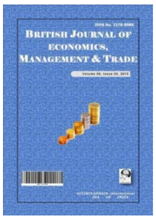

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The Enigma of the Income Tax

M. M. Khoshyaran[1*]

[1] *Economics Traffic Clinic- ETC, 34 Avenue des champs Elyses, 75008 Paris, France.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The objective of this paper is to propose an alternative formulation of income tax that is based on simultaneous utility maximization of both the taxpayer and the government. This method is different from both the fixed income tax method which applies a fixed rate irrespective of the income level, and the graduated income tax formulation which is based on incremental tax rates. Welfare properties of the simultaneous utility maximization income tax formulation such as Pareto optimality, majority voting, social optimality, and unproductive taxation are studied. It is proven that the utility based taxation satisfies all welfare properties.

Keywords: Utility based tax; differential topology; life band model; simultaneous utility maximization; income's moment of inertia.

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Corresponding author: E-mail: megan.khoshyaran@wanadoo.fr;

1 INTRODUCING THE UTILITY BASED TAX SYSTEM

Many countries face a dilemma when it comes to their income tax system [1], [2], [3], [4]. In many countries the tax rate is calculated in a complicated fashion. It is not evident that tax codes raise either the taxpayer's utility or the government's utility. It seems resources are squandered. Many government projects in many countries suffer from lack of sufficient funding. Taxpayers are burdened with the disutility of the income tax. The surplus derived from many of the government's social projects is lost to the taxpayer. The most recent answer to the problem of burdensome tax codes is the "flat tax". For many countries such as Lativa, Lithuania, and Estonia, and some other Baltic countries the "flat tax" seems to be a much preferred option than the progressive tax that requires taxpayers to forfeit a bigger share of their income. The flat tax is based on the theory introduced by Arthur Laffer, defining a relationship between tax rate (τ) , and tax revenue (R_τ) shown in Fig. 1, [5], [6], [7].

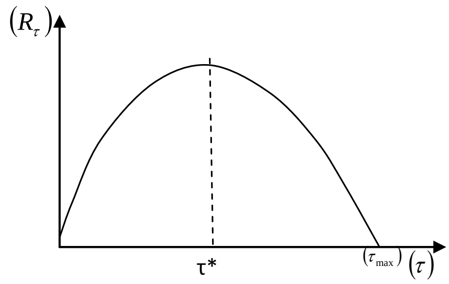

Fig. 1. Laffer tax revenue curve

At tax rate (τ^*), tax revenue (R_τ) is at maximum. As (τ) increases beyond the point (τ^*), (R_τ) decreases and eventually goes to zero at (τ_{max}). This is usually due to the fact that a lower tax rate can be applied to a larger population base, [8], [9].

Though on the surface the idea of imposing a "flat-tax" seems reasonable there are still many aspects of social welfare criteria that are not addressed. There is no evidence that the flat rate taxation brings adequate revenue to see to the operational and investment needs of many government projects. There is no relationship between the flat-tax and government projects. It is not clear how this tax is redistributed among various government projects, and even if this distribution is equitable [10], [11], [12], [13], [14]. It is also lopsided when it comes to equitability among tax payers. A flat tax seems to burden the lower income categories more than the higher income categories. Progressive taxation is an extension of the idea of "equal marginal sacrifice" put forward by [15], [16], [17]. The model of equal marginal sacrifice seemed justifiable in view of wars and patriotic sentiments and scarce government public projects. It was clear where the taxpayer contributions went and the consumer surplus was clearly quantified since war loots were distributed among taxpayers. The efficiency of the few government public projects was easy to estimate as these projects were just a handful. The general idea of the progressive taxation is to maximize public utility given that certain tax revenue is needed [18], [19]. Progressive taxation can be formulated as:

$$\begin{aligned} &F(w) = \max_{U_w} \int_1^N U_{w_i}(Z_{w_i} - T_{w_i})dw \\ &\sum_{i=0}^N T_{w_i} = R_0 \end{aligned} \tag{1.1}$$

In Equation (1.1), (w) is the individual's earning ability. (Z_{w_i}) is the before-tax earnings of each individual (i) in the (w) income category, and (T_{w_i}) is the income tax paid of each individual (i) in the (w) category. The after-tax utility for individual(i) in category (w) is denoted by ($U_{w_i}(Z_{w_i} - T_{w_i})$). The utility formulation is chosen that maximizes F(w). F(w) is the cumulative distribution of taxpayers of type (w). (N) indicates the total number of taxpayers in category (w). The aim in progressive taxation is to maximize social utility which is the utility of all the taxpayers in the (w) category subject to a tax level imposed by government, itself subject to raising a predetermined government revenue level (R_0). Problems with such taxation methods are obvious. There is no guarantee that individual utilities are maximized. To remedy this problem many models of taxation are devised that modify the type and the amount of taxes levied based on many criteria such as marital status, number of children and other dependants per couple, property owned, capital and stock holdings, and many other factors. With all the modifications, and improvements, the progressive tax system has not been able to maximize either the taxpayer's utility or the government's utility [20], [21], [22], [23].

The surplus derived from public projects has diminished considerably. The diminishing consumer surplus has the effect that less and less people now enjoy the benefits of many public projects. On the government side, many projects are underfunded, and it is more and more difficult to acquire the financial support needed to run these projects [24], [25], [26]. Many government remedies such as decentralized fiscal regimes have not yet been successful in providing adequate funding for local public projects. The fundamental reason for this failure is that there is no relation between the utility of taxpayer and the utility of local public projects. Even though on the local level there are fewer projects, still the distribution of the local tax revenue is the result of some hierarchical decision process and not the maximization of local public project benefits and taxpayer utilities. The main idea that the local government decides on a pre-determined local tax revenue level (R_0) remains the dogmatic criterion. Local taxes are adjusted to reach this level.

It is the aim of this paper to show that the only valid tax formulation is the one that takes into account both the maximizing effect of the utility function of the taxpayer and the government through its social projects. Though this concept may come across as more complicated than the existing tax systems; in fact it is to the contrary. The simultaneous utility based income tax system is an alternative method of taxation that is based on the particular situation of each tax payer. Each tax payer is represented by his income or profit profile. It is assumed that the taxpayer's income has an absolute lower and upper limit. The income level of the taxpayer is denoted by (w). Change in the income level is represented by an income vector ($\vec{w}$). The absolute lower income limit is denoted by (w_l) and the absolute upper income limit is denoted by (w_u). It is assumed that during the life time of any economic agent income changes are within this lower and upper bounds. But within each year the taxpayer's income vector ($\vec{w}$) has a specific inclination inside this rigid band of upper and lower income level. This inclination of the income vector signifies both the level, and the moment of inertia of income. The moment of inertia is the force required for the income level to change direction. The individual's lifetime income band is demonstrated in Fig 2.

The individual's active lifetime (K) is divided into one year intervals. The active years of an individual are assumed to be anywhere between one and hundred plus/minus some years (k), ($1 \leq K \leq 100 \pm k; k = 1, ..., 20$). The word active is used to indicate not only the working years of an individual but also after retirement years when the individual can still make investment decisions either in the stock market, real estate market, or any such similar financial markets, and subsequently receive dividends, equities, interests, or any other type of profits, and therefore augment his monetary assets. In Figure 2, the lower boundary (ab) of the surface represents the absolute lower income level (w_l), and the upper boundary (cd) represents the absolute upper income level (w_u). Edge (ac) is denoted by (δA), and Edge (bd) is denoted by (δB). Both edges (δA) and (δB) are of equal distances. (δA) and (δB) are the distances between the absolute lower income (w_l) and the absolute higher income (w_u) formulated as ($\delta A = \delta B =\mid w_u - w_l \mid$). It is possible to have one or more twists along the lifetime income band. These twists represent any drastic change (either positive or negative) in the income level during the individual's lifetime.

For example an unexpected promotion that suddenly raises the income level significantly or an unexpected stock market boom that delivers high dividends on stock assets or even winning a lottery are twists in the income life band of an individual. The trajectory of the individual's monetary evolution from the beginning of the individual's professional life to the end is represented by the income vector ($\vec{w}$) that moves within the life band surface. Income here refers both to earned income and other monetary assets. It is the movement of the income vector ($\vec{w}$) that creates twists in the income life band. In Figure 3, several possible twists of the income vector ($\vec{w}$) for a specific year (k) are shown.

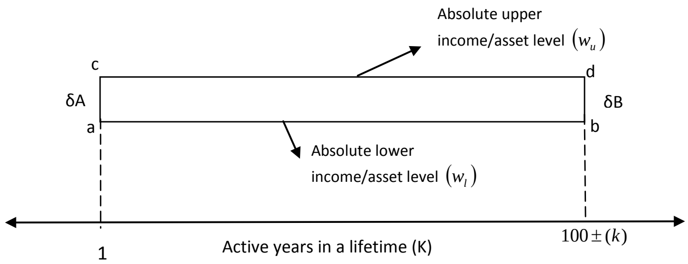

Fig. 2. Band like representation of an individual's lifetime income

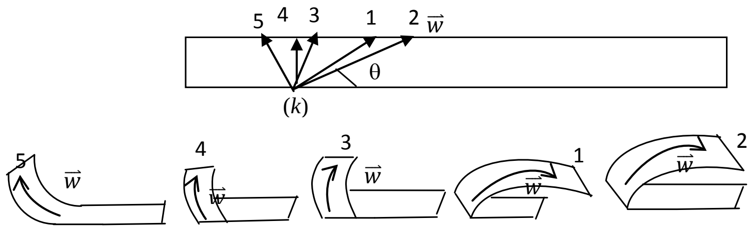

Fig. 3. Some examples of possible income vector during year (k)

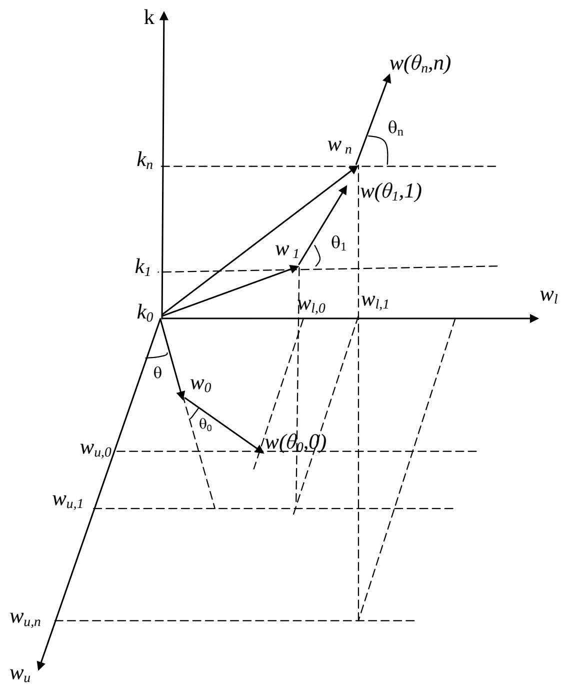

Fig. 4. Hypothetical trajectory of an individual's income vector during years (k_0), (k_1, and (k_n)

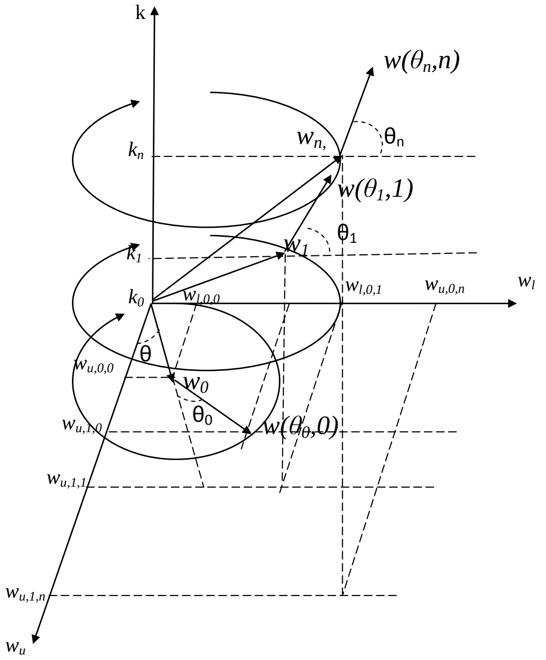

Fig. 4a. Moment of inertia of income vector (M_w)

The angle of rotation ($\theta \in (0, 2\pi)$) represents the inclination of the income vector ($\vec{w}$) during any year (k). With each income vector ($\vec{w}$) a moment of inertia (M_w) is associated. The significance of the moment of inertia is that it gives a quantitative measure of the potential mobility of ($\vec{w}$). An example of the moment of inertia is the investment based income. Obviously, the investment based income has a higher mobility potential than the salary based income and therefore has a higher moment of inertia. The moment of inertia (M_w) is used to modify the income vector($\vec{w}$), as ($\vec{w_\theta} = M_w \times \vec{w}$), where ($\vec{w_\theta}$) is the income vector after an angular rotation. The technical aspect of this element will be discussed in the modelling section of the paper.

Each individual taxpayer's utility is formulated based on the length of the income vector ($\vec{w}$), the angle of rotation (θ), and the moment of inertia (M_w). In Figure 4, possible trajectories of an individual's income vector ($\vec{w}$) for several active years (k_0), (k_1, and (k_n)) are depicted. An individual is constrained by his absolute lower and upper bound income levels (w_l) and (w_u). The income vector's position represents the situation of the individual at a particular point in his professional life. Angle (θ) gives the rotation movement of the income vector ($\vec{w}$). The moment of inertia of the income vector (M_w) is the force that causes an evolution in the income vector ($\vec{w}$) as is shown in Figure 4a. In Figure 4a, the initial income vector ($\vec{w_0}$) during year (0), given the the absolute lower income level ($w_{l,0,0}$), where the indices (l,0,0) represent lower, initial level, and year; and the absolute upper level ($w_{u,0,0}$). During the same year, the income vector ($\vec{w_0}$) evolves into income vector ($\vec{w_{\theta,0}}$), due to the moment of inertia of the income vector ($\vec{w_0}$). The moment of inertia of the income vector ($\vec{w_0}$) can be expressed as ($M_{w_0} = \int_0^{w_{u,1,0}} (\vec{w_0}^2) \times w_{l,1,0}$). The income vector ($\vec{w_0}$) is equivalent to the radius of rotation, and ($w_{l,1,0}$) is equivalent to a mass of a rigid body in movement. In general, the moment of inertia for any year (k) given the income vector ($\vec{w_0},$) can be expressed as ($M_{w_0} = \int_0^{w_{u,1,k}} (\vec{w_0}^2) \times w_{l,1,k}$). In a general way, the moment of income vector can be expressed as ($M_{w_j} = \int_0^{w_{u,j,k}} (\vec{w_j}^2) \times w_{l,j,k}$), where (j) represents any subsequent levels. The individual taxpayer's utility is modelled in the next section of the paper. Similar structure is adopted for the utility function of government projects. Each project can be represented by the projects' investment/profit life band. Figure 5 gives a graphical representation of a government projects' investment/profit life band.

The lifetime of a project is denoted by ($L =| l_n - l_0 |$). The length (L) is the total anticipated duration of a project from its point of conception (l_0) to the point when government puts a stop to the project (l_n), ($l_0 \leq L \leq l_n$). This length is divided into one year intervals denoted by ($l_0, l_1, ..., l_n$). The lower boundary ($a_1 b_1$) of the life band represents the absolute minimum required investment level (ι) of the project. The upper boundary ($c_1 d_1$) represents the absolute maximum expected profit level (ρ). Profit in this case is the social wealth produced by the project. Both edges (δA_1), and (δB_1) represent the difference between profit and investment levels formulated as: ($\delta A_1 = \delta B_1 =| \rho - \iota |$). The projects' investment/profit movement is represented by a vector, ($\vec{v}$). Vector ($\vec{v}$) behaves in a similar manner as the income vector with the exception that for the first few years ($\vec{v}$) represents the investment vector. Fig 6 demonstrates the behaviour of vector ($\vec{v}$) for each year (l_0), (l_1), and (l_n).

(ρ, ι) represent the absolute minimum investment and absolute maximum profit respectively. Angle ($\phi \in [0, 2\pi]$) indicates possible degrees of inclination and rotation of the investment/profit vector ($\vec{v}$) during any year of the project. A major factor in the investment evolution and profit generation of a project is the moment of inertia (M_v) of a project. During the first years (M_v) determines the potential need for investment, and during the following years the moment of inertia (M_v) determines the profitability potential of a project. Figure 6a, depicts the moment of inertia of investment/profitability potential (M_v). The formulation of the moment of inertia of investment/profitability potential (M_v) is similar to that of income vector ($\vec{w}$). As an example, the moment of inertia (M_{v_0}) for initial year(0) can be expressed as ($M_{v_0} = \int_0^{\rho_{1,0}} (\vec{v_0}^2) \times \rho_{0,0}$). The indices (0,0) represent initial level and year. The moment of inertia of investment/profitability potential for any year (k) given initial level (1), (M_{v_l}) can be expressed as ($M_{v_l} = \int_0^{\rho_{1,k}} (\vec{v_l}^2) \times \rho_{1,k}$). This expression can be extended to include subsequent levels as (M_{v_l}) can be expressed as ($M_{v_j} = \int_0^{\rho_{j,k}} (\vec{v_{j,k}}^2) \times \rho_{j,k}$), where (j) represents any level.

For any tax model to be valid it should pass a number of social welfare credibility tests. A number of tests are chosen and tried in order to show the validity of the life band structure tax modelling. The first test is the Pareto optimality test. The objective of this test is to show that no other income tax modelling can increase either the utility of tax payers or the utility of government projects. Test number two is the social acceptability or the majority voting test. The objective is to apply the May's theorem and show that it holds true for the life band structure tax modelling. Test number three is the test of social optimality. The objective of this test is to show that there is no other utility based tax model that can optimize social utility. The final test is the test of unproductive taxation. In this test it is demonstrated that the utility based taxation is a productive tax system, since it is based on the utility of taxpayer, and government projects. Any alternative tax method is unproductive.

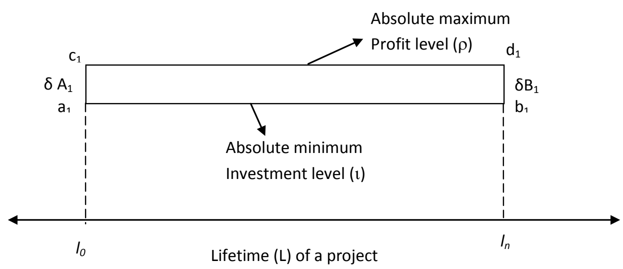

Fig. 5. Life band form of a government project

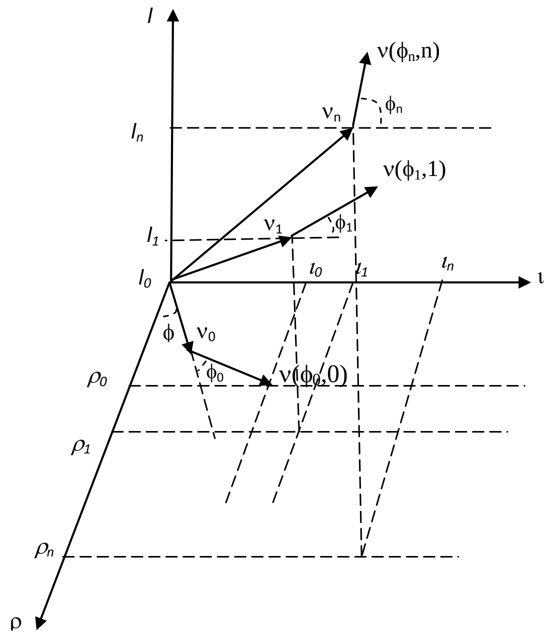

Fig. 6. ($\vec{v}$) as a function of investment, profit, and year

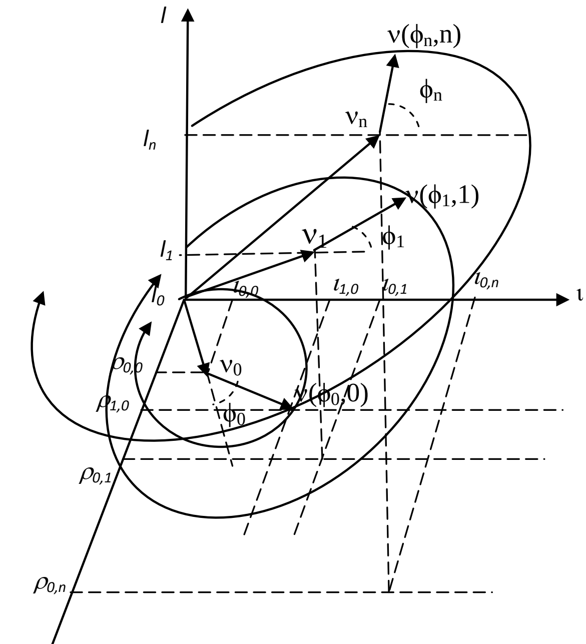

Fig. 6a. Moment of inertia (M_v) of investment/profitability potential

2 MATHEMATICAL FORMULATION OF THE UTILITY BASED INCOME TAX SYSTEM

Let (x) represent a finite set of taxpayer population such that, ($x = x_1, x_2, ..., x_n$), where (n) is the total number of taxpayers. Let (y) be the finite set of (m) government projects for which income tax money is used for financing, ($y = y_1, y_2, ..., y_m$). Let's denote the utility of set (x) by (U_x), where ($U_x = u_{x_1}, u_{x_2}, ..., u_{x_n}$) represents the utility functions of each of the (n) taxpayers. Let's denote the utility of set (y) by (U_y), where ($U_y = u_{y_1}, u_{y_2}, ..., u_{y_m}$) represents the utility functions of each of the (m) government projects. Both (U_x), and (U_y) are manifolds and set (U_x) is not homeomorphic to set (U_y). This means that each element of the set (U_y) corresponds to (n) elements of set (U_x). The utility function of each taxpayer is formulated as a life band described in the previous section, and is represented by ($U_x = (u_{x_i}(\vec{w}_{x_i}, \theta_{x_i})), i = 1, ..., n$). ($\theta_{x_i}$) represents the degree of the inclination of the income vector ($\vec{w}$), and ($\vec{w}_{x_i}$) is the individual taxpayer's (x_i) income vector. The life band structure is the most basic geometric representation possible that can be used to formulate this particular type of utility. Using notation used in the previous section the taxpayer utility can be expressed as: ($u_{x_i}(\vec{w}_{x_i}, \theta_{x_i})) = (\delta A_{x_i} \times \cos(\theta_{x_i}) + \mid \vec{w}_{x_i} \mid \times \cos(\alpha_c \times \theta_{x_i}) \times \cos(\theta_{x_i})) \times M_w, i = 1, ..., n$). ($\alpha_c$) is a fraction between (0) and (1), ($\alpha_c \in [0, 1]$). Here, the term ($\delta A_{x_i} \times \cos(\theta_{x_i})$ signifies the start of an individual's professional life. It represents the initial income or asset that an individual has accumulated or earned before starting a regular professional activity. The term ($\mid \vec{w}_{x_i} \mid \times \cos(\alpha_c \times \theta_{x_i}) \times \cos(\theta_{x_i})$)) indicates the movement of the income vector ($\vec{w}$) with respect to the axis (w_u), the absolute higher income. This term also represents any twists in the movement of the income vector ($\vec{w}$) with respect to the axis (w_u). (δA_{x_i}) is the taxpayer (x_i)'s difference between the absolute upper and lower income level ($\delta A_{x_i} = \mid w_u - w_l \mid$). The moment of inertia of the income vector (M_w) represents the generic potential of the income vector to evolve. This evolution is usually in a positive direction meaning that individuals improve their income and assets as they advance in their professional activities. The only factor that can delay this evolution is professional drawbacks resulting in twists. In that case, it is assumed that suite to any professional drawbacks an individual rebounds from it and can pick up either a new professional activity or modify the old one.

Similarly, with respect to the absolute lower income (w_l) axis, the utility function can be formulated as: ($u_{x_i}(\vec{w}_{x_i}, \theta_{x_i})) = (\delta A_{x_i} \times \sin(\theta_{x_i}) + \mid \vec{w}_{x_i} \mid \times \cos(\alpha_c \times \theta_{x_i}) \times \sin(\theta_{x_i})) \times M_w, i = 1, ..., n$). The term (($\delta A_{x_i} \times \sin(\theta_{x_i})$)) represents the initial income/asset position of an individual taxpayer with respect to the absolute lower income level, (w_l)axis. The term ($\mid \vec{w}_{x_i} \mid \times \cos(\alpha_c \times \theta_{x_i}) \times \sin(\theta_{x_i})$))), signifies the movement or twists in the direction of the income vector ($\vec{w}$) with respect to the (w_l) axis. With respect to (k), the time axis, the utility function is formulated as: ($u_{x_i}(\vec{w}_{x_i}, \theta_{x_i})) = (\mid \vec{w}_{x_i} \mid \times \sin(\alpha_c \times \theta_{x_i}) \times M_w$).

The utility function of a government project (U_y) is formulated in the same way as the taxpayer's utility function, ($U_y = (u_{y_j}(\vec{v}_{y_j}, \phi_{y_j})), j = 1, ..., m$). The utility function of a government project with respect to investment is expressed as: ($u_{y_j}(\vec{v}_{y_j}, \phi_{y_j})) = (\delta A_{y_j} \times \cos(\phi_{y_j}) + \mid \vec{v}_{y_j} \mid \times \cos(\beta_c \times \phi_{y_j}) \times \cos(\phi_{y_j})) \times M_v, j = 1, ..., m$). ($\beta_c$) is a fraction between (0) and (1),($\beta_c \in [0, 1]$). . The term (($\delta A_{y_j} \times \cos(\phi_{y_j})$)) signifies the initial investment/profit position of the project with respect to the investment axis (ρ). And the term ($\mid \vec{v}_{y_j} \mid \times \cos(\beta_c \times \phi_{y_j}) \times \cos(\phi_{y_j})$))) signifies the initial rotation of the investment/profit vector ($\vec{v}$) with respect to the axis (ρ). The utility function of a government project with respect to the profit axis (τ) is formulated as: ($u_{y_j}(\vec{v}_{y_j}, \phi_{y_j})) = (\delta A_{y_j} \times \sin(\phi_{y_j}) + \mid \vec{v}_{y_j} \mid \times \cos(\beta_c \times \phi_{y_j}) \times \sin(\phi_{y_j})) \times M_v, j = 1, ..., m$). The term ($\delta A_{y_j} \times \sin(\phi_{y_j})$) represents the initial investment/profit position of the project with respect to the profit axis(τ). The term ($\mid \vec{v}_{y_j} \mid \times \cos(\beta_c \times \phi_{y_j}) \times \sin(\phi_{y_j})$))) signifies the rotation of the investment/profit vector ($\vec{v}$) with respect to the profit axis(τ). Finally, with respect to the time axis (l), the utility function of a government project is expressed as: ($u_{y_j}(\vec{v}_{y_j}, \phi_{y_j})) = (\mid \vec{v}_{y_j} \mid \times \sin(\beta_c \times \phi_{y_j}) \times M_v$). ($\delta A_{y_j}$) is given earlier as: ($\delta A_{y_j} = \mid \rho - \iota \mid$), the magnitude of the difference between absolute maximum profit and absolute minimum investment. The moment of inertia (M_v) signifies the potential of the investment/profit vector ($\vec{v}$) to evolve, given the initial conditions of investment and profitability of a project. The evolution of the profitability is usually in a positive direction.

To calculate tax rate (τ_{x_i}), the change in each tax payer's utility with respect to change in each government project's utility is calculated and is then summed up over the total number of projects. This change in utility is denoted by ($\prod_{x_i}$), the subscript (x_i) represents each taxpayer (i). ($\prod_{x_i}$), is referred to as C-Utility. The C-Utility is expressed mathematically as: ($\prod_{x_i} = \sum_{j=1}^{m}(\frac{\partial u_{x_i}(\vec{w}_{x_i},\theta_{x_i}}{\partial u_{y_j}(\vec{v}_{y_j},\phi_{y_j}}), i = 1, ..., n$). If this sum is zero then the utility of each taxpayer is not positively changed with respect to any change in the government's project utility and thus no tax should be imposed on the tax payer, ($\prod_{x_i} = 0, \rightarrow \tau_{x_i} = 0, i = 1, ..., n$). If the sum is between (0) and (1), ($0 < \prod_{x_i} < 1$), then it is assumed that the change in taxpayer utility is less than that of the change in government project utility, therefore a low tax rate should be applied to each tax payer, ($0 < \prod_{x_i} < 1, \rightarrow \tau_{x_i} = Low, i = 1, ..., n$). If the sum is equal to (1, ($\prod_{x_i} = 1$)), then the change in tax payer utility is equal to the change in the government project utility, and therefore, a moderate tax should be applied, ($\prod_{x_i} = 1, \rightarrow \tau_{x_i} = Moderate, i = 1, ..., n$). If the sum is greater than (1), ($\prod_{x_i} > 1$), then the change in taxpayer utility is greater than the government project utility, and therefore, a higher tax should be applied, ($\prod_{x_i} > 1, \rightarrow \tau_{x_i} = High, i = 1, ..., n$).

3 THE VALIDITY TEST OF THE UTILITY BASED INCOME TAX MODEL

To validate the utility based taxation model four social welfare tests are chosen. These tests are: 1) Pareto optimality, 2) majority voting or social acceptability, 3) social optimality, and finally 4) unproductive taxation. The utility based taxation model is examined and analysed in view of these four tests. The aim is to show that the utility based modelling of income tax can be justified and used. The aim of the research discussed in this paper is to show that the utility based income tax modelling approach will open a door to a more efficient income tax application which would have the least adverse impact on the tax payer and the government.

Theorem 3.1. *Utility based taxation is Pareto optimal.*

Proof. Let's assume that there are three alternatives (x,y,z) corresponding to each of the three tax models, 1) a utility based tax rate (x), 2) a flat tax rate (y) , which is usually set at a moderate level, and 3) a progressive tax rate (z), which starts at a moderate level and increases as a function of time. There are a total of (N) taxpayers. Each taxpayer's preference among the three alternatives can be stated by a profile, $(\alpha_1, ..., \alpha_N)$, where $(\alpha_i, i = 1, ...N)$ takes value (1) if taxpayers prefer alternative (x) to alternatives (y) and (z), (+1) if taxpayers prefer alternative (y) to alternatives (x) and (z), (-1) if taxpayers prefer alternative (z) to alternatives (x) and (y). Let an aggregate welfare functional $(W(\alpha_1, ..., \alpha_N))$ be a rule that assigns aggregate social preference, $(W(\alpha_1, ..., \alpha_N) \in -1, 0, 1, +1)$. Let $(n(\alpha) = \sharp i : \alpha_i = 1)$ be the number of taxpayers who prefer alternative (x), $(n^+(\alpha) = \sharp(i : \alpha_i = +1))$ be the number of taxpayers who prefer alternative (y), $(n^-(\alpha) = \sharp(i : \alpha_i = -1))$ be the number of taxpayers who prefer alternative (z). Let (τ_{x_i}) be the imposed income tax under alternative (x), (τ_{y_i}) be the imposed income tax under alternative (y), and (τ_{z_i}) be the alternative imposed income tax under alternative (z). 4 cases are considered. In each case it is shown that the aggregate social functional $(W(\alpha_i), i = 1, ..., N)$ is Pareto optimal with respect to alternative (x) which means that there exists a unanimous preference on the part of taxpayers with respect to utility based taxation.

Case 1. If the C-Utility is equal to zero, $(\prod_{x_i} = 0)$, then the aggregate social functional $(W(\alpha_1, ..., \alpha_N) = W(1, ..., 1))$, and $(n(\alpha) = N)$, naturally each taxpayer strictly prefers alternative (x), the utility based taxation. No tax is imposed on taxpayers.

Case 2. If the C-Utility is between zero and one, $(0 < \prod_{x_i} < 1)$, has a normal cumulative distribution which is skewed to the left. The skewness is due to the majority of the taxpayer's changes of income not being compatible with investment/profit changes of public projects. More precisely stated the income/asset of the taxpayer changes at a slower rate than the investment/profit changes of public projects. In this case the tax rate of alternative (x) by definition is lower than the the flat tax rate and the progressive tax rate ($\tau_{x_i} < \tau_{y_i} < \tau_{z_i}$). Since the inequality strictly holds then, the utility based tax model, alternative (x) is Pareto optimal. All tax payers prefer the utility based tax model, ($n(\alpha) > n^+(\alpha) > n^-(\alpha)$), and ($n(\alpha) = N$), all taxpayers are better off. In this case the aggregate social functional ($W(\alpha_i) = 1, i = 1, ..., N$) is true for the utility based tax model.

Case 3. The C-Utility, ($\prod_{x_i} = 1$), has a uniform cumulative distribution. The change in the income/asset of each tax payer is equal to the change in the investment/profit of public projects. (3) conditions can occur. Condition (1) is if ($\tau_{x_i} < \tau_{y_i} < \tau_{z_i}$), the tax rate of the utility based regime is lower than the other two regimes. In this case the inequality strictly holds, alternative (x), the utility based regime is Pareto optimal since ($n(\alpha) > n^+(\alpha) > n^-(\alpha)$), and ($n(\alpha) = N$). Condition (2), all tax rates are equivalent, ($\tau_{x_i} = \tau_{y_i} = \tau_{z_i}$), then tax payers are indifferent among the three alternatives (x,y,z), ($n(\alpha) = n^+(\alpha) = n^-(\alpha) = N$). Any of the tax models are Pareto optimal. Condition (3), ($\tau_{x_i} > \tau_{y_i}$), or ($\tau_{x_i} > \tau_{z_i}$), ($W(\alpha_i) = +1, i = 1, ..., N$), or ($W(\alpha_i) = -1, i = 1, ..., N$), preference is either for alternative (y), or alternative (z). Condition (3) can not be realized. Let ($\kappa_{x_i} \ |_l^{x_i}$) be the monetary or service benefits of each project for each taxpayer. No taxpayer is worse off by switching to tax rate (τ_{x_i}) of alternative (x), the utility based taxation. This is because the utility of each taxpayer is a function of profit gained from each project, ($\kappa_{x_i} \ |_l^{x_i}$). Each taxpayer receives a surplus. A taxpayer is considered to be a consumer with respect to public projects. Tax rate (τ_{x_i}) is a price of consumption of a product of a public project. The monetary or service benefits, ($\kappa_{x_i} \ |_l^{x_i}$) is the consumer surplus over the set price. All taxpayers are better off by adopting alternative (x), and thus the utility based taxation is Pareto optimal.

Case 4. The C-Utility, ($\prod_{x_i} > 1$), has an exponential cumulative probability distribution that is skewed to the right. The majority of the taxpayer's change in income is compatible with investment/profit changes of public projects. Following the same reasoning as in case (3), for conditions (2), and (3), taxpayers will choose alternative (x) over alternatives (y) and (z) without anyone being worse off. Therefore the utility based taxation is Pareto optimal. ☐

Theorem 3.1 can be demonstrated graphically, as in Figure 7. The taxpayer surplus is denoted by (δ). (δ) is a function of the income vector ($\vec{w_{x_i}}$), and the monetary or service benefits, ($\kappa_{x_i} \ |_l^{x_i}$). (δ) is equal to ($\delta = f(w_{x_i} + \kappa_{x_i} \ |_l^{x_i})$), if ($\kappa_{x_i} \ |_l^{x_i} > 0$). Taxpayer surplus ($\delta$), is equal to the triangle ($\Delta(\tau_{x_i} cd)$). All taxpayers are better off with the utility based taxation system.

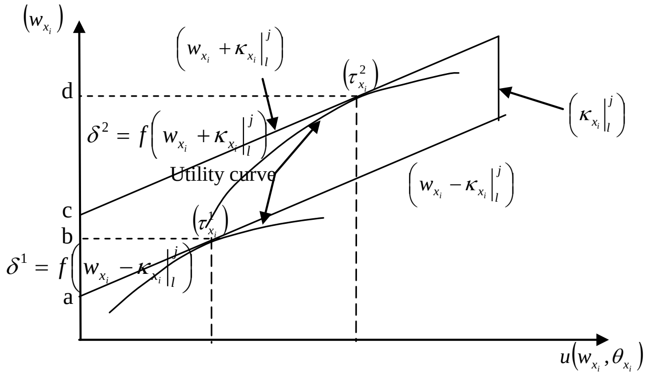

Fig. 7. Pareto optimality of the utility based taxation

Theorem 3.2. *The utility based taxation is a majority voting social welfare functional which is symmetric and neutral among agents and is positive responsive*

Proof. The C-utility, ($\prod_{x_i}$) does not change when the order or permutation within the summation changes. The outcome of the summation therefore does not alter the social preference among the three alternatives (x,y,z). The agents are considered anonymous, and therefore are not ranked based on income levels. Thus, let ($\mu : \{1, ..., N\} \rightarrow \{1, ..., N\}$) be an onto function such that for any (i) there is (h) such that ($\mu(h) = i$). An example of such permutation is: ($\{1, 2, ..., N\}$) is permuted to ($\{2, N, N-1, N-2, ..., 1\}$). This means that ($\mu(1) = 2, \mu(2) = N, \mu(N) = 1$). The C-utility as a function of ($\mu(h)$) is expressed as ($\prod^{\mu(h)} = \sum_{j=1}^{m}(\frac{\partial u_{\mu(h)}(\vec{w}_{\mu(h)}, \theta_{\mu(h)})}{\partial u_{y_j}(\vec{v}_{y_j}, \phi_{y_j})}), h = 1, ..., N$). Since all taxpayers are equal, the C-utility which is based on a summation over all taxpayers, ($\prod_{x_i}$) is equal to the permuted summation over all taxpayers ($\prod^{\mu(h)}$), ($\prod_{x_i} = \prod^{\mu(h)}$). Thus welfare functional and the permuted welfare functional of tax payers are equal, ($W(\alpha_i) = W(\alpha_{\mu(h)}), i = 1, ..., N; h = 1, ..., N$). The taxpayers' choice of alternative (x) is not affected by the ordering of taxpayers. The utility based taxation is symmetric.

Neutrality: In the utility based tax modelling, each taxpayer receives monetary or service benefits of public projects ($\kappa_{x_i} \mid_l^{x_i}$). The probability of the acceptance of the utility based tax rate is conditional on ($\kappa_{x_i} \mid_l^{x_i}$). Let event (B) represent the event that each taxpayer receives ($\kappa_{x_i} \mid_l^{x_i}$), and let event (A) be that each taxpayer chooses the utility based tax rate (τ_{x_i}), then event (A) strongly depends on event (B). Given that ($P(B) > 0$) is the probability that a taxpayer receives ($\kappa_{x_i} \mid_l^{x_i}$), the conditional probability of accepting the utility based taxation is positive and equal to ($P(A \cap B) = P(A \setminus B) \times P(B) > 0$). If (P(B) = 0), the taxpayer does not receive any benefits from public projects ($\kappa_{x_i} \mid_l^{x_i} = 0$), then the conditional probability of accepting the utility based taxation is zero, ($P(A \cap B) = 0$). Since the probability of acceptance of tax rate (τ_{x_i}) depends on ($\kappa_{x_i} \mid_l^{x_i}$), then positive ($\kappa_{x_i} \mid_l^{x_i} > 0$) results in all taxpayers choosing tax rate (τ_{x_i}), and no benefits, ($\kappa_{x_i} \mid_l^{x_i} = 0$) results in all taxpayers reversing their preference and choosing other tax rates. The welfare functional changes as the taxpayer preference changes from, ($W(\alpha_i)$) to ($-W(\alpha_i)$). Negative sign signifies that the social preference is reversed when the preference of each tax payer is reversed.

Positive responsive: by Theorem 3.1, the utility based taxation is Pareto optimal. Majority of taxpayers prefer tax rate (τ_{x_i}), ($n(\alpha) > n^+(\alpha)$), ($n(\alpha) > n^-(\alpha)$). Therefore, (($N - n(\alpha) > (n^+(\alpha) > n^-(\alpha)$))). If the preference of a certain number of taxpayers changes in favour of alternative tax rates, then (($N - n(\alpha) < (n^+(\alpha) > n^-(\alpha)$))). The utility based taxation loses its majority standing, and the majority moves to the alternative tax rate.

□

Theorem 3.3. *The utility based taxation is socially optimal.*

Proof. The utility set of taxpayers is given as ($U_X = \{u_{x_i}\}, i = 1, ..., N$). Let ($\bar{u_w}$) be the average utility of each tax payer. Let ($n_+(u_{x_i}) = \sharp\{i : u_{x_i} > \bar{u_w}; i = 1, ..., a$) be the number of taxpayers whose utilities are greater than the average utility ($\bar{u_w}$), and let ($n_-(u_{x_i}) = \sharp\{i : u_{x_i} < \bar{u_w}; i = a+1, ..., N$) be the number of taxpayers whose utilities are less than ($\bar{u_w}$). By Theorem 3.1, the utility based taxation is Pareto Optimal. The majority of taxpayers have their utilities maximized, ($n_+(u_{x_i}) > n_-(u_{x_i})$). However, at the limit it is possible to have a situation where ($n_+(u_{x_i}) = n_-(u_{x_i}$). Let the utility set of public projects be given as: ($U_Y = \{u_{y_j}\}, j = 1, ..., M$). Let ($\bar{u_\rho}$) be the average utility of each public project. Let ($m_+(u_{y_j}) = \sharp\{j : u_{y_j} > \bar{u_\rho}; j = 1, ..., b$) be the number of public projects whose utilities are greater than ($\bar{u_\rho}$), and let ($m_-(u_{y_j}) = \sharp\{j : u_{y_j} < \bar{u_\rho}; j = b+1, ..., M$) be the number of taxpayers whose utilities are less than ($\bar{u_\rho}$). Naturally, it is possible to observe two situations, ($m_+(u_{x_i}) > m_-(u_{x_i})$), and ($m_+(u_{x_i}) = m_-(u_{x_i})$). Four combinations are possible: combination (1) is when both the utilities of taxpayers and public projects have the same pattern, (($n_+(u_{x_i}) = n_-(u_{x_i})) \cup (m_+(u_{x_i}) = m_-(u_{x_i})$)). Combinations (2), and (3), the utilities of taxpayers and public projects have opposite patterns, $((n_+(u_{x_i}) = n_-(u_{x_i})) \cup (m_+(u_{x_i}) > m_-(u_{x_i})))$, and $((n_+(u_{x_i}) > n_-(u_{x_i})) \cup (m_+(u_{x_i}) = m_-(u_{x_i})))$. In combination (4), the utility of taxpayers and public projects have the same trend, $((n_+(u_{x_i}) > n_-(u_{x_i})) \cup (m_+(u_{x_i}) > m_-(u_{x_i})))$. In combination (1), the C-utility $(\prod_{x_i} = \sum_{j=1}^{b} \frac{\partial u_{x_i}^{n_+}}{\partial u_{y_j}^{m_+}} = \sum_{j=b+1}^{M} \frac{\partial u_{x_{i'}}^{n_-}}{\partial u_{y_j}^{m_-}} = 0; i = 1, ..., a; i' = a+1, ..., N)$ is equal to zero, and accordingly, the tax rate $(\tau_{x_i} = 0)$. The welfare functional $(W(\alpha_i) = 1, i = 1, ..., N)$ is Pareto optimal. Evidently, in this case, the utility based taxation is socially optimal since the tax rate is at the point where demand for the tax rate, $(n_+(u_{x_i}))$ intersects the C-utility at tax rate $(\tau_{x_i} = 0)$, where $(n_+ = N)$. In combinations (2), and (3), the C-utility is between (0), and (1), $(0 < \prod_{x_i} < 1)$, $(\prod_{x_i} = \sum_{j=1}^{b} \frac{\partial u_{x_i}^{n_+}}{\partial u_{y_j}^{m_+}} < \sum_{j=b+1}^{M} \frac{\partial u_{x_{i'}}^{n_-}}{\partial u_{y_j}^{m_-}} < 1; i = 1, ..., a; i' = a+1, ..., N)$. By definition the tax rate (τ_{x_i}) is low to moderate. By Theorem 3.1, the welfare functional, $(W(\alpha_i) = 1, i = 1, ..., N)$ is Pareto optimal. The tax rate (τ_{x_i}) is socially optimal, since it is the intersection point of the C-utility, and demand for the tax rate $(n_+(u_{x_i}))$. This is demonstrated in Figure 8. By Theorem 3.1, the utility based taxation is Pareto optimal, therefore, by Theorem 3.2, it is majority voting, meaning that the C-utility has a normal distribution. The intersection point of the C-utility and the demand for tax rate, is the socially optimal point.

In combination (4), the C-utility is greater than (1), $(\prod_{x_i} > 1)$, $(\prod_{x_i} = \sum_{j=1}^{b} \frac{\partial u_{x_i}^{n_+}}{\partial u_{y_j}^{m_+}} < \sum_{j=b+1}^{M} \frac{\partial u_{x_{i'}}^{n_-}}{\partial u_{y_j}^{m_-}} > 1; i = 1, ..., a; i' = a+1, ..., N)$. By definition the tax rate (τ_{x_i}) is high. By Theorem 3.1, the welfare functional, $(W(\alpha_i) = 1, i = 1, ..., N)$ is Pareto optimal. The tax rate (τ_{x_i}) is socially optimal, since it is the intersection point of the C-utility, and demand for the tax rate $(n_+(u_{x_i}))$. This is demonstrated in Fig. 9.

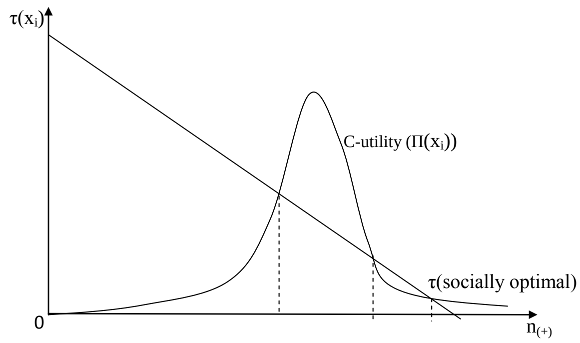

Fig. 8. Socially optimal low tax rate

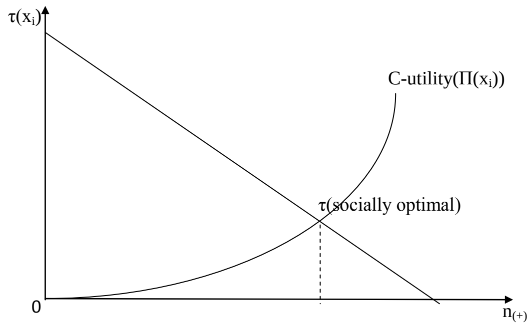

Fig. 9. Socially optimal high tax rate

Theorem 3.4. *The utility based taxation is a productive taxation system.*

Proof. The proof consists of showing that the utility based taxation is not an unproductive taxation. Productive taxation is defined as taxation that promotes social welfare. If no tax is imposed the taxpayer spends all his income on consumption/leisure and does not contribute to public goods. This leads to the reduction of social welfare. Under the utility based taxation the tax rate is a function of both the utilities of taxpayers and public projects. It is adjusted with respect to the evolution of these utilities. Thus any reduction of the purchasing power is remedied by adding social benefits in the form of monetary subsidies or services due to public projects. To show that the utility based taxation is a productive taxation, let's assume there is one taxpayer and one public project. Let the utility of the tax payer be the function of income (w), and tax rate, (τ), ($u(\tau, w) = (1 - \tau) \times w$). Let the utility of the public project be the function of the tax payer's utility and profit, formulated as ($u(\tau, w, \rho) = (f(u(\tau, w)), \rho) = \tau \times w + \rho$) where ($\rho$) denotes profit derived from the operation of the public project. The profit (ρ) is assumed to be positive and monotonically increasing. As the public project becomes more profitable the taxpayer's contribution through taxes will go down. This is due to the C-utility function, ($\prod = \frac{\partial u(\tau,w)}{\partial u(\tau,w,\rho)} < 1$). By the definition given earlier, in this case the tax rate (τ), is low to moderate. To calculate this tax rate, let's put the utility of the tax payer equal to the utility of the public project, ($u(\tau, w) = u(\tau, w, \rho)$), tax rate ($\tau$) is then calculated as ($(1 - \tau) \times w = \tau \times w + \rho$) to be equal to ($\tau = \frac{\rho}{1-2w}$). To simplify, let's take the tax rate to be equal to the ratio of profit (ρ) to income (w), ($\tau = \frac{\rho}{w}$), and call it a (τ-curve). The (τ-curve) is demonstrated in Figure 10. In Figure 10, the (τ-curve) shows the evolution of the tax rate (τ) with respect to ($\frac{\rho}{w}$) profit-income ratio changes. When ($\rho = w$), profit is equal to income, this ratio is equal to (1), the tax rate is at point (τ_1). Below point (1), ($\rho < w$), and the tax rate is high as is expected, at point (τ_2). Above point (1,) ($\rho > w$), the tax rate is low conforming to the definition given earlier, at point (τ_3). The tax rate (τ) is a function of public project's profit and therefore is a productive tax.

Now, must show that under the utility based taxation system the taxpayer benefits from a lower tax contribution due to the project's profit generating ability as compared to the two regimes of (y) and (z), the flat tax method, and the progressive taxation method. In other words, since in the utility based taxation, the tax rate is productive, the taxpayer's tax rate is almost always similar to (τ_3), as in Figure 10. The relations that prove this statement are put together graphically, in Figure 11.

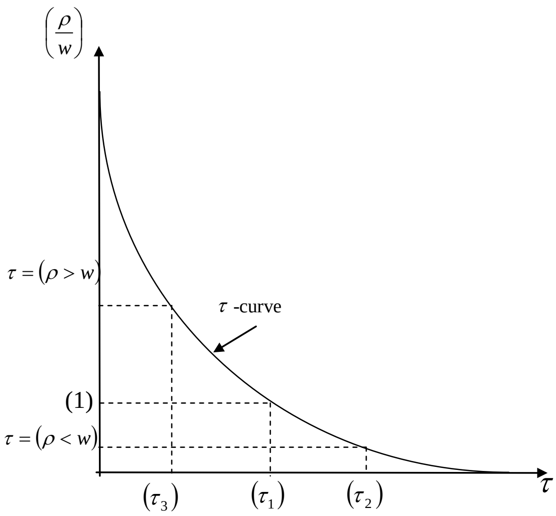

Fig. 10. Productive tax rate

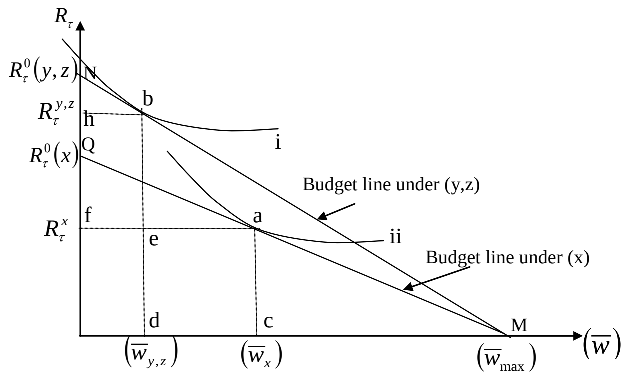

Fig. 11. Comparison of taxpayer surplus increase under regimes (x), (y),and (z)

In Fig 11, the (x) axis ($\bar{w}$) is the the net income. This is the money available to the taxpayer for consumption and savings. The y-axis, (R_τ) is the tax revenue. Under regimes (y), and (z) this revenue is set based on the spending needs of the government. Curve (i), represents the indifference curve of the tax payer under regimes (y),and (z). Curve (ii) represents the indifference curve of the taxpayer under regime (x). Under regimes (y), and (z) the taxpayer chooses an indifference curve tangent to the line MN, the budget line under (y),and (z) at point(b). It is assumed that the tax payer has a preference for public projects. At point (b) tax taxpayer spends most of his income on taxes, and still the tax revenue does not reach the required level, ($R_\tau^0(y,z)$). At point (b) tax taxpayer spends most of his income on taxes, and still the tax revenue does not reach the required level ($R_\tau^0(y,z)$), since the revenue generated at point (b) is ($R_\tau^{y,z}$). In contrast under regime (x) the indifference curve is optimized at point (a). At point (a) the utility of the taxpayer is higher than point (b). The tax revenue (R_τ^x) is less than the maximum tax revenue. But the taxpayer is not penalized; tax revenue is supplemented by the difference between the tax revenue generated by the tax rate (τ_x), (R_τ^x), and maximum tax revenue generated under regime (x), ($R_\tau^0(x)$), ($R_\tau^0(x) - R_\tau^x = \rho$) which is the profit gained from the public project. The taxpayer's surplus under regime (x), area (Qfa) is larger than the surplus under regimes (y),and (z), area (Nhb). It is shown that the taxpayer's surplus under regime (x) is higher than under regimes (y), and (z). The taxpayer has a higher propensity for consumption due to the state of his surplus, proving that the utility based taxation is a productive taxation.

4 CONCLUSION

In this paper a new methodology for calculating an income tax rate is proposed that is based on simultaneous maximization of the utility of taxpayers and the utility of public projects. The taxpayer utility is formulated as a function of the movement of the income vector. The utility of the public project is formulated as a function of the movement of the investment/profit vector. Both vectors move within a life band structure boundaries of which are determined by the lowermost and uppermost income level for the taxpayer, and minimum investment and maximum profit limits for the public project. Any drastic change in the direction of the movement of the vectors is represented by twists within the life band structure. The income tax rate imposed is based on analysing the changes and the potential for raises in the income level of the taxpayer with respect to changes in the investment/profit level of the public project. The social welfare acceptability of the utility based taxation is examined by applying four social welfare tests to the model.

The model is shown to pass the four welfare tests and has a high social acceptability potential. A possible extension of the work done in this paper is to conduct an experiment on the validity and acceptability of this type of taxation. A random sample without replacement of at least 100 salaried individuals from all walks of life and a few public projects should be taken. Three tax rates should be calculated based on three methods of 1) the simultaneous utility based taxation introduced in this paper, 2) the flat tax, and 3) the progressive taxation. The input used in calculating tax rates based on methods (2) and (3) is the pre-determined revenue level set by the government. Input into the simultaneous utility based taxation model is the income history of each taxpayer, and the professional situation of each tax payer. Input into the public project is the investment/profit history of the project, the number of years the project has to be operational, and the starting date of the project. Each participant in the experiment is then presented with three tax rates without being told which tax rate belongs to which tax regime. Each participant then chooses the tax rate they perceive to be most adapted to their situation. The choice of each taxpayer is then registered and analysed. Based on the tests of social acceptability it is expected that the majority of the participants in such an experiment will choose the utility based taxation method.

DISCLAIMER

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COMPETING INTERESTS

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