head loss coefficient*	Head loss (m)
1 (A to B)	0.45	Q	6.0	350	109479	0.0043	$1.623Q^2$	350mm elbow	2	0.16	$0.880Q^2$ x 2
								350mm tee	1	0.28	$1.540\ Q^2$
2 (B to C)	0.30	0.667Q	4.0	300	85150	0.0045	$1.089Q^2$	350mm x 300mm reducer	1	0.06	$0.272\ Q^2$
								300mm x 225mm reducing tee	1	0.28	$1.000\ Q^2$
3 (C to Outlet at C)	0.15	0.333Q	1.2	225	56767	0.0049	$0.374Q^2$	-	-	-	-
							$3.086Q^2$				$4.572Q^2$

**Source: J. J. Barton (1964)*

Table 2. Summary of head loss calculations for distribution system for 19 outlets (Figs. 1 and 2)

Duct section (in Figs. 1 and 2)	**Flow rate, q (m^3/s)**	**Fractional flow with respect to total fan discharge Q**	**Length, l (m)**	**Diameter, d (mm)**	**Reynolds number, Re**	**Friction factor, f**	**Frictional head loss (m)**	**Fitting**			
								Type	**Number in duct section**	**Head loss coefficient***	**Head loss (m)**
1 (A to B)	2.85	Q	6.0	850	285503	0.0036	$0.016Q^2$	850mm radius elbow	2	0.16	$0.025Q^2$ x 2
								850mm radius tee	1	0.28	$0.044Q^2$
2 (B to C)	2.70	0.947Q	4.0	850	270476	0.0036	$0.010Q^2$	850mm tap-in	1	0.20	$0.012Q^2$
3 (C to D)	2.40	0.842Q	4.0	800	255450	0.0037	$0.011Q^2$	850mm x 800mm reducer	1	0.06	$0.086Q^2$
								800mm tap-in	1	0.20	$0.012Q^2$
4 (D to E)	2.10	0.737Q	4.0	750	238420	0.0037	$0.011Q^2$	800mm x 750mm reducer	1	0.06	$0.009Q^2$
								750mm tap-in	1	0.20	$0.012Q^2$
5 (E to F)	1.80	0.632Q	4.0	700	218957	0.0038	$0.012Q^2$	750mm x 700mm reducer	1	0.06	$0.008Q^2$
								700mm tap-in	1	0.02	$0.012Q^2$
6 (F to G)	1.50	0.526Q	4.0	700	182464	0.0039	$0.008Q^2$	700mm tap-in	1	0.20	$0.012Q^2$
7 (G to H)	1.20	0.421Q	4.0	650	157200	0.0040	$0.008Q^2$	700mm x 650mm reducer	1	0.06	$0.005Q^2$
								650mm tap-in	1	0.20	$0.012Q^2$
8 (H to I)	0.90	0.316Q	4.0	550	139336	0.0041	$0.011Q^2$	650mm x 550mm reducer	1	0.06	$0.005Q^2$
								550mm tap-in	1	0.20	$0.012Q^2$
9 (I to J)	0.60	0.211Q	4.0	450	113533	0.0043	$0.014Q^2$	550mm x 450mm reducer	1	0.06	$0.005Q^2$
								450mm tap-in	1	0.20	$0.012Q^2$
10 (J to K)	0.30	0.105Q	4.0	350	72986	0.0047	$0.013Q^2$	450mm x 350mm reducer	1	0.06	$0.004Q^2$
								350mm x 275mm reducing tee	1	0.28	$0.017Q^2$
11(K to L)	0.15	0.053Q	1.2	275	46445	0.0051	$0.004Q^2$	-	-	-	-
							$0.118Q^2$				$0.329Q^2$

**Source: J. J. Barton(1964)*

Table 3. Summary of head loss estimates

No. of air conditioned rooms or supply outlets	Length of index duct run (m)	Frictional head loss (m)	Head loss due to fittings (m)	Total head loss (static pressure) (m)	Fraction of loss due to fittings
3	11.2	$3.086Q^2$	$4.572Q^2$	$7.658Q^2$	0.60
5	15.2	$1.314Q^2$	$1.804Q^2$	$3.118Q^2$	0.58
7	19.2	$0.826Q^2$	$1.379Q^2$	$2.205Q^2$	0.63
9	23.2	$0.369Q^2$	$0.729Q^2$	$1.098Q^2$	0.66
11	27.2	$0.273Q^2$	$0.464Q^2$	$0.737Q^2$	0.63
13	31.2	$0.218Q^2$	$0.416Q^2$	$0.634Q^2$	0.66
15	35.2	$0.186Q^2$	$0.361Q^2$	$0.547Q^2$	0.66
17	39.2	$0.151Q^2$	$0.291Q^2$	$0.442Q^2$	0.66
19	43.2	$0.118Q^2$	$0.329Q^2$	$0.447Q^2$	0.74

Table 4. Compilation of statistical variables and terms for x_1 and x_2

x_1 No. of air conditioned rooms or supply outlets	x_2 length of index duct run (m)	Frictional head loss (m)	Head loss due to fittings (m)	Total head loss (m)	y loss thru fittings total loss	yx_1	x_1^2	yx_1^2	x_1^3	x_1^4	yx_2	x_2^2	yx_2^2	x_2^3	x_2^4
3	11.2	$3.086Q^2$	$4.572Q^2$	$7.658Q^2$	0.60	1.80	9	5.40	27	81	6.720	125.44	75.2640	1404.928	15735.1936
5	15.2	$1.314Q^2$	$1.804Q^2$	$3.118Q^2$	0.58	2.90	25	14.50	125	625	8.816	231.04	134.0032	3511.808	53379.4816
7	19.2	$0.826Q^2$	$0.379Q^2$	$2.205Q^2$	0.63	4.41	49	30.87	343	2401	12.096	368.64	232.2432	7077.888	135895.4496
9	23.2	$0.369Q^2$	$0.729Q^2$	$1.098Q^2$	0.66	5.94	81	53.46	729	6561	15.312	538.24	355.2384	12487.168	289702.2976
11	27.2	$0.273Q^2$	$0.464Q^2$	$0.737Q^2$	0.63	6.93	121	76.23	1331	14641	17.136	739.84	466.0992	20123.648	544363.2256
13	31.2	$0.218Q^2$	$0.416Q^2$	$0.634Q^2$	0.66	8.58	169	111.54	2197	28561	20.592	973.44	642.4704	30371.328	947585.4336
15	35.2	$0.186Q^2$	$0.361Q^2$	$0.547Q^2$	0.66	9.90	225	148.50	3375	50625	23.232	1239.04	817.7664	43614.208	1535220.1220
17	39.2	$0.151Q^2$	$0.291Q^2$	$0.442Q^2$	0.66	11.22	289	190.74	4913	83521	25.872	1536.64	1014.1824	60236.288	2361262.4900
19	43.2	$0.118Q^2$	$0.329Q^2$	$0.447Q^2$	0.74	14.06	361	267.14	6859	130321	31.968	1866.24	1381.0176	80621.568	3482851.7380
$\sum=99$	$\sum=244.8$			$\sum=$	5.82	65.74	1329	898.38	19899	317337	161.744	7618.56	5118.2848	259448.832	9368995.4320

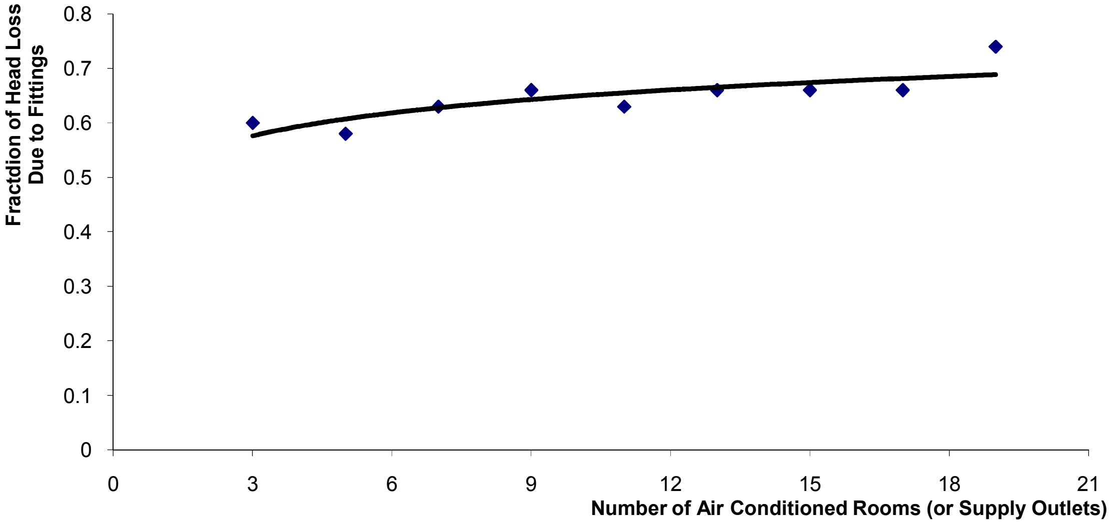

Fig. 11. Graph of fraction of head loss due to fittings versus number of conditioned rooms (or supply outlets)

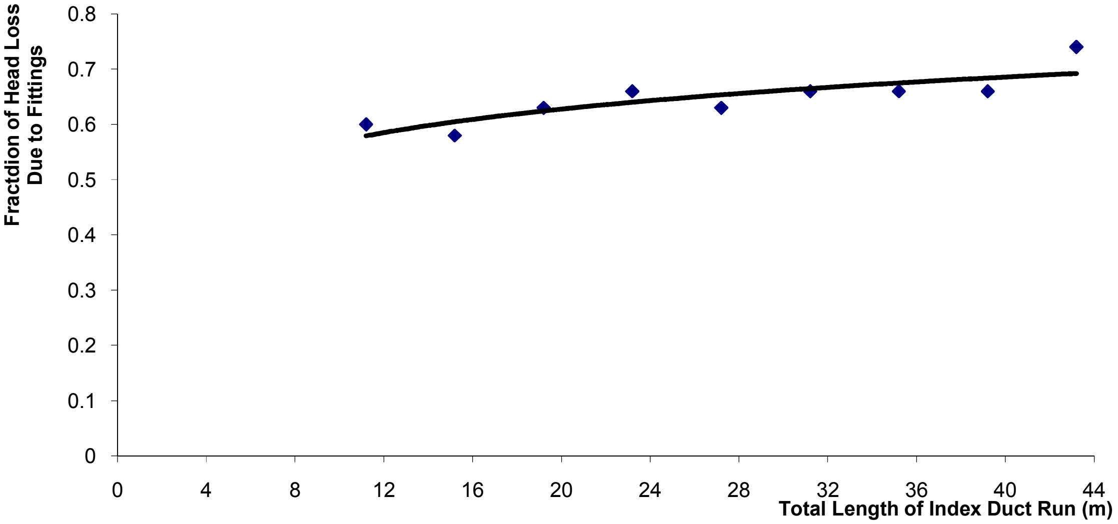

Fig. 12. Graph of fraction of head loss due to fittings versus length of index run

n-3 is the number of degrees of freedom, as the number of regression parameters to be estimated in Eqn. 7 is three : a_0, a_1 and a_2.

$$s_y = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}{n-1}} \quad - - - (13)$$

= sample standard deviation of y , $\overline{y}$ being the mean of y_i

The calculation of the coefficient of correlation r for x_1, which is facilitated by Table 5, is illustrated as follows:

The regression equation is

$$y_{ic} = 0.5893 + 2.167 \times 10^{-3}\ x_i + 2.273 \times 10^{-4}\ x_i^{\ 2} \quad (14)$$

where y_{ic} is the calculated i^{th} value of y and x_i is the i^{th} value of x .

Table 5. Values for calculating correlation coefficient for x_1

i	x_i	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$ x 10^{-4}	y_{ic}	$y_i - y_{ic}$	$(y_i - y_{ic})^2$ x 10^4
1	3	0.60	- 0.047	22.09	0.598	0.002	0.04
2	5	0.58	- 0.067	44.89	0.606	- 0.026	6.76
3	7	0.63	- 0.017	2.89	0.616	0.014	1.96
4	9	0.66	0.013	1.69	0.627	0.033	10.89
5	11	0.63	- 0.017	2.89	0.641	- 0.011	1.21
6	13	0.66	0.013	1.69	0.656	0.004	0.16
7	15	0.66	0.013	1.69	0.673	- 0.013	1.69
8	17	0.66	0.013	1.69	0.692	- 0.032	10.24
9	19	0.74	0.093	86.49	0.713	0.027	7.29
		$\sum = 5.82$ $\bar{y} = 0.647$		$\sum = 166.01$			$\sum = 40.24$

In Table 5 y_i is the actual value of y obtained from the analysis of losses of pressure head, and $\bar{y}$ is the mean of these y values.

Then $s_{y.x} = \sqrt{\dfrac{40.24x10^{-4}}{6}}$ = 0.02590 from Eqn. 12

$s_y = \sqrt{\dfrac{166.01x10^{-4}}{8}}$ = 0.04555 from Eqn. 13

and coefficient of correlation

$$r = \sqrt{1 - \left(\frac{0.02590}{0.04555}\right)^2} = 0.823$$

Similarly, the coefficient of correlation for x_2 is obtained as 0.823.

From statistical data, the coefficient of 0.823 corresponds to a 95% confidence interval value of $0.760 \le r \le 0.886$ [10].

4. DISCUSSION OF RESULTS

There is, thus, 95% confidence that the variation of the fraction of head loss through fittings is dependent, in turn, on variations of number of conditioned air distribution outlets and length of index run of ductwork. It also follows that estimates of the fraction can be made using the derived regression equations.

The fractions of loss through fittings calculated from the regression equations show second order increases from 0.598 to 0.713 for an increase in number of outlets from 3 to 19 and for a corresponding increase of 11.2 m to 43.2 m in index duct length. Thus, within the limits of system parameters used in the analysis, needed estimates of the fittings loss fraction may be made by interpolating between computed values and adding a safety margin.

5. CONCLUSION

Regression analyses have been done to model the head loss fraction due to duct fittings for index duct runs in a set of conditioned air distribution systems. Such derived regression equations are useful in system head loss estimates. For instance, for a given number of outlets and length of duct run, the total head loss can quickly be estimated by adding the relevant fraction to the frictional loss; thus facilitating the air conditioning fan selection process. Furthermore, similar analyses can be done for more extensive duct systems to obtain relevant regression model equations for wider applications.

COMPETING INTERESTS

Author declares that there are no competing interests.

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