



## Influence of Sampling Period on Harmonics of Three-Phase Space Vector Modulated Inverter

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### Authors' contributions

This work was carried out in collaboration between all authors. Author JAG investigated and designed the study including the analytical approach, and wrote the first draft of the manuscript. Authors MAAK and SAJ managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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### ABSTRACT

The three-phase inverter is driven by a number of semiconductor switches for obtaining three-phase output of nearly sinusoidal fundamental voltage. The purpose of the Space Vector Modulation (SVM) technique is to produce three-phase load voltages of fewer harmonics. It can be reached by selecting an appropriate sampling period for the desired circular locus. This paper endeavors to present the influence of the sampling period on inverter output voltages and their harmonics. Simulation results are presented to assess the inverter performance for different values of sampling periods. The results conclude that an optimum inverter output voltage is achieved when the given circular locus of space vector is sampled at a period equals the half interval between of the two adjacent space vectors.

*Keywords:* Inverters; PWM inverter; SVM inverter; power electronics.

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## 1. INTRODUCTION

The three-phase inverter has three legs where each leg has two series switches. Each leg produces one phase of the three-phase output voltages [1,2]. Fig. 1 shows the three-phase bridge inverter.

The two switches of any leg cannot be switched at same time to prevent the short circuit across the D.C source [3]. A proper sequence of gating signals for the semiconductor switches is applied to assure a positive and negative output voltage and to obtain the three-phase balanced fundamental output voltages. The state of each bottom switch is the complementary state of the top switch in the corresponding leg of the bridge inverter. The three top switches form 3-bit binary number, leading to an eight different states (000, 001, ..., 111). Six of these states are active (001, 010, ..., 110) and two of them are non-active because they cause a short circuit across three terminal outputs and produce a zero line voltages [4].

The six active states of the inverter are generated in the way to produce balanced three-phase output voltages. The six active states can be transformed into a single complex variable called a space vector which represents the effect of three-phase instantaneous voltage values [5]. The transformed space vector produces six vectors corresponding to the six active states of swishes.

The relationship between the carrier based Pulse-Width-Modulation (PWM) and Space Vector Modulation (SVM) was examined by [6,7], independent of the load type. The authors have developed a relation for providing a standard for implementing transformation between carrier based PWM and SVM. An analyzed investigation for multi-phase space

vector PWM for three-phase inverter was made by [8], based on harmonic analysis and output waveform modifications. Such technique that combines the advantages of both the symmetrical SVM and harmonic elimination SVM method was proposed in [9]. A space vector modulation technique for a single-phase multi-level converter was suggested by [10], to minimize the commutation losses on converter and to obtain high quality output waveforms. The undesirable narrow pulse in three-level space vector PWM is investigated and minimized in [11]. The multi-level SVM techniques that are used with the single-phase multi-level converter can be applied to a converter of any number of levels and phases as d proved by [12]. For harmonic investigation, the optimal harmonic profile for a space vector modulation occurs when the two middle space vectors are centered in each switching cycle as discussed by [13]. The torque and current steady state of an induction motor is improved using space vector modulation technique [14].

One cause of inverter losses is the generation of harmonics in the inverter output, due to the switching technique used with the space vector modulation. The paper investigates the space vector modulation applied on three-phase inverter and determines the optimum sampling period of the given circular locus of space vector based on high fundamental frequency output voltage and less Total Harmonic Distortion (THD).

The paper is organized as follows: Section 2 gives the detailed space vector representation. The space vector pulse width modulated inverter is discussed in Section 3. Section 4 discusses the influence of the sampling period of the circular locus on the output voltage of the inverter and the simulated results. Finally, conclusions are given in Section 5.

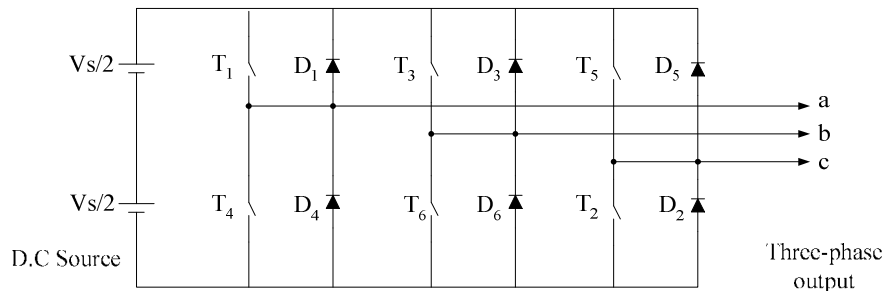
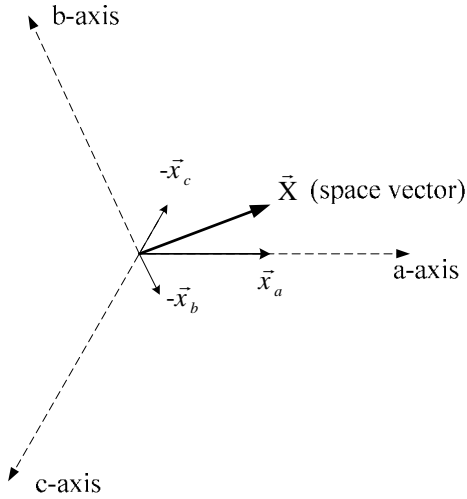


Fig. 1. Three-phase inverter

## 2. SPACE VECTOR MODULATION (SVM)

The space vector technique enables the three-phase quantities of a balanced system to be represented by a single vector rotating in the complex plane. It is the total effect of a set of instantaneous values in all balanced three variables into a single complex variable [15].



**Fig. 2. Space vector representation**

The space vector of three variables  $x_a$ ,  $x_b$  and  $x_c$  can be represented by:

$$\vec{x}_s = x_a + a x_b + a^2 x_c \quad (1)$$

Where;

$$a = e^{j\frac{2\pi}{3}}, \quad a^2 = e^{j\frac{4\pi}{3}}$$

$x_a$ ,  $x_b$ ,  $x_c$  : are the time-domain variables.

By substituting 'a' and 'a<sup>2</sup>' in Eqn. (1), we get:

$$\vec{x}_s = x_a + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) x_b + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) x_c$$

or;

$$\vec{x}_s = x_a - \frac{1}{2} (x_b + x_c) + j\frac{\sqrt{3}}{2} (x_b - x_c) \quad (2)$$

but for a balance case:  $x_a + x_b + x_c = 0$

or;

$$x_a = - (x_b + x_c) \quad (3)$$

By substituting Eqn. (3) in Eqn. (2), we get:

$$\vec{x}_s = \frac{3}{2} x_a + j\frac{\sqrt{3}}{2} (x_b - x_c)$$

or;

$$\vec{x}_s = x_a + j\frac{1}{\sqrt{3}} (x_b - x_c) \quad (4)$$

Fig. 2 shows the resultant space vector of three instantaneous variables:  $x_a$ ,  $x_b$  and  $x_c$  at  $t=6.111$  msec.

## 3. PULSE WIDTH MODULATED THREE-PHASE INVERTER

A balanced three-phase out put voltage can be obtained from the three- phase bridge inverter configuration shown in Fig. 1. There are six semiconductor switches are used in the configuration, divided into three legs. Three of six switches are on at any instant of time. The three upper switches are symbolized as:  $T_1$ ,  $T_3$  and  $T_5$  respectively. The three lower switches are symbolized as:  $T_4$ ,  $T_6$  and  $T_2$  respectively. For a 180-degree conduction inverter, a balanced output is obtained if the gating signals of the six switches are generated in the sequence of  $g_1$ ,  $g_2$ ,  $g_3$ ,  $g_4$ ,  $g_5$  and  $g_6$  respectively at phase shift of  $60^\circ$ . Any three switches are turning on for an interval of  $60^\circ$ , leading to six modes of operation. These six modes of operation can be obtained by applying the six states of binary codes of; 101, 100, 110, 010, 011 and 001. The two binary states of 000 and 111 cause a short circuit for the three output terminals, leading to zero line voltages. Therefore, they are not applied and lie in the origin of the complex plane.

### 3.1 Space Vector Modulated Inverter

The six states form a hexagon locus of six space vectors of:  $\vec{V}_1$ ,  $\vec{V}_2$ , ...,  $\vec{V}_6$  of binary codes of: 101, 100, 110, 010, 011 and 001 respectively. Each two adjacent active vectors form an equilateral triangle which is called a sector.

A balanced sinusoidal three-phase output voltage is obtained if the locus becomes circular.

This circular locus has a maximum amplitude at modulation index of  $m=1$ .

The modulation index:

$$m = \frac{A_r}{A_c} \quad (5)$$

Where;

$A_r$  is the amplitude of reference in the complex plane.

$A_c$  is the amplitude of carrier phase voltage in the three-phase space.

The space vector ( $\vec{V}_d$ ) in the circular locus is called the desired or reference space vector, as shown in Fig. 3. Each two adjacent active space vectors form an equilateral triangle called a sector.

The operation at reference space vector is the desired operation because the inverter output is sinusoidal at no harmonics. This desired operation cannot be achieved due to the switching method applied on the inverter circuit that produces the six active space vectors of hexagon locus ( $\Gamma_x$ ). This locus can be controlled on the way to be closed to the desired circular locus ( $\Gamma_d$ ), by representing the reference space vector into a linear combination of two adjacent active vectors and the zero vectors. The approximation can be made as closely as possible to the desired circular locus by adjusting the sampling period ( $T_s$ ) of the given circular locus. The sampling period must be not very small, to assure less inverter losses. The pattern of eight segments is formed across the two sampling periods. The time of the pattern is the sum of intervals of the two neighboring active vectors and the zero vectors ( $\vec{V}_0$  &  $\vec{V}_7$ ). For example, the time of the pattern in the two states: 100 and 110 is divided into intervals of eight segments: 000, 100, 110, 111, 111, 110, 100 and 000. Therefore, the volt-second of the desired vector can be written as below:

$$\vec{V}_d \cdot T_s = \vec{V}_n \cdot t_n + \vec{V}_{n+1} \cdot t_{n+1} + \vec{V}_z \cdot t_0 \quad (6)$$

Where:

$T_s$  is the sampling period of the desired locus (circular locus).

$\vec{V}_n$ ,  $\vec{V}_{n+1}$  are any two adjacent space vectors in the hexagon locus.

$\vec{V}_z$  is any of zero vectors ( $\vec{V}_0$  or  $\vec{V}_7$ ).

$t_n$ ,  $t_{n+1}$  are the intervals assigned to  $\vec{V}_n$  and  $\vec{V}_{n+1}$ .

$t_0$  is the interval assigned to  $\vec{V}_z$ .

There are six sectors in the hexagon area. For instance, the second sector in Fig. 3 is formed by the two vectors:  $V_3$  of angle of  $60^\circ$  and  $V_4$  of angle of  $120^\circ$ , so Equation (6) becomes:

$$\vec{V}_d \cdot T_s = \vec{V}_3 \cdot t_3 + \vec{V}_4 \cdot t_4 + \vec{V}_0 \cdot t_0 \quad (7)$$

Magnitude of  $V_3$  or  $V_4$  is given by  $\frac{2}{3} V_s$ , where  $V_s$  is the D.C input voltage of the inverter.

The modulation index is defined in terms of  $V_d$  and  $V_s$  as below:

$$m = \sqrt{3} \frac{V_d}{V_s} \quad (8)$$

The real part of Equation (7) is:

$$V_d T_s \cos(60^\circ + \alpha) = V_3 t_3 \cos 60^\circ + V_4 t_4 \cos 120^\circ \quad (9)$$

$\alpha$ : is the angle of desired space vector (angle between  $\vec{V}_d$  and  $\vec{V}_n$  ( $\vec{V}_3$ )).

After some arrangements:

$$t_3 = 3 \frac{V_d}{V_s} T_s \cos(60^\circ + \alpha) + t_4 \quad (10)$$

The imaginary part of Equation (7) is:

$$V_d T_s \sin(60^\circ + \alpha) = V_3 t_3 \sin 60^\circ + V_4 t_4 \sin 120^\circ \quad (11)$$

After some arrangements:

$$t_4 = \frac{\sqrt{3}}{V_s} T_s V_d \sin(60^\circ + \alpha) - t_3 \quad (12)$$

Substituting Eq. (12) in Eq. (10), we obtain:

$$t_3 = \frac{3T_s}{2V_s} V_d \cos(60^\circ + \alpha) + \frac{\sqrt{3}}{2V_s} T_s V_d \sin(60^\circ + \alpha)$$

or;

$$t_3 = \frac{3T_s}{2V_s} [V_{x2} + \frac{1}{\sqrt{3}} V_{y2}] \quad (13)$$

Where;

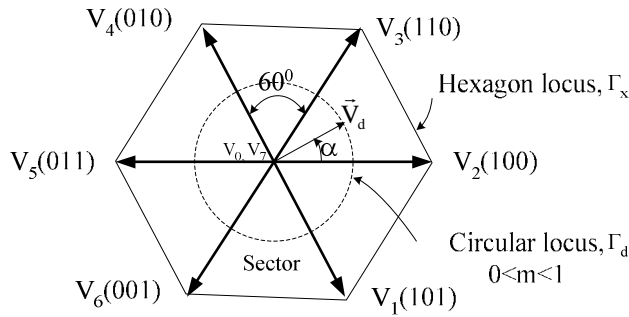
$$V_{x2} = V_d \cos(60^\circ + \alpha) \text{ and}$$

$$V_{y2} = V_d \sin(60^\circ + \alpha)$$

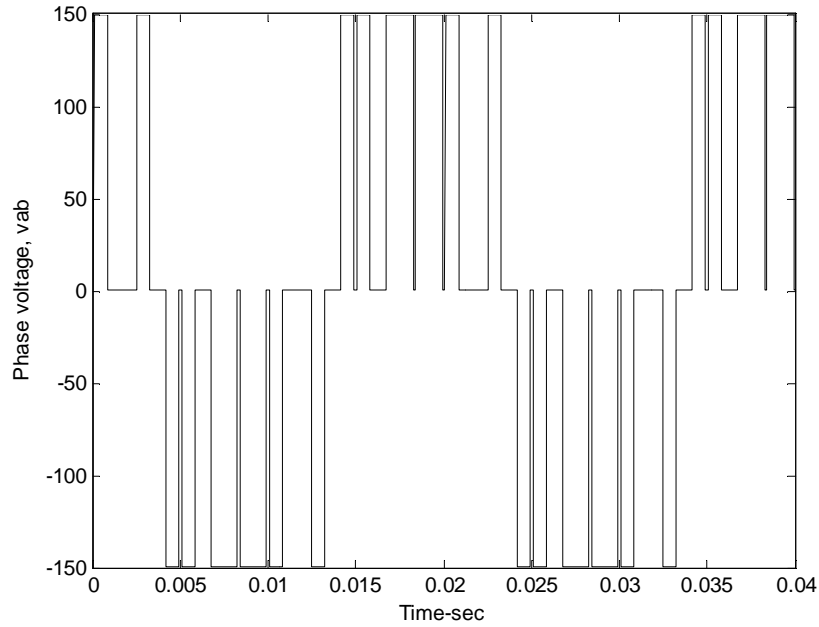
Substituting Eq. (10) in Eq. (12), we obtain:

$$t_4 = \frac{\sqrt{3} T_s}{2V_s} [\frac{1}{2} V_{y2} + \frac{\sqrt{3}}{2} V_{x2}] \quad (14)$$

Similarly,  $t_n$  and  $t_{n+1}$  can be obtained for any sector in the hexagon area. Fig. 4 shows the inverter line voltage  $v_{ab}$  for space vector-PWM.



**Fig. 3. Space vector for 3-phase voltages**



**Fig. 4. Line voltage  $v_{ab}$  for space vector PWM inverter**

#### 4. RESULTS AND DISCUSSION

The influence of the sampling period of the given circular locus on the performance of inverter based on fundamental frequency output voltage and Total Harmonic Distortion (THD) of output voltage is examined in this work. A model file is developed using Simulink library to assess the inverter performance. To obtain a desired output

voltage of an inverter, a suitable sampling period of the circular locus has to be chosen. The angle of the desired space vector ( $\alpha$ ) is taken  $30^\circ$ . Fig. 5 shows output voltage and its frequency spectrum, at  $T_s=1.666$  msec ( $30^\circ$  for 50Hz frequency), at which there is one pattern per a sector. The sampling period equals the half interval between the two adjacent space vectors. A fundamental frequency output voltage of 92%

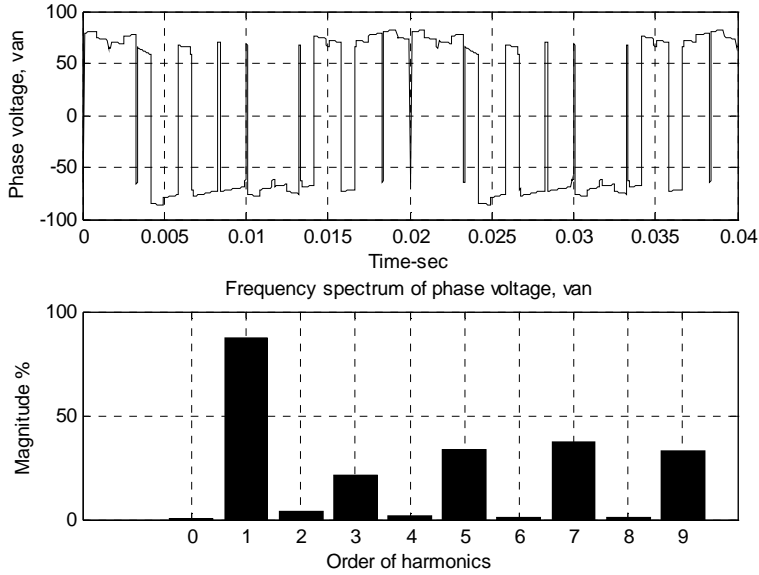


Fig. 5. Output phase voltage and its frequency spectrum  $T_s = 1.666$  msec

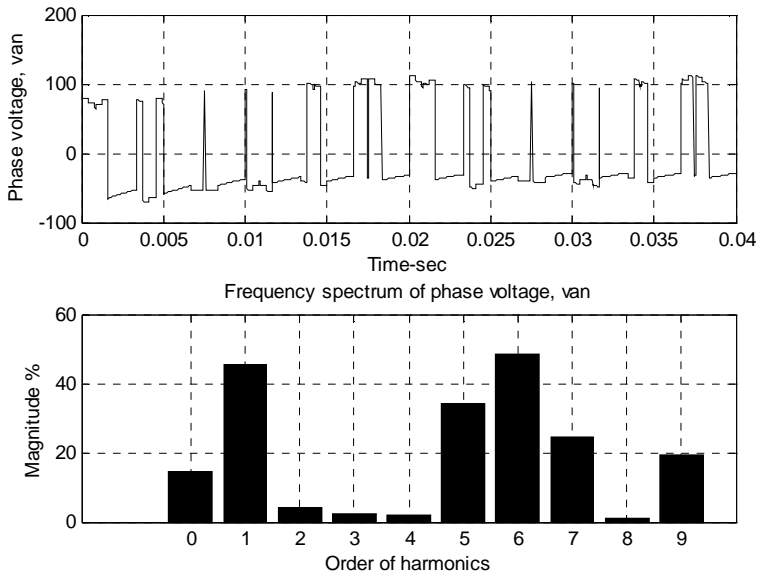
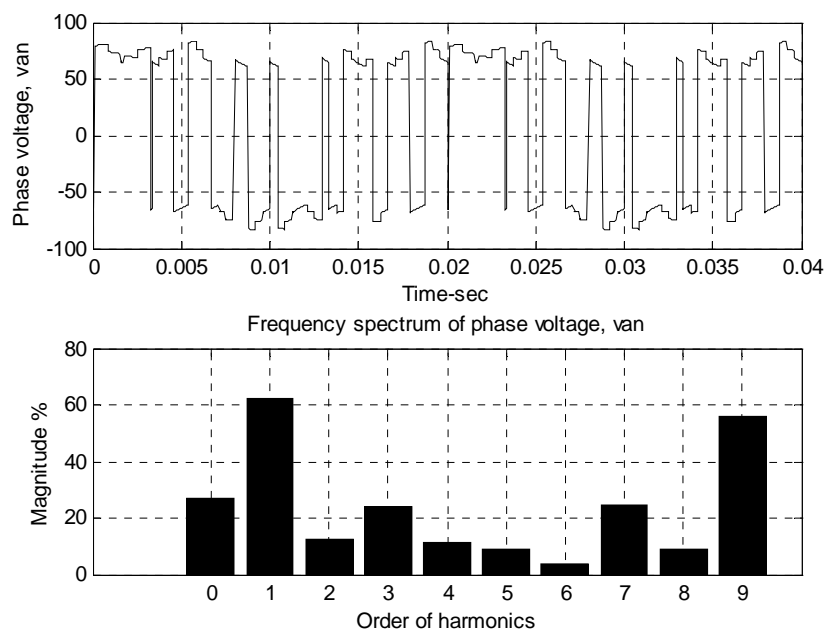


Fig. 6. Output phase voltage and its frequency spectrum  $T_s = 0.833$  msec



**Fig. 7. Output phase voltage and its frequency spectrum at  $T_s = 2.222$  msec**

and THD of 9.85% are obtained. Figs. 6 and 7 show output voltages and their frequency spectrum at  $T_s=0.833$  msec (more patterns per a sector) and  $T_s=2.222$  msec (less patterns per a sector), respectively. The fundamental output voltage for Fig. 6 is 42.50% at THD of 19.55% and for Fig. 7 is 63.33% at THD of 16.21%. It is clear that a superior fundamental output voltage is obtained when the given circular locus of space vector is sampled at a period equals the half interval between the two adjacent space vectors as shown in Fig. 5.

## 5. CONCLUSION

A balance sinusoidal three-phase output voltage is obtained if the locus of space vectors becomes a circular. The hexagon locus of space vectors can be approximated to the desired circular locus as closely as possible by adjusting the sampling period ( $T_s$ ) of the given circular locus. This paper examines the effect of the sampling period of the circular locus on the performance of inverter based on percentage of fundamental frequency output voltage and Total Harmonic Distortion (THD) in the output. It is concluded that a preferable fundamental output voltage is obtained when the given circular locus of space vector is sampled at a period equals the half interval between the two adjacent space vectors.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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