



# **Non-linear Stability in Transitional and Rotatory Regime of the Cooking of Gari, Food Based on Cassava Tubers**

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## **Authors' contributions**

*This work was done in collaboration among all authors. Author VEC designed the study, carried out the statistical analysis, drafted the protocol and drafted the first draft of the manuscript. Authors DAA and SGC managed analyzes of the study, judged the relevance of the study carried out and verified all the established equations. Author DG managed the documentary research. All authors read and approved the final manuscript.*

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## **ABSTRACT**

The present study focuses on the non-linear stability in the transient rotary regime of the cooking process of gari. The process of cooking gari consists of a rotating rectangular cavity filled with grated cassava flour, pressed, retted and considered to be an anisotropic porous medium in a permeably saturated viscoelastic fluid. The cavity is heated from below to a constant temperature. The lower wall of the cavity is impermeable and the upper wall is permeable. Using a numerical method, we have established the transient expressions of the Nusselt number, the flow and temperature fields as a function of the anisotropy parameters of the porous medium and of the Taylor number. The results obtained showed that the anisotropy of the porous medium and the Taylor number greatly influenced the cooking of gari over time.

**Keywords:** *Anisotropic porous rotating medium; nonlinear instability; transient regime; Nusselt number.*

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## NOMENCLATURES

$\mu$ : dynamic viscosity of the fluid saturating the porous medium;  
 $\nu = \mu/\rho_0$  : kinematic viscosity of the fluid;  
 $\vec{g}$  : Gravity field vector;  
 $p'$  : Saturation vapor pressure;  
 $\vec{\Omega} (0, 0, \Omega)$  : Vector speed of rotation of the enclosure;  
 $\vec{r}'$  : Position vector;  
 $\vec{K}$  : Third order permeability tensor;  
 $\sigma$  : Thermal capacity ratio;  
 $\alpha$  : thermal diffusivity of the porous medium;  
 $\vec{V}$  : Vector fluid filtration speed in the porous medium;  
 $\Delta T = T_H - T_C$  : Temperature difference between the two surfaces;  
 $u'$ ;  $v'$  et  $w'$  : Respective components of the speed vector along the axes  $(o, x)$ ,  $(o, y)$  and  $(o, z)$  ;  
 $k_y$  and  $k_z$  : Numbers of waves describing the periodicity of the disturbance in the  $y$  and  $z$  directions respectively;  
 $k_y^2 + k_z^2 = k^2$  : Wave number;  
 $\delta$  : porosity;  
 $\varphi$  : Orientation angle of the main directions of the permeability tensor;  
 $K_1, K_2$  and  $K_3$  : Permeabilities along the main directions;  
 $K^* = K_1/K_2$  : Anisotropy ratio of permeability in the horizontal plane;  
 $\xi = K_1/K_3$  : Anisotropy ratio of permeability in the vertical direction;  
 $\rho$  and  $\rho_0$  : the densities of the fluid respectively at temperatures  $T$  and  $T_0$ ;  
 $\hat{e}_x$ ;  $\hat{e}_y$  and  $\hat{e}_z$  : Unit vectors in the principal directions;  
 $\lambda_2$  : the delay time constant;  
 $\lambda_1$  : the expansion or release time constant;  
 $Y$ : thermal capacity ratio;  $(\rho c_p)_m/(\rho c_p)_f$ .

## 1. INTRODUCTION

The present study focuses on the non-linear stability in the transient rotary regime of the cooking process of gari. The process of cooking gari consists of a rotating rectangular cavity filled with grated cassava flour, pressed, retted and considered to be an anisotropic porous medium in a permeably saturated viscoelastic fluid. The study of viscoelastic fluids is of great interest in many areas of modern engineering sciences and technology such as materials processing, petroleum, chemistry and nuclear industries, geophysics, biology and of bio-mechanical engineering.

Vadasz P [1]. Conducted three-dimensional analytical research on the flow of a fluid through a heterogeneous porous medium confined in a rotating rectangular cavity. The permeability of the porous medium varies in the vertical direction. The results of his work showed that for a pressure gradient applied to the lateral faces, there appears a main flow of the fluid in the horizontal direction. The analytical solution found remains valid for large numbers of Ekman (Ek),

which confirms the conditions of practical applications.

The same author also sought an analytical solution to the problem of natural convection generated by centrifugal force in a rotating porous medium heated from above. For having assumed the vertical component of the flow velocity and the temperature independence of the horizontal coordinate, he found that the domain of validity of this analytical solution must be restricted. The Nusselt number varies linearly with the modified Rayleigh number  $Ra_w$  for low values of the latter. It appears that apart from the heat flow associated with the flow of the fluid in the horizontal direction which remains important, there is a flow of heat in the vertical direction.

Enock P. and Tyvand A. [2] on the basis of the Darcy-Boussinesq equations studied a porous medium in rotation and in a steady state. They have shown that this problem is equivalent to that of the anisotropic porous medium with respect to the parameter of mechanical anisotropy, characterizing the permeability ratio of the medium. According to the results they

obtained, one can deduce basic results on thermal convection in a rotating porous layer from the analysis made on thermal convection in an anisotropic porous medium not set in rotational motion.

Jong J.J. and Jian S. L. [3], made a study on thermal convection in transient regime in a porous layer whose free upper and lower surfaces, initially at the same temperature, are subjected to constant heating from below. They were also interested in the analytical and numerical study of the criteria for the appearance in permanent and transient conditions of two-dimensional thermal convection in a rotating porous medium. The anisotropic porous medium in permeability is such that its upper and lower limit boundaries are rigid. The lower rigid wall is heated at a constant rate so as to generate a linear distribution of temperature in the vertical direction. The instability related to the anisotropy in permeability of the porous medium, saturated with a fluid was analyzed by the technique of calculating the mean flow volume. They determined the critical Rayleigh numbers  $R_c$  and the critical wave numbers  $a_c$  for the occurrence of convection in the anisotropic medium.

Govender S. [4] studied natural convection in a rotating anisotropic porous layer subjected to centrifugal force. He used Darcy's equation to describe the flow and found that convection is stabilized as the ratio of thermal parameters and mechanical anisotropies increases in amplitude.

Nield and Bejan [5] and Bejan [6] have established comprehensive reviews of the fundamentals of heat convection in porous media.

Dègan G. [7] conducted a numerical and analytical investigation of natural convection in a rectangular cavity confined by an anisotropic porous medium in permeability and isothermally

heated from the sides. The results showed that the permeability anisotropy ratio and the inclination angle of the major axes both had a great influence on the system. In particular, the maximum (minimum) heat transfer is obtained when the orientation of the main axis of the anisotropic porous medium having the permeability is parallel (perpendicular) to the gravitational field.

This review shows the authors did not address the effect of anisotropy in all directions. Also, they did not discuss the effects of the anisotropy ratio in permeability in the horizontal and vertical directions.

This study is part of the exploration of the effects of the orientation angle of the principal directions of permeability in the horizontal plane containing the porous medium by varying the angle of orientation. Also, the effects of the permeability anisotropy ratio and the effects of mechanical anisotropy on the transient behavior of the Nusselt number, flow and temperature fields over time are investigated.

The application of the present study is to perform a cooking modeling of gari based on the influence of anisotropy on the permeability of flour retted in the horizontal and vertical directions.

## 2. MATERIALS AND METHODS

### 2.1 Description of the Physical Model

The physical model considered in Fig. 1 is that of a parallelepipedal enclosure with flat walls. The lower horizontal wall is the one symbolizing the pan which is heated from below to a constant temperature  $T_H$  while the upper face is at the constant temperature  $T_C$ , such as ( $T_H > T_C$ ).

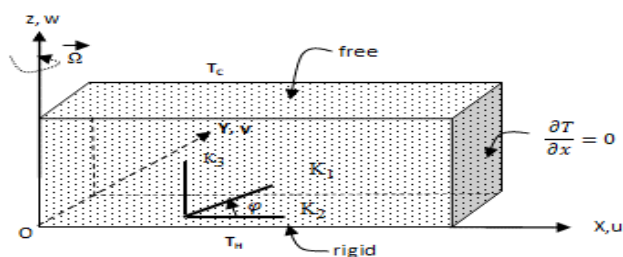


Fig. 1. Physical model: Rectangular cavity in rotation and coordinate axes, containing the anisotropic porous medium

The cassava paste contained in the enclosure constitutes a porous medium saturated with starch which can be assimilated to a viscoelastic fluid.

The porous medium-enclosure system is subjected to a sustained rotational movement, of constant frequency  $N$ .

The porous medium is anisotropic in permeability, the directions of which are oriented obliquely with respect to the vertical axis at an angle  $\varphi$ .

From initiation, the anisotropic porous medium is the site of unsteady thermo-convective phenomena that we will study.

## 2.2 Governing Equations

The equations governing our system are written:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left[ \frac{\rho_o}{\delta} \frac{\partial \vec{V}}{\partial t} + \frac{2\rho_o}{\delta} (\vec{\Omega} \wedge \vec{V}) \right] + \mu(\bar{K})^{-1} \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \vec{V} = \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) (-\vec{\nabla}p + \rho \vec{g}) \tag{2}$$

$$\left(\gamma \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla}\right) T = k \nabla^2 T \tag{3}$$

$$\text{With } \bar{K} = \begin{bmatrix} K_1 \sin^2 \varphi + K_2 \cos^2 \varphi & (K_2 - K_1) \sin \varphi \cos \varphi & 0 \\ (K_2 - K_1) \sin \varphi \cos \varphi & K_1 \cos^2 \varphi + K_2 \sin^2 \varphi & 0 \\ 0 & 0 & K_3 \end{bmatrix}$$

## 2.3 The Equations in the Disturbance State

At the start of cooking (basic or resting state) characterized by pure conduction, we have:

$$\left. \begin{aligned} u_b = v_b = w_b = 0 \\ T = T_b(z) \\ P = P_b(z) \end{aligned} \right\} \tag{4}$$

The initial state of equations (2) and (3) are :

$$\left. \begin{aligned} \frac{d^2 T_b}{dz^2} = 0 \\ \frac{dp_b}{dz} = -\rho_b g \\ \text{where } T_b = T_H - \frac{\Delta T}{H} z \end{aligned} \right\} \tag{5}$$

The experiment of the stability of the thermo-convective phenomenon consists in disturbing the basic solution and in observing under which conditions the imposed disturbance increases in amplitude [8]. Thus, by substituting the system of equations below (6) in equations (1), (2) and (3), we obtain the following equations in the disturbed state:

$$\begin{aligned}
 T(x, y, z, t) &= T_b(z) + T'(x', y', z', t) \\
 v(x, y, z, t) &= 0 + v'(x', y', z', t) \\
 u(x, y, z, t) &= 0 + u'(x', y', z', t) \\
 w(x, y, z, t) &= 0 + w'(x', y', z', t) \\
 p(x, y, z) &= \underbrace{p_b(z)} + \underbrace{p'(x', y', z')}
 \end{aligned} \tag{6}$$

Transient base disturbance

$$\begin{aligned}
 \vec{\nabla} \cdot \vec{V}' \\
 = 0
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 \left(1 + \lambda_1 \frac{\partial}{\partial t'}\right) \left[ \frac{\rho_o}{\delta} \frac{\partial \vec{V}'}{\partial t'} + \frac{2\rho_o}{\delta} (\vec{\Omega} \wedge \vec{V}') \right] + \mu(\bar{K})^{-1} \left(1 + \lambda_2 \frac{\partial}{\partial t'}\right) \vec{V}' \\
 = \left(1 + \lambda_1 \frac{\partial}{\partial t'}\right) (-\vec{\nabla} p' + \rho_o \beta T' \vec{g})
 \end{aligned} \tag{8}$$

$$\left(\gamma \frac{\partial}{\partial t'} + \vec{V}' \cdot \vec{\nabla}\right) T' + w' \frac{dT_b}{dz} = k \nabla^2 T' \tag{9}$$

## 2.4 A Dimensionalisation of the Equations

By introducing the following dimensionless variables:

$$\left. \begin{aligned}
 (x^*, y^*, z^*) &= \frac{(x', y', z')}{d}; t^* = \frac{k}{d^2} t' \\
 (u^*, v^*, w^*) &= (u', v', w') \frac{d}{k}; T^* = \frac{T'}{\Delta T} \\
 P^* &= \frac{K_3}{\mu k} P'; \lambda_1^* = \frac{k}{d^2} \lambda_1; \lambda_2^* = \frac{k}{d^2} \lambda_2
 \end{aligned} \right\} \tag{10}$$

in equations (6), (7) and (8), we get the following:

$$\vec{\nabla} \cdot \vec{V}^* = 0 \tag{11}$$

$$\begin{aligned}
 \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \left[ \frac{1}{Pr_D} \frac{\partial \vec{V}^*}{\partial t^*} + \sqrt{Ta} (\hat{e}_z \wedge \vec{V}^*) \right] + K_3(\bar{K})^{-1} \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \vec{V}^* \\
 = - \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) (\vec{\nabla} P^* - Ra T^* \hat{e}_z)
 \end{aligned} \tag{12}$$

$$\left(\gamma \frac{\partial}{\partial t^*} + \vec{V}^* \cdot \vec{\nabla}\right) T^* - w^* = \nabla^2 T^* \tag{13}$$

With

$$(\bar{K})^{-1} = \frac{1}{K_3} \begin{bmatrix} \frac{a}{\xi} & -\frac{c}{\xi} & 0 \\ -\frac{c}{\xi} & \frac{b}{\xi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

From the above,  $K^* = K_1/K_2$  denotes the anisotropy ratio of permeability in the horizontal plane,  $\xi = K_1/K_3$  denotes the anisotropy ratio in the vertical direction,  $Da = K_3/d^2$  denotes the Darcy number,  $Ta = (2\Omega K_3/\delta v)^2$  denotes the Taylor number,  $Ra = K_3 g \beta \Delta T d / \nu k$  denotes the

Rayleigh number,  $Pr = d/k$  denotes the Prandtl number and  $Pr_D = \delta Pr/Da$  indicate the number of Vadasz.

By eliminating the pressure terms in equation (12) through rotating this equation, we obtain the system of equations (14):

$$\left\{ \begin{array}{l} \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \left[ \frac{1}{Pr_D} \frac{\partial \omega_{x^*}}{\partial t^*} - \sqrt{Ta} \frac{\partial u^*}{\partial z^*} \right] + \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \left( \frac{\partial w^*}{\partial y^*} + \frac{c}{\xi} \frac{\partial u^*}{\partial z^*} - \frac{b}{\xi} \frac{\partial v^*}{\partial z^*} \right) = \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) Ra \frac{\partial T^*}{\partial y^*} \\ \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \left[ \frac{1}{Pr_D} \frac{\partial \omega_{y^*}}{\partial t^*} - \sqrt{Ta} \frac{\partial v^*}{\partial z^*} \right] - \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \left( \frac{\partial w^*}{\partial x^*} - \frac{a}{\xi} \frac{\partial u^*}{\partial z^*} + \frac{c}{\xi} \frac{\partial v^*}{\partial z^*} \right) = - \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) Ra \frac{\partial T^*}{\partial x^*} \\ \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \left[ \frac{1}{Pr_D} \frac{\partial \omega_{z^*}}{\partial t^*} - \sqrt{Ta} \frac{\partial w^*}{\partial z^*} \right] + \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \left( \frac{b}{\xi} \frac{\partial v^*}{\partial x^*} - \frac{c}{\xi} \frac{\partial u^*}{\partial x^*} - \frac{a}{\xi} \frac{\partial u^*}{\partial y^*} + \frac{c}{\xi} \frac{\partial v^*}{\partial y^*} \right) = 0 \end{array} \right. \quad (15)$$

Equations (16), (17) and (18) becomes:

$$\left[ \frac{c}{\xi} \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) - \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \sqrt{Ta} \right] \frac{\partial^2 \psi^*}{\partial z^{*2}} - \frac{b}{\xi} \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \frac{\partial v^*}{\partial z^*} = 0 \quad (19)$$

$$\left[ \frac{c}{\xi} \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) + \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) \sqrt{Ta} \right] \frac{\partial v^*}{\partial z^*} - \left(1 + \lambda_2^* \frac{\partial}{\partial t^*}\right) \left[ \frac{\partial^2}{\partial x^{*2}} + \frac{a}{\xi} \frac{\partial^2}{\partial z^{*2}} \right] \psi^* - \left(1 + \lambda_1^* \frac{\partial}{\partial t^*}\right) Ra \frac{\partial T^*}{\partial x^*} = 0 \quad (20)$$

$$\frac{\partial T^*}{\partial t^*} + \left[ \frac{\partial \psi^*}{\partial z^*} \frac{\partial T^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial T^*}{\partial z^*} \right] + \frac{\partial \psi^*}{\partial x^*} = \left( \frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial y^{*2}} + \frac{\partial^2}{\partial z^{*2}} \right) T^* \quad (21)$$

## 2.5 Analysis of Non-Linear Stability in Transitional Regime

### 2.5.1 The transient equations

Considering the minimum Fourier series expressed in the directions where thermoconvective motion is preponderant, the current function  $\psi^*$ , the temperature fields  $T^*$  and the component  $v^*$  of the speed are defined by [9]:

$$\left\{ \begin{array}{l} \psi^* = A_1(t^*) \sin(kx^*) \sin(\pi z^*) \\ T^* = B_1(t^*) \cos(kx^*) \sin(\pi z^*) + C_1(t^*) \sin(2\pi z^*) \\ v^* = D_1(t^*) \sin(kx^*) \cos(\pi z^*) + E_1(t^*) \sin(2\pi x^*) \end{array} \right. \quad (22)$$

where  $A_1(t^*)$ ,  $B_1(t^*)$ ,  $C_1(t^*)$  and  $D_1(t^*)$  are the amplitudes of these series and depend on time. By introducing the system of equations (21) into equations (18), (19) and (20), we obtain the following equations (22), (23) and (24):

$$-\pi^2 \left[ \frac{c}{\xi} \left( A_1(t^*) + \lambda_2^* \frac{dA_1(t^*)}{dt^*} \right) + \left( A_1(t^*) + \lambda_1^* \frac{dA_1(t^*)}{dt^*} \right) \sqrt{Ta} \right] + \frac{b}{\xi} \left( D_1(t^*) + \lambda_2^* \frac{dD_1(t^*)}{dt^*} \right) = 0 \quad (23)$$

$$-\pi \left[ \frac{c}{\xi} \left( D_1(t^*) + \lambda_2^* \frac{dD_1(t^*)}{dt^*} \right) + \left( D_1(t^*) + \lambda_1^* \frac{dD_1(t^*)}{dt^*} \right) \sqrt{Ta} \right] + \frac{b}{\xi} \left( A_1(t^*) + \lambda_2^* \frac{dA_1(t^*)}{dt^*} \right) \left( k^2 + \frac{a}{\xi} \pi^2 \right) + \left( B_1(t^*) + \lambda_1^* \frac{dB_1(t^*)}{dt^*} \right) Ra \cdot k = 0 \quad (24)$$

$$\begin{aligned} & \left[ \frac{dB_1(t^*)}{dt} + B_1(t^*)(\eta k^2 + \pi^2) + kA_1(t^*) + 2A_1(t^*) \cdot C_1(t^*) \cdot k\pi \right] \cos kx^* \\ & \quad - 4A_1(t^*) \cdot C_1(t^*) k\pi \cos^2 \pi z^* \cos kx^* \\ & \quad + \left[ 2 \frac{dC_1(t^*)}{dt^*} + 8\pi^2 C_1(t^*) - A_1(t^*) \cdot B_1(t^*) k\pi \right] \cos \pi z \\ & = 0 \end{aligned} \tag{25}$$

From equations (23), (24) and (25) we obtain the following equations:

$$\begin{aligned} \frac{dA_1(t^*)}{dt^*} = J & \left\{ \left[ \pi^2 \left( \frac{c}{\xi} - \sqrt{Ta} \right) (\lambda_2^* c + \xi \lambda_1^* \sqrt{Ta}) - b\lambda_2^* \left( k^2 + \frac{a}{\xi} \pi^2 \right) + Ra \cdot b \cdot k^2 \cdot \lambda_1^* \cdot \lambda_2^* \right] A_1(t^*) \right. \\ & \quad - Ra \cdot b \cdot k \cdot \lambda_2^* [1 - \lambda_1^* (k^2 + \pi^2)] B_1(t^*) + 2\pi Ra \cdot b k^2 \cdot \lambda_1^* \cdot \lambda_2^* \cdot A_1(t^*) \cdot C_1(t^*) \\ & \quad + \pi b \left[ \lambda_2^* \left( \frac{c}{\xi} + \sqrt{Ta} \right) \right. \\ & \quad \left. \left. - \left( \lambda_2^* \frac{c}{\xi} + \lambda_1^* \sqrt{Ta} \right) \right] D_1(t^*) \right\} \end{aligned} \tag{26}$$

$$\begin{aligned} & \frac{dB_1(t^*)}{dt^*} \\ & = -kA_1(t^*) - (k^2 + \pi^2)B_1(t^*) \\ & \quad - 2k\pi A_1(t^*) \cdot C_1(t^*) \end{aligned} \tag{27}$$

$$\begin{aligned} & \frac{dC_1(t^*)}{dt^*} \\ & = \frac{k}{2} \pi A_1(t^*) B_1(t^*) \\ & \quad - 4\pi^2 C_1(t^*) \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{dD_1(t^*)}{dt^*} = \frac{\pi}{b\lambda_2^*} & \left( \lambda_2^* \frac{c}{\xi} - \lambda_1^* \sqrt{Ta} \right) \frac{dA_1(t^*)}{dt^*} + \frac{\xi \pi}{b\lambda_2^*} \left( \frac{c}{\xi} - \sqrt{Ta} \right) A_1(t^*) \\ & \quad - \frac{1}{\lambda_2^*} D_1(t^*) \end{aligned} \tag{29}$$

With

$$J = \frac{1}{\pi^2 (\lambda_1^{*2} \xi Ta - \lambda_2^{*2} c^2) + b\lambda_2^{*2} \left( k^2 + \frac{a}{\xi} \pi^2 \right)} \tag{30}$$

The initial conditions are :

$$\left. \begin{aligned} & A_1(0) = 0; B_1(0) = cte; C_1(0) = 0 \\ & D_1(0) = \frac{-Ra \cdot b \cdot k \cdot \lambda_2^* [1 - \lambda_1^* (k^2 + \pi^2)] B_1(0)}{\pi b \left[ \lambda_2^* \left( \frac{c}{\xi} + \sqrt{Ta} \right) - \left( \lambda_2^* \frac{c}{\xi} + \lambda_1^* \sqrt{Ta} \right) \right]} \end{aligned} \right\} \tag{31}$$

The boundary conditions :

$$\left. \begin{aligned} & z = 0, \quad \frac{\partial \psi^*}{\partial x} = 0 \\ & z = 1, \quad \frac{\partial^3 \psi^*}{\partial x \partial z^2} = 0 \end{aligned} \right\} \tag{32}$$

and

$$\left. \begin{array}{l} x = 0, \quad \frac{\partial T}{\partial x} = 0 \\ x = 1, \quad \frac{\partial T}{\partial x} = 0 \end{array} \right\} \quad (33)$$

Equations (26), (27), (28) and (29) will be solved numerically by "Mathcad version 15.0".

### 2.5.2 Expression of the Nusselt number in transient regime

If H denotes the rate of heat transported per unit area, we have [10]:

$$H = -K_T \left( \frac{\partial T_{total}}{\partial z^*} \right)_{z=0} \quad (34)$$

Where the term between square brackets represents the horizontal mean and  $T_{total}$  has the expression:

$$T_{total} = T_o - \Delta T \frac{z}{d} + T(x, z, t) \quad (35)$$

The expression of H therefore becomes:

$$H = K_T \frac{\Delta T}{d} [1 - 2\pi C_1(t^*)] \quad (36)$$

The Nusselt Nu number is defined by:

$$Nu = \frac{H}{K_T \frac{\Delta T}{d}} = [1 - 2\pi C_1(t^*)]$$

$$Nu = [1 - 2\pi C_1(t^*)] \quad (37)$$

## 3. RESULTS AND DISCUSSION

To illustrate the evolution of the cooking of gari as a function of time, we have in general distinguished two cases of anisotropy in permeability, namely the first, anisotropy in permeability in the vertical direction (essentially controlled by the parameter  $\xi$ ) and the second, the dynamic anisotropy in the direction of the horizontal plane (mainly controlled by the parameters  $K^*$  and  $\varphi$ ).

The behavior in unsteady state of the heat transfer generated by the phenomenon of cooking of gari, is analyzed by numerically solving the system of nonlinear ordinary differential equations, by the calculation software "MATLAB version 15.0". Using the initial conditions and the appropriate boundary conditions, the heat transfer predicted by the Nusselt number Nu was calculated as a function of time t.

Considering the case of anisotropy in dominant permeability in the vertical direction, that is to

say, when ( $\xi < K^* < 1$ ), Fig. 2.a illustrates the effects of the Taylor number Ta on the transfer of Nu heat as a function of time t, for different values of the control parameters  $K^* = 0.9$ ,  $\varphi = 15^\circ$ ,  $\xi = 0.6$ ,  $Ra = 400$ ,  $\lambda_1^* = 0.8$  and  $\lambda_2^* = 0.2$ . We observe from this figure that Nu initially oscillates with time to reach the steady state as gradually increases. In addition, it is observed that, when Ta increases, the Nusselt number Nu therefore decreases the heat transfer gradually decreases to dampen the convection.

Then, the effects of the Rayleigh number Ra on the transient behavior of the heat transfer are shown in Fig. 2.b for the same values of the control parameters as before and for Ta = 25. It is noted that, apart from of the oscillatory character of Nu during the initial cooking times to reach the steady state, it increases with increase in Ra, therefore with the increase in the rate of heat transfer.

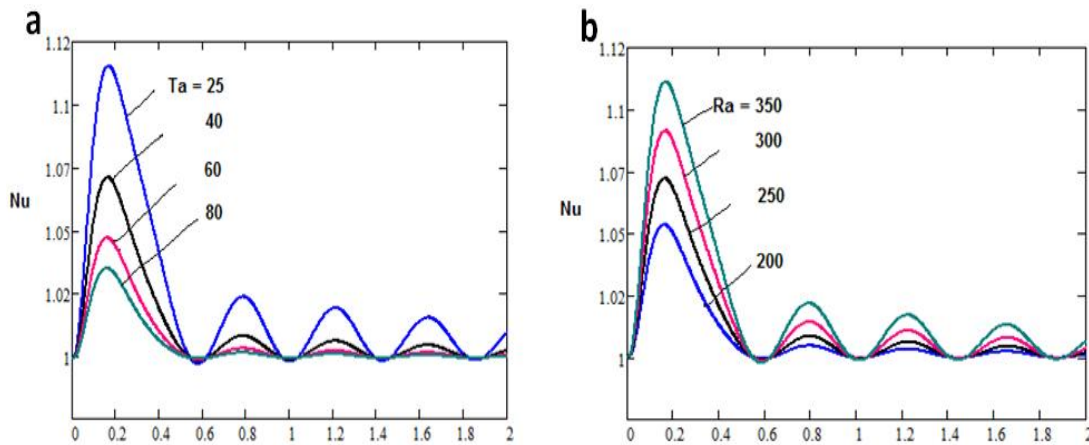
Similarly, dealing with the case of anisotropy in dominant permeability in the horizontal plane, that is to say, when ( $1 < K^* < \xi$ ), we distinguish



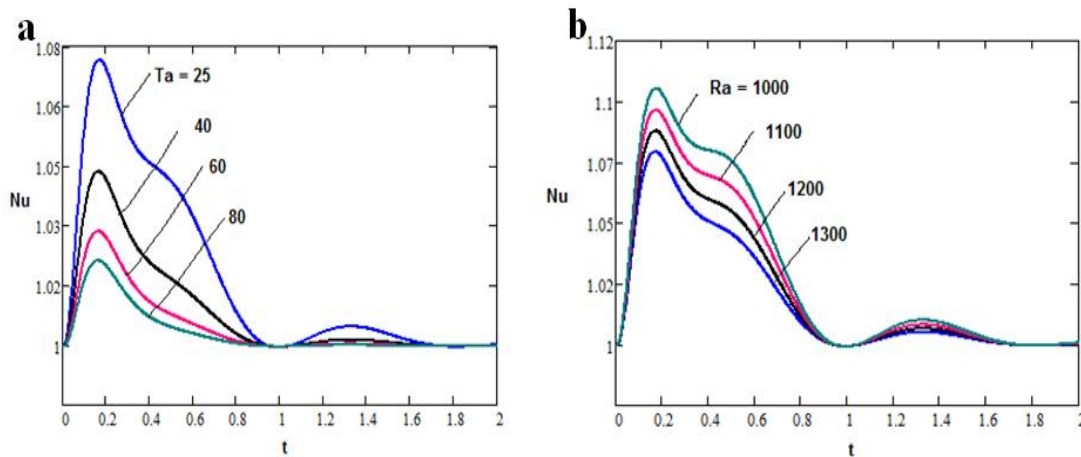
Fig. 3.a and 3.b respectively illustrating the effects of the Taylor number  $Ta$ , the Rayleigh number  $Ra$  on the rate of heat transfer as a function of time. As already noted in the previous case, the Nusselt number oscillates at the start of cooking gari over time and gradually reaches a steady state as the time gets higher and higher. In Fig. 3.a, the variation of  $Nu$  as a function of  $t$  for different values of  $Ta$  and for  $K^* = 1.3$ ,  $\varphi = 60^\circ$ ,  $\xi = 1.6$ ,  $Ra = 1200$ ,  $\lambda_1^* = 0.8$  and  $\lambda_2^* = 0.2$ , showed that the heat transferred by convection decreases with the increase in the Taylor number, as was noted previously in the case of

anisotropy previously studied. The difference observed here in this case of anisotropy lies in the flattening of the peaks of the curves in general; which results in the fact that the hydrodynamic anisotropy occurs mainly in the horizontal direction.

As for Fig. 3.b illustrating the evolution of  $Nu$  as a function of  $t$  for different values of  $Ra$  and for  $K^* = 1.3$ ,  $\varphi = 60^\circ$ ,  $\xi = 1.6$ ,  $Ta = 25$ ,  $\lambda_1^* = 0.8$  and  $\lambda_2^* = 0.2$ , the heat transfer rate increases as the number of  $Ra$  decreases.



**Fig.2 Effect of dimensionless quantities (a) Effect of Taylor number on anisotropy in the vertical direction** ( $\xi < K^* < 1$ ) with  $K^* = 0.9$ ;  $\xi = 0.6$ ;  $Ra = 400$ ;  $\varphi = 15^\circ$ ;  $\lambda_1^* = 0.8$ ;  $\lambda_2^* = 0.2$  (b) Effect of Taylor number on anisotropy in the vertical direction ( $\xi < K^* < 1$ ) with  $K^* = 0.9$ ;  $\xi = 0.6$ ;  $Ra = 400$ ;  $\varphi = 15^\circ$ ;  $\lambda_1^* = 0.8$ ;  $\lambda_2^* = 0.2$



**Fig.3 Effect of dimensionless quantities (a) Effect of the Taylor number  $Ta$  on anisotropy in the horizontal plane** ( $1 < K^* < \xi$ ) with  $\xi = 1.6$ ;  $Ra = 1200$ ;  $K^* = 1.3$ ;  $\lambda_1^* = 0.8$ ;  $\lambda_2^* = 0.2$ ;  $\varphi = 60^\circ$  (b) Effect of Rayleigh number  $Ra$  on anisotropy in the horizontal plane ( $1 < K^* < \xi$ ) with  $\xi = 1.6$ ;  $Ta = 25$ ;  $K^* = 1.3$ ;  $\lambda_1^* = 0.8$ ;  $\lambda_2^* = 0.2$ ;  $\varphi = 60^\circ$

#### 4. CONCLUSION

The study of the non-linear stability in a transient rotating regime of an anisotropic porous medium saturated with a viscoelastic fluid was carried out. We have in general distinguished two cases of anisotropy in permeability, namely the first, the anisotropy in permeability in the vertical direction (controlled mainly by the parameter  $\xi$ ) and the second, the dynamic anisotropy in the direction of the horizontal plane (mainly controlled by parameters  $K^*$  and  $\varphi$ ). Two salient points emerge from this study :

- In both cases of anisotropy, it is observed that, when the Taylor number  $Ta$  increases, the Nusselt number  $Nu$  therefore decreases the heat transfer rate gradually decreases to dampen the convection. We can therefore conclude that the heat transfer rate decreases with the speed of rotation of the cavity.
- The Nusselt  $Nu$  number increases with an increase in the Rayleigh  $Ra$  number, so increasing  $Ra$  increases the rate of heat transfer in the case of permeability anisotropy in the vertical direction. On the other hand, in the case of dynamic anisotropy in the direction of the horizontal plane we observed the opposite effect, that is to say that the heat transfer rate increases when the Rayleigh number  $Ra$  decreases.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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