



# Finite Element Modeling of Timoshenko Micro-beam Based MEMS Sensor Behavior against Variation in Poisson's Ratio

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## Authors' contributions

This work was carried out in collaboration between both authors. Author HS performed the simulations and wrote the first draft of the manuscript. Author MT designed the study, wrote the protocol, managed literature searches and managed the analyses of the study. Both authors read and approved the final manuscript.

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## ABSTRACT

Epoxy Micro-beams are used in microelectromechanical systems (MEMS) for sensing applications in two operating mode of deflection and resonant frequency shift. One of the main questions in micro-beam based MEMS sensors behavior is effect of Poisson's ratio on the deflection and resonant frequency of the micro-beam. In this study, two epoxy Timoshenko microbeams with different dimensions were modeled based on the finite element method considering the effects of variation in Poisson's ratio. The results of this analysis indicated that change in the Poisson's ratio of the microbeams does not significantly affect the deflection and resonant frequency. Therefore, in the design of the microbeam based microelectromechanical systems where Poisson's ratio is one of the system variables, the FEM analysis ensures that changes in the environmental conditions affecting Poisson's ratio would not affect the system outputs. There was a good agreement between the results of this study and those obtained based on the strain gradient elasticity theory, classical theory, and the couple stress theory.

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## 1. INTRODUCTION

Micro electromechanical systems (MEMS) have had increasing applications in various fields in recent decades. These systems have various applications including sensing applications in the medical and chemical fields; Ionic species, explosives, pollutants, and gases. A significant portion of the microelectromechanical sensors is based on the use of microbeams. These systems are critically important due to their high cost and power consumption saving, small dimensions, low weight, and high sensitivity and accuracy.

Since the experiments needed to test and identify the behavior of these sensors are costly and time-consuming, researchers model and simulate these systems to predict their behavior. Several studies have been conducted to analyze the behavior of microbeam based microelectromechanical systems. Some of the most important studies in this field are listed here. Ansari et al. [1] investigated the vibrational behavior of piezoelectric microbeams based on the modified couple stress theory. Numerical results were obtained for two microbeams, and the piezoelectric effect and the size effect were examined. Results showed that the piezoelectric effect and the size effect are considerable at lower microbeam lengths. Bekir Akgöz et al. [2] presented a shear deformation beam model and new shear correction factors for nonhomogeneous microbeams. A new microstructure-dependent sinusoidal beam model for buckling of microbeams using modified strain gradient theory was developed by Bekir Akgöz et al. [3]. Kahrobaiyan et al. [4] studied the effect of size on the mechanical behavior of microbeams based the strain gradient theory and the non-classical continuum theory. Static deformations and resonant frequency of a microbeam were studied analytically considering a distributed axial loading applied on the microbeam. Rahaeifard et al. [5] investigated the static and dynamic behavior of a nonlinear Euler-Bernoulli beam made of functionally graded materials (FGMs) using the strain gradient theory. Equations and boundary conditions were established for a simply-supported microbeam. Static and free vibration analysis was carried out. The results obtained based on the nonlinear strain gradient theory were compared with the results of the linear strain gradient theory, linear and nonlinear modified couple stress theory, and linear and the non-linear classical models.

Kahrobaiyan et al. [6] studied the effect of size on the mechanical behavior of Euler-Bernoulli beams using the strain gradient theory and a non-classical theory. Free vibration and static behavior of a simply-supported microbeam were examined, and the results were compared with the results obtained based on the modified couple stress and the classical continuum theories. Asghari et al. [7] analyzed the effect of size in a Timoshenko beam made of FGMs using the modified couple stress theory. Asghari et al. [8] presented size-dependent Timoshenko beam on the basis of the couple stress theory. The results indicated that modeling on the basis of the couple stress theory causes more stiffness than modeling by the classical beam theory. Kahrobaiyan et al. [9] studied the nonlinear dependence of size in an Euler-Bernoulli beam based on the strain gradient theory and numerically investigated the size-dependent static bending in a simply-supported beam. These results were compared with those obtained by the linear strain gradient theory, linear and nonlinear modified couple stress theory, and linear and non-linear classical models. A model based on a modified couple stress theory for the free vibration and buckling analyses of beams was suggested by J.V.A Dos Santos et al. [10]. Kahrobaiyan et al. [11] modeled the sensitivity and resonance frequency of the AFM microcantilever using the modified couple stress theory. It was shown that when the ratio of cantilever thickness to material length scale parameter is less than 10, the difference between the sensitivity and resonance frequency values obtained using the classical theory and couple stress theory is considerable. Results suggested that the effect of size on the behavior of AFM microcantilever is significant. A micro scale Timoshenko beam model based on strain gradient elasticity theory was suggested by Binglei Wang et al. [12]. Asghari et al. [13] studied the effect of size in the Timoshenko microbeams. The modified couple stress theory was used to numerically analyze the static bending. Sahmani et al. [14] investigated size effect on dynamic stability of piezoelectrically actuated microbeams using strain gradient elasticity theory. Analysis of electrostatically and piezoelectrically excited micro-cantilever switches with considering curvature and piezoelectric nonlinearities effects was carried out by Bahrami et al. [15]. Vibrational analysis of micro-beam and Micro-cantilever using Homotopy Perturbation Method (HPM) was presented by

Moeenfard et al. [16]. Mojahedi et al. [17] was reported an analytical model to analyze nonlinear behavior of the micro-cantilever based gyroscope. Stability of micro-beams under various boundary conditions was analyzed by Yaylı [18] based on the strain gradient elasticity theory.

The aim of this study was to evaluate the effect of variation in Poisson's ratio on the maximum deflection and natural frequency of Timoshenko microbeams using the finite element method. Vibrational mode shapes of the microbeams in their natural frequencies were also simulated. Results of this research were compared with those obtained using the couple stress model, and the strain gradient elasticity and classical model theories. There was a good agreement between results of the present research and the results obtained based on other theories.

## 2. MATERIALS AND METHODS

### 2.1 FEM Analysis

Two epoxy Timoshenko microbeams were considered in order to simulate the effects of variation in Poisson's ratio on their mechanical behavior. Epoxy is a polymer used in microelectromechanical systems. Its mechanical properties such as elastic modulus, Poisson's ratio, and density are listed in Table 1 [19].

**Table 1. Material properties of the epoxy microbeam**

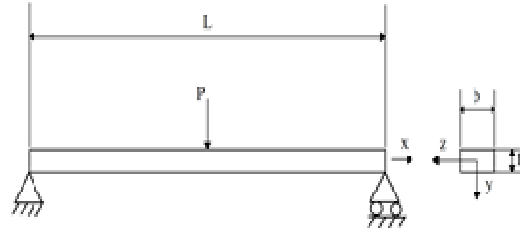
Young's modulus (GPa)	1.44
Poisson's ratio	0 - 0.4
Material density (kg/m <sup>3</sup> )	1.22

Structure, boundary conditions, support and loading conditions of the microbeams are shown in Fig. 1. As can be seen, the microbeam has a hinge support at one side and a roller support on the other side. A 100 μN load was applied at the midpoint of the microbeam.

Dimensions of the two microbeams are listed in Table 2. The thicknesses of the microbeams were 17.6 μm and 35.2 μm. The width and length of the microbeams were 2 and 20 times their thicknesses, respectively.

ABAQUS software was used for microbeams through the finite element method, in this research. Two Timoshenko microbeams with dimensions and mechanical properties listed in Tables 1 and 2 were modeled. All degrees of

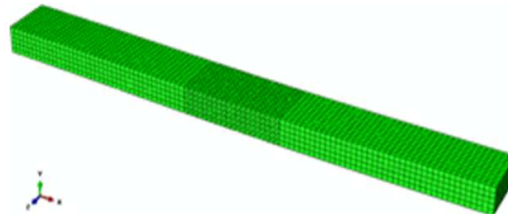
freedom, except for z-axis rotation, were closed for one end of the microbeam, while all degrees of freedom, except for z-axis rotation and x-axis motion, were closed for the other end. A 100 μN load was applied at the microbeam midpoint. The microbeams were then meshed using the elements with approximate dimensions of 3.5×3.5×3.5 μm. The meshed view of one of the microbeams is shown in Fig. 2.



**Fig. 1. Schematic diagram of the Timoshenko microbeam and its Boundary conditions**

**Table 2. Geometric configurations for the timoshenko microbeam**

Sets	1	2
Thickness, h (μm)	17.6	35.2
Width, b=2h (μm)	35.2	70.4
Length, L=20h (μm)	352	704

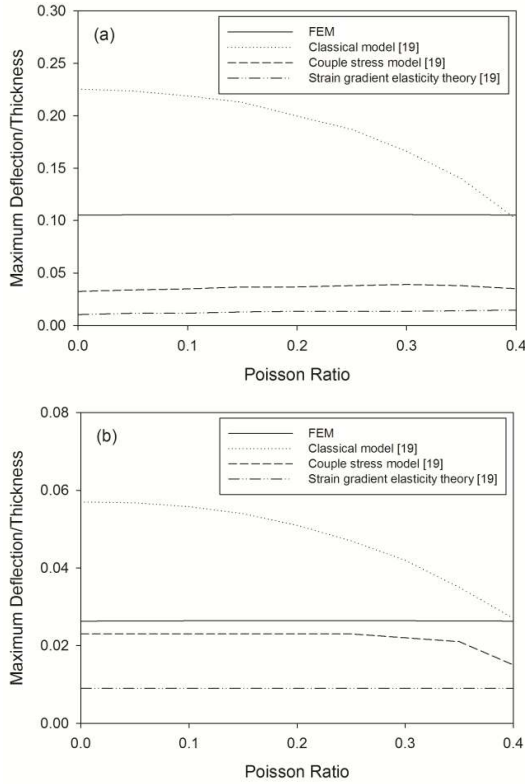


**Fig. 2. A meshed view of the timoshenko microbeam**

## 3. RESULTS AND DISCUSSION

After obtaining the results of finite element analysis, diagrams of the effect of Poisson's ratio on the resonant frequency and the maximum deflection were investigated for microbeams with different thicknesses. As can be seen in Fig. 3, finite element analysis showed that variation in Poisson's ratio did not significantly affect the maximum deflection to thickness ratio in the microbeams. This has been somehow observed in the results obtained for Timoshenko microbeam through the strain gradient elasticity theory and the couple stress model [19]. The analysis results for the Timoshenko microbeam

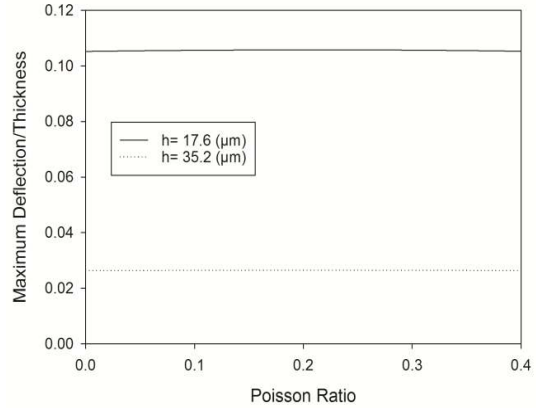
using the classical model has been indicated that increase in the Poisson's ratio reduces the deflection to thickness ratio. As evident in the diagrams, the analyses performed using the finite element method is consistent with the theoretical analysis of couple stress theory. The maximum deflection to thickness ratio obtained using the finite element method and classical model were approximately equal at the Poisson's ratio of 0.4.



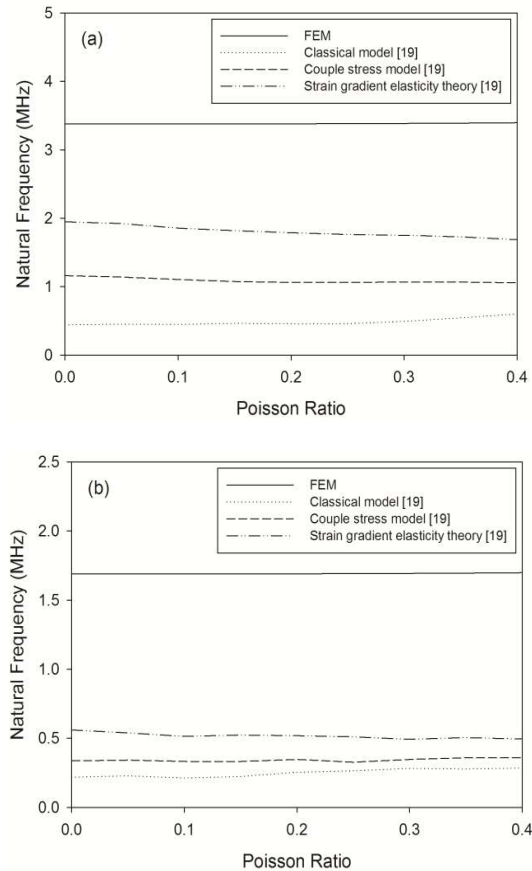
**Fig. 3. The effect of Poisson's ratio variation on the maximum deflection of the Timoshenko microbeam based on four different models: a)  $h=17.6 \mu\text{m}$ , b)  $h=35.2 \mu\text{m}$**

Diagram of the effect of Poisson's ratio variation on the maximum deflection to thickness ratio of the microbeams for the two studied thicknesses (17.6 and 35.2  $\mu\text{m}$ ) is shown in Fig. 4. As can be seen, the ratio is lower for greater thicknesses.

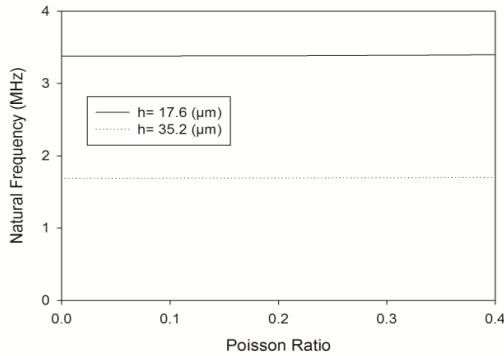
The effects of Poisson's ratio on the resonant frequencies of the microbeams with different thicknesses were also investigated. Fig. 5 shows the effect of Poisson's ratio on the resonant frequency of the microbeams with thicknesses of 17.6 and 35.2  $\mu\text{m}$ .



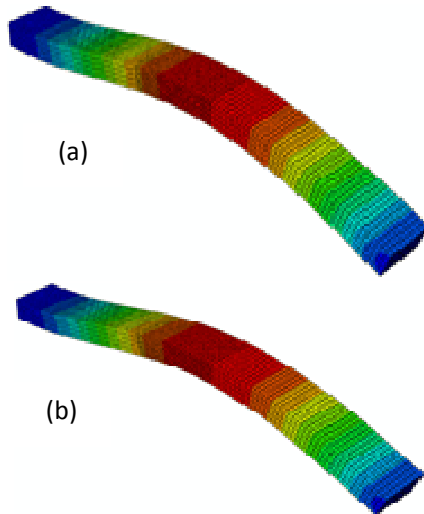
**Fig. 4. The effect of Poisson's ratio variation on the maximum deflection of the Timoshenko microbeam based on the FEM model with  $h=17.6 \mu\text{m}$ ,  $35.2 \mu\text{m}$**



**Fig. 5. The effect of Poisson's ratio variation on the natural frequency of the microbeam based on four different models: a)  $h=17.6 \mu\text{m}$ , b)  $h=35.2 \mu\text{m}$**



**Fig. 6. The effect of Poisson’s ratio variation on the natural frequency of the timoshenko microbeam based on FEM model with h=17.6 μm, 35.2 μm**



**Fig. 7. The first resonant shape mode of the timoshenko microbeams for the Poisson’s ratio of 0.1; a) h=17.6 (μm), b) h=35.2 (μm)**

Increase in Poisson’s ratio slightly decreases or does not affect the resonant frequency calculated by the couple stress model, strain gradient elasticity theory, and the classical model methods [19]. This study showed that the results of the FEM method are closer to those obtained by the strain gradient elasticity theory, in resonant frequencies analysis. Fig. 6 shows that an increased thickness decreases the resonance frequency in the two studied microbeams. A change in Poisson’s ratio in the two studied microbeams does not affect their resonance frequency.

The first resonant mode shapes of the microbeams were also investigated using the

finite element method (Fig. 7). Considering the unchanged resonance frequency for different Poisson’s ratios, the first resonant mode shapes of the microbeams are listed in Fig. 7. As can be seen, the maximum deflection in this frequency mode occurred in the microbeam midpoint. The general conclusion drawn from this study is that changes in the Poisson’s ratio of the microbeams do not change the deflection and resonant frequency values obtained through the FEM method. Therefore, factors that result in changes in the Poisson’s ratio do not affect the deflection and resonant frequency of the microbeams.

#### 4. CONCLUSION

In this study, two epoxy Timoshenko microbeams with known dimensions and geometries were modeled using the FEM method by taking into account all boundary conditions and initial conditions of the problem. The results of this study were compared with those obtained based on the couple stress, the strain gradient elasticity, and the classical model theories. This study aimed at investigating the effects of variation in Poisson’s ratio on the resonance frequency and maximum deflection of the microbeams. Modeling results showed that increase in Poisson’s ratio does not significantly change the resonant frequency and maximum deflection of the microbeams. Therefore, changes in environmental conditions that alter the Poisson’s ratio do not affect the desired output parameters. As a result, in the design of the microbeam based microelectromechanical systems where Poisson’s ratio is one of the system variables, the FEM analysis ensures that changes in the environmental conditions affecting Poisson’s ratio would not affect the system outputs (resonant frequency and deflection).

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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