



Global Stellar Budget for LIGO Black Holes

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Abstract

The binary black hole mergers observed by Laser Interferometer Gravitational-Wave Observatory (LIGO)–Virgo gravitational-wave detectors pose two major challenges: (i) how to produce these massive black holes from stellar processes; and (ii) how to bring them close enough to merge within the age of the universe? We derive a fundamental constraint relating the binary separation and the available stellar budget in the universe to produce the observed black hole mergers. We find that $\lesssim 14\%$ of the entire budget contributes to the observed merger rate of $(30+30) M_{\odot}$ black holes, if the separation is around the diameter of their progenitor stars. Furthermore, the upgraded LIGO detector and third-generation gravitational-wave detectors are not expected to find stellar-mass black hole mergers at high redshifts. From LIGO’s strong constraints on the mergers of black holes in the pair-instability mass gap ($60\text{--}120 M_{\odot}$), we find that $\lesssim 0.8\%$ of all massive stars contribute to a remnant black hole population in this gap. Our derived separation–budget constraint provides a robust framework for testing the formation scenarios of stellar binary black holes.

Unified Astronomy Thesaurus concepts: [Astrophysical black holes \(98\)](#); [Gravitational wave astronomy \(675\)](#)

1. Introduction

So far, the network of the Laser Interferometer Gravitational-Wave Observatory (LIGO; Aasi et al. 2015)–Virgo (Acernese et al. 2015) detectors have publicly announced over 40 binary black hole (BBH) events, of which 10 are confirmed detections (Abbott et al. 2019). From this list of 30 confirmed black holes (primary, secondary, remnant), 80% are much heavier than the black hole candidates found in X-ray binaries, thus suggesting a distinct evolutionary path for such binaries. Additionally, LIGO–Virgo has released a list of marginal triggers found in their different searches that may have an astrophysical origin but cannot be confirmed due to their relatively low detection statistics.

Among this rich gravitational-wave data set lie two cases that hint at a new population of black holes: (i) GW170729, whose primary black hole mass lies in the range $[40.4\text{--}67.2] M_{\odot}$ with 90% confidence (Chatziioannou et al. 2019; Kimball et al. 2019), and (ii) IMBHC-170502, the loudest marginal trigger published so far by LIGO–Virgo (Udall et al. 2019; The LIGO Scientific Collaboration & The Virgo Collaboration 2019a), which has a primary black hole mass in the range $[66\text{--}138] M_{\odot}$ with 90% confidence (The LIGO Scientific Collaboration & The Virgo Collaboration 2019a). Further, a new gravitational-wave trigger with a similar primary black hole mass to GW170729 has been claimed by an independent search (Zackay et al. 2019). While mass constraints of such candidates may be influenced by the choice of prior (Fishbach et al. 2019), persistent observations of such black holes may challenge the mass gap from the pair-instability supernovae ($60\text{--}120 M_{\odot}$; Woosley 2017).

Observations of these black holes have further complicated the question of how massive stars end up as tight BBH systems. A non-exhaustive list of proposed scenarios to tackle this includes mass exchange in binary stars (Belczynski et al. 2016; de Mink & Mandel 2016), multiple mergers of young stars (Di Carlo et al. 2019), dynamical segregation of black holes in star clusters (Rodríguez et al. 2016), binary formation within a

single rapidly rotating star (D’Orazio & Loeb 2018) and AGN disks (Bartos et al. 2017; Yang et al. 2019). While each scenario has a distinct observable signature (involving spin orientations, mass ratios) and predicted merger rate, they ought to be fundamentally constrained by the number of stars in the universe. We take this as a starting point and derive a global budget for BBH mergers. We show that regardless of the proposed scenario, the progenitor star budget already imposes a strict limit on the BBH separation.

2. Methodology

As shown diagrammatically in Figure 1, the maximum budget for producing mergers of BBHs from stellar evolution can be calculated as

$$\mathcal{B}_{\max} = \underbrace{n_s(M_s)}_{\text{Massive stars in binary}} \times \underbrace{\left(\frac{f_{\text{bs}}}{2}\right)}_{\text{Formation channel efficiency}} \times \underbrace{\epsilon_{\text{BBH}}}_{\text{Formation channel efficiency}} \times \underbrace{\psi(z_{\text{star-form}})}_{\text{Comoving star formation rate}} \times \underbrace{\left(\frac{1 + z_{\text{BBH-merge}}}{1 + z_{\text{star-form}}}\right)}_{\text{Comoving star formation rate}} \quad (1)$$

with three astrophysical constraints,

$$\mathcal{C}_1(M_s/M_{\odot} \leftrightarrow Z_s/Z_{\odot}) \ , \quad \mathcal{C}_2(z_{\text{star-form}} \leftrightarrow a_0) \ , \quad \mathcal{C}_3(\max\{a_0\} \leftrightarrow z_{\text{obs}}) \quad (2)$$

Mass of progenitor from metallicity Birth rate of progenitor from separation
 Separation from GW detection

Here,

$$n_s = N_{>M_s} / M_{\text{tot}} = \int_{M_s}^{M_{\max}} \xi(m) dm / \int_{0.01 M_{\odot}}^{1000 M_{\odot}} m \xi(m) dm \quad (3)$$

is the number of progenitor stars per unit stellar mass that will result as black holes. We utilize the piecewise initial stellar-

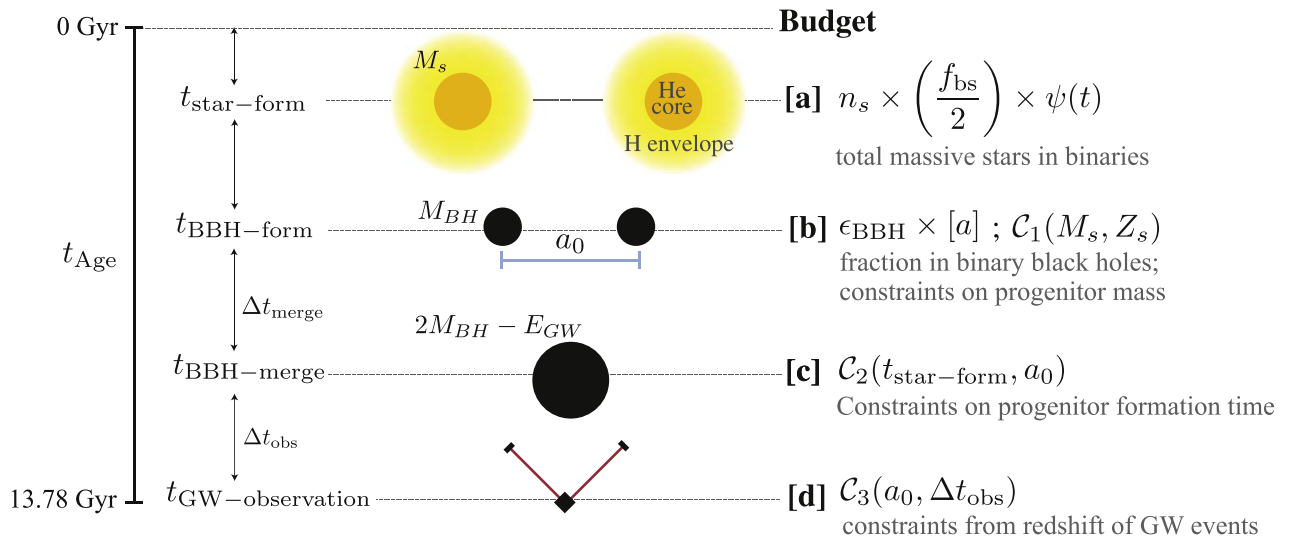


Figure 1. Diagram of stellar budget for merging black holes at different snapshots of time.

mass function $\xi(m)$ of Kroupa (2001) and include the numerical treatment that finds maximum star mass M_{max} , given a fundamental cutoff at $1000 M_{\odot}$ (Kroupa & Weidner 2005). The choice for progenitor mass M_s is determined by the desired black hole mass M_{BH} . We use results from the population synthesis code of Spera & Mapelli (2017) to find the mapping between the mass of progenitor star and remnant black hole (for simplicity, excluding the treatment of the pair-instability mass gap on M_{BH}). For the massive stars that produce LIGO black holes, this mapping can vary significantly with the assumed metallicity, Z/Z_{\odot} , of the progenitor stars that were born at epoch $t_{\text{star-form}}$. This dependence is related to the assumed BBH formation channel, and sets a relation, \mathcal{C}_1 , between metallicity and the number of massive stars.

The fraction of progenitor stars in a tight binary is set by the parameter $f_{\text{bs}} \in [0, 1]$. While 70% of O-stars within the Milky Way are essentially binaries, this fraction is lower for the ones that are tightly bounded with orbital period of days (Sana et al. 2012). Such binaries provide a favorable chance for LIGO black holes. Therefore, unless stated otherwise, we adopt $f_{\text{bs}} = 0.1$ throughout this study.

The efficiency for converting binaries of massive stars into gravitationally bound BBH sources is captured by $\epsilon_{\text{BBH}} \in [0, 1]$. This free parameter solely depends on the assumed formation channel. From the asymmetric collapse, the remnant formed as black hole can get a natal kick (Hoogerwerf et al. 2001), thus decreasing the fraction (ϵ_{BBH}) of BBHs from two progenitor massive stars. If LIGO’s BBHs are formed through dynamical capture (Rodríguez et al. 2016), then a_0 could be the separation at the last encounter (provided there is no third-body interaction), and ϵ_{BBH} will be the fraction of black holes that will find such a coalescing partner.

From the LIGO observations, we cannot infer the initial separation a_0 at $t_{\text{BBH-form}}$, the instance when the two black holes become gravitationally bound. Therefore, in this study, we adopt a delta function for a given choice of the initial BBH separation a_0 (au). This simple assumption allows us to provide an upper limit on the maximum a_0 , regardless of an underlying astrophysical distribution for binary separation.

Assuming a circular orbit and absence of any external influence, the separation a_0 sets a bound on Δt_{merge}

(Peters 1964). As the time from the birth to collapse, $\Delta t_{\text{col}} \ll \Delta t_{\text{merge}}$, we can assume that the progenitor stars were formed at $t_{\text{star-form}} \sim t_{\text{BBH-form}}$. The number of progenitor stars in the universe at this instance can be found through stellar formation rate, $\psi(z)$ (see Equation (16) of Madau & Dickinson 2014). This sets the second constraint, \mathcal{C}_2 , which relates a_0 with the production of progenitor stars. As we are using cosmic star formation rate at an earlier epoch, $(1 + z_{\text{BBH-form}})$, to calculate the LIGO event rate at $(1 + z_{\text{BBH-merge}})$ there will be a time lag for the corresponding ψ . Furthermore, the observed redshift $z_{\text{BBH-merge}}$ of LIGO detections provides a constraint, \mathcal{C}_3 , on the maximum value of a_0 such that the merger time, Δt_{merge} , is less than the Hubble time at that redshift.

3. Results and Discussion

Based on Equations (1)–(3), we can compute the fraction of stellar budget that is being utilized for \mathcal{B}_{obs} —the observed merger on BBH from LIGO. If $\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}} > 1$, then there are not enough stars in the universe to produce these merging black holes. This global budget calculation allows us to put fundamental limits on a_0 , regardless of the mechanism that brings them close enough. Figure 2 shows the fraction of the total allowed budget being utilized for four distinct populations of BBH mergers that are strongly constrained by LIGO observations.

Stellar-mass BBHs: To produce black holes $10\text{--}30 M_{\odot}$, the typical mass of the progenitor star ranges from 25 to $35 M_{\odot}$. For metallicities $\lesssim 10^{-3} Z_{\odot}$, this mass range remains fairly unaffected. Thus, the constraint \mathcal{C}_1 can be ignored for the simple BBH events detected in LIGO. We take the upper limit $\mathcal{B}_{\text{obs}} = 111.7 \text{ Gpc}^{-3} \text{ yr}^{-1}$ across the stellar BBH mass range $\lesssim 100 M_{\odot}$ (The LIGO Scientific Collaboration & The Virgo Collaboration 2019b).

For a $(30 + 30) M_{\odot}$ source (such as GW150914), we find the stellar budget sets stringent constraints on binary separation. Adopting $a_0 = 0.2 \text{ au}$ can max out the entire stellar budget (see the top right panel of Figure 2). If these BBHs are separated by the diameter of one of their progenitor stars of $35 M_{\odot}$ (Demircan & Kahraman 1991), which corresponds to $\sim 10^5$ Schwarzschild radii (r_s), the progenitor star budget

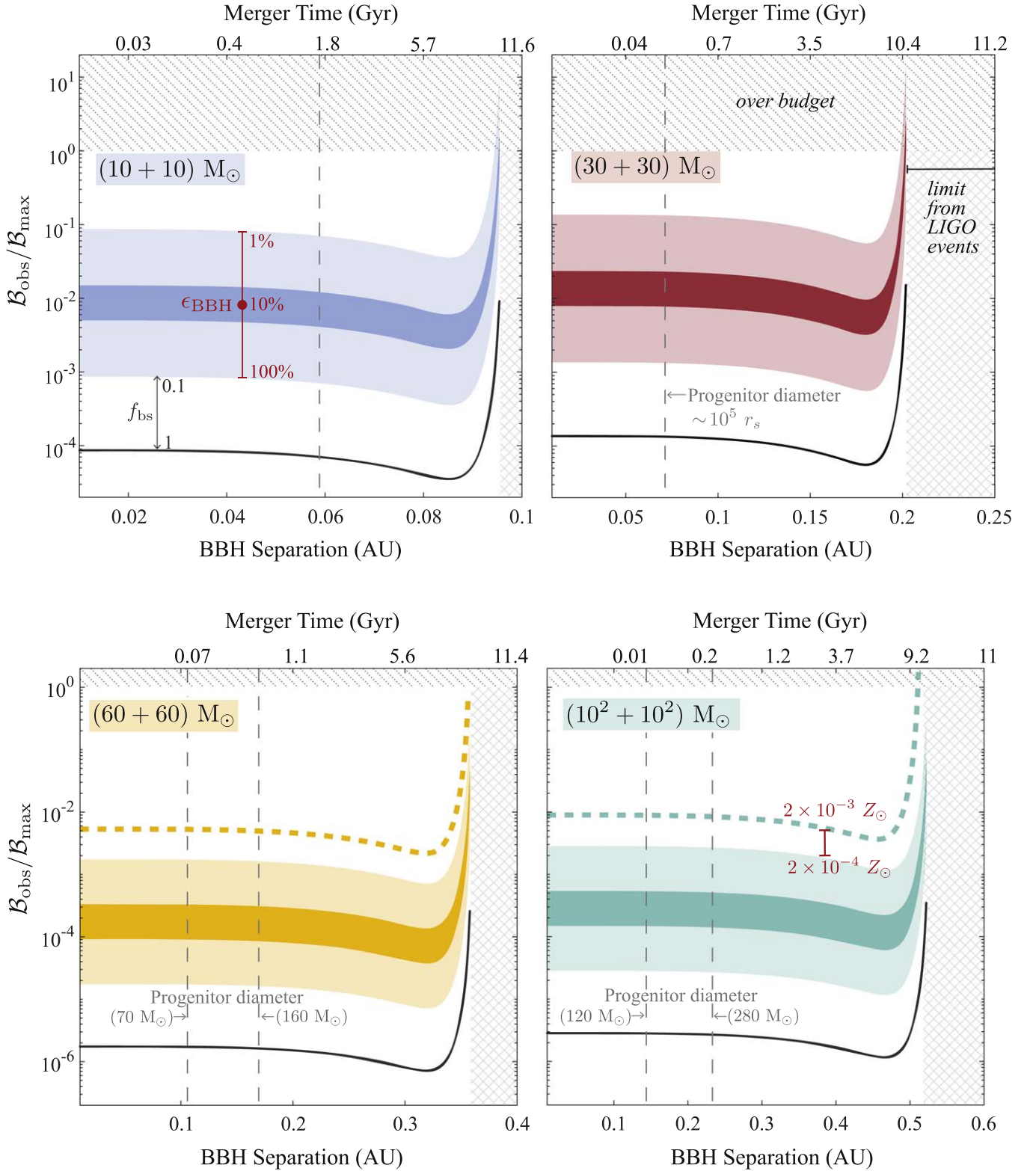


Figure 2. Progenitor star budget for different LIGO–Virgo BBHs as a function of initial separation a_0 . The vertical axis shows the observed fraction $\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}}$ of progenitor remnants. Top panel: stellar BBHs currently observed by LIGO. Bottom panel: binaries of black holes in the pair-instability mass gap. The colored patches refer to the assumed stars-to-BBH efficiency parameter, ϵ_{BBH} . The thick line in their center corresponds to $\epsilon_{\text{BBH}} = 0.1$, i.e., 10% of all massive stars turn into LIGO black holes. The black curve in each case refers to the theoretical maximum ($\epsilon_{\text{BBH}} = 1$; $f_{\text{bs}} = 1$). The thick dotted line in the bottom panel shows the effect of lower metallicity. The shaded area at the top of each subplot highlights the over budget ($\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}} > 1$). While the shaded area on the right shows the constraint a_0 , which will result in merger time equal to the Hubble time at an average redshift of LIGO observations.

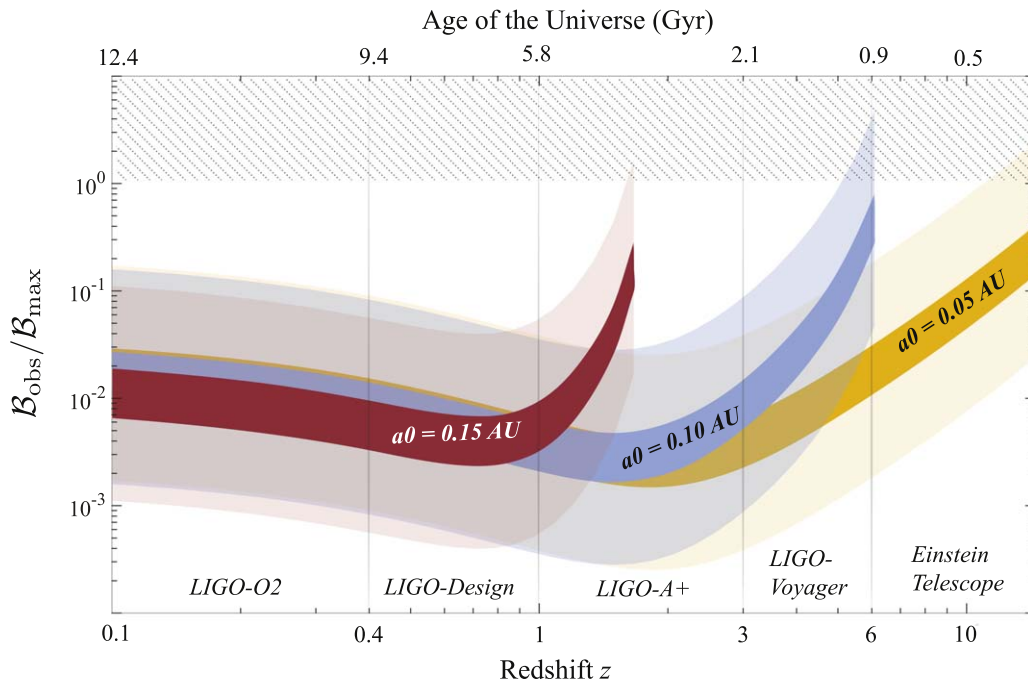


Figure 3. Progenitor star budget for $(30 + 30) M_{\odot}$ events as a function of the maximum observed redshift and the corresponding age of the universe (upper horizontal axis label). The vertical axis $\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}}$ with the colored patches refer to three values of binary separation a_0 . Like Figure 2, their width refers to the assumed efficiency parameter ϵ_{BBH} . The thick line in their center is for $\epsilon_{\text{BBH}} = 0.1$. The hashed area near the top highlights the over budget ($\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}} > 1$). The redshift has been highlighted for different epochs of gravitational-wave experiments based on their sensitivity for a $(30 + 30) M_{\odot}$ (Jani et al. 2019).

utilized would be $\lesssim 14\%$ (at low efficiency, $\epsilon_{\text{BBH}} = 0.01$). The budget remains fairly constant for any lower separation from the progenitor’s diameter.

Furthermore, we find a tipping point in a_0 , which assures the lowest stellar budget is being utilized. This separation corresponds to $t_{\text{BBH-form}}$ at the peak of star formation. A small increase in a_0 from this tipping point leads to an almost exponential rise in the budget, eventually maxing out ($\mathcal{B}_{\text{obs}}/\mathcal{B}_{\text{max}} > 1$). The merger time at such a separation, which is close to the Hubble time, is competing with the decrease in star formation rate at the high redshift. For $(30 + 30) M_{\odot}$ binary black hole mergers in LIGO, this tipping point is found at $a_0 = 0.18$ au.

For a $(10 + 10) M_{\odot}$ source (such as GW151226), we find that a binary separation of just $a_0 = 0.094$ au can max out the entire progenitor star budget. If these BBHs are separated by the diameter of one of their $25 M_{\odot}$ progenitor stars, the budget utilized would be $\lesssim 7\%$ (at low efficiency, $\epsilon_{\text{BBH}} = 0.01$). The tipping point for these BBH masses happen at $a_0 = 0.085$ au.

Pair-Instability BBHs: From the detection threshold set by the trigger 170502, The LIGO Scientific Collaboration & The Virgo Collaboration (2019a) provided upper limits on the population of merging black holes in the pair-instability gap. From their published list, we take two cases $(60 + 60) M_{\odot}$ and $(100 + 100) M_{\odot}$, which have $\mathcal{B}_{\text{obs}} = 0.56$ and $0.44 \text{ Gpc}^{-3} \text{ yr}^{-1}$ respectively. Because their progenitor mass can vary significantly with the assumed metallicity, the constraint C_1 becomes important in determining their budget.

For these black holes, we note an interesting result—at no binary separation they tend to exceed the stellar budget (see the bottom panel of Figure 2). When separated by the diameter of one of their progenitor stars, $(100 + 100) M_{\odot}$ would at most utilize a 0.8% (0.3%) budget for a metallicity of $2 \times 10^{-3} Z_{\odot}$ ($2 \times 10^{-4} Z_{\odot}$). The maximum budget is spent when

$a_0 = 0.52$ au. At this separation, high metallicity of progenitor stars may just max out the budget, but it is unlikely for stars that early in the universe. The tipping point for these BBH masses happens at $a_0 = 0.46$ au. In the case of the $(60 + 60) M_{\odot}$ BBH merger, the budget utilized would be 0.2% (for $2 \times 10^{-3} Z_{\odot}$). The maximum budget for such BBHs is spent when $a_0 = 0.36$ au. From this analysis, we conclude that more confident detections of pair-instability black holes by LIGO would likely correspond to rare progenitors whose population is at least an order of magnitude less than the entire progenitor population. This constraint is consistent with the expectation of no BBH in this mass range based on stellar evolution (Woosley 2017).

Future Detectors: As the sensitivity improves, LIGO-like detectors can detect BBH mergers from earlier cosmic times (The LIGO Scientific Collaboration & The Virgo Collaboration 2013). For mergers of $(30 + 30) M_{\odot}$, the upgraded versions of LIGO (mid-2020s) would permit detection up to redshift $z \sim 6$, while third-generation detectors in 2030s such as the Einstein Telescope (Punturo et al. 2010) and Cosmic Explorer (Reitze et al. 2019) can find these events up to $z \lesssim 40$ (Jani et al. 2019). If mergers are found at high redshift, the constraint C_3 on maximum a_0 becomes even more stringent. Figure 3 shows the budget that would be utilized for different choices of a_0 as a function of detection redshift. If BBHs are separated initially at 0.1 au (0.15 au), then their detection by redshift 5 (1.5) would max out the entire budget. We can start constraining ϵ_{BBH} if the current LIGO facility sees mergers at $z \gtrsim 1$. The next-generation detectors would constraint a_0 to the lowest realistic values.

Conclusion: We derived global constraints on binary black hole separation and the stellar budget of their progenitor stars. In particular, we find that at most 14% of the available cosmic budget contributes to the observed merger rate of

$(30 + 30) M_{\odot}$ black holes. For the black hole mergers accessible to LIGO, we find a general trend, where up to a tipping point a_0 dictated by their mass, the progenitor star budget remains fairly constant, and thereafter gets maxed out exponentially. Our results indicate that observations beyond redshift $\gtrsim 1$ of $(30 + 30) M_{\odot}$ BBHs—the most common source for current ground-based gravitational-wave astronomy—can strongly constrain the efficiency (ϵ_{BBH}) of converting stars into merging black holes. These global considerations provides a model-independent framework for testing all possible formation channels for BBHs.

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