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Fixed Point Results for "Two Pairs" of OWC- Maps in S - Spaces

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

Our aim in this paper, prove a 'unique common fixed point' result for two pairs of OWC (Occasionally Weakly Compatible) - maps in S-Spaces. Our results are generalizing and improving the known main results in the references.

Keywords: Fixed points; fixed point theorem; OWC (Occasionally Weakly Compatible); S-Space (Symmetric Space).

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1. INTRODUCTION

In prior to 1968 basic result in 'fixed point theory' is a 'Banach' contraction principle. And in 1968, R.Kannan [1] proved fixed point theorems for a self-maps which satisfies the contractive condition and there is no need to continuity. After that so many authors were extends, improves and generalizes the results in fixed point theory in various types (E.x. [2-18]). Hicks and Rhoades [11] in 1999, proved unique commixed fixedpoint results in S- Spaces and semi metric spaces. Recently, Abbas and Rhoades [5], obtained unique common fixed point theorems for OWC(Occasionally Weakly Compatible) maps which satisfies the generalized contractive

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condition in S-Spaces. In the present research paper we proved a unique common fixed point theorem for two pairs of OWC(Occasionally Weakly Compatible) self- maps in S-Spaces.

The following are useful in our main results and which are in [5].

Definition 1.1. Let X_1 be a set , and u, v be self - maps of X_1. A point x_1 in X_1 is said to be a 'coincidence point' of u and v iff $ux_1 = vx_1$.
$w_1 = ux_1 = vx_1$ is said to a point of coincidence of u and v.

Definition 1.2. Let u, v be self- maps of a set X_1. A point x_1 in X_1 are said to be OWC(Occasionally Weakly Compatible) iff there exists a point x_1 in X_1 which is a 'coincidence' point of u and v at which they are 'commute' to each other.

Lemma 1.1. Let X_1 be a set, and u, v are OWC (Occasionally Weakly Compatible) self- maps of X_1. If u and v have a 'unique point coincidence' $w_1 = ux_1 = vx_1$, then w_1 is said to be a 'unique common fixed point' of u and v.

Note: Our results are proved in symmetric spaces, which are more general than metric spaces.

Definition 1.3. Let X_1 be a set. A symmetric ρ on X_1 be a map $\rho: X_1 \times X_1 \rightarrow [0, \infty)$ such that

$\rho(\alpha, \beta) = 0$ iff $\alpha = \beta$, and $\rho(\alpha, \beta) = \rho(\beta, \alpha)$ for $\alpha, \beta \in X_1$.

Let $A\epsilon[0,\infty)$, $R_A^+ = [0, A)$. Let H: $R_A^+ \rightarrow R$ satisfy

(i) $H(0) = 0$ and $H(s) > 0$ for each $s\epsilon(0, A)$and

(ii) H is not-decreasing on R_A^+.

We define, $H\{0,A) = \{H: R_A^+ \rightarrow R$: H satisfies (i) - (ii)}.

Let $A\epsilon[0,\infty)$. Let $\psi: R_A^+ \rightarrow R$ satisfies the following:

(a) $\psi(s) < s$, for each $s \in (0, A)$ and

(b) ψ is not-decreasing.

Now we define, $\psi\ \{0, A)=\{\ \psi{:}R_A^+ \rightarrow R$: H satisfies (i) - (ii) above}.

Some examples of mappings $H{:}R_A^+ \rightarrow R$:
H satisfies (i) - (ii) , we refer toZhang [18] .

Definition 1.4. A control function Φ is defined by $\Phi: R^+ \rightarrow R^+$ which satisfies $\Phi(s) = 0$ iff $s = 0$.

2. FIXED POINT THEOREM

We obtained a unique 'common fixed point theorem' for Two Pairs of OWC-maps in S-Spaces.

Theorem 2.1. Let X be a set with symmetric ρ. Let $D = \text{Sup}\ \{\rho(u, v) : u, v \in X\}$. Suppose that A, B, M and N are Two pairs of self- maps of X satisfying the following conditions:

(i) $H((\rho(Au, Bv))) < \psi(H(L(u, v)))$,

where,

$$L(u, v) = \text{Max}\ \{\alpha[\rho(Mu, Nv) + \rho(Mu, Au) + \rho(Nv,Bv)]\} + \text{Max}\ \{\beta[\rho(Mu,Bv)) + \rho(Nv, Au)/2)]\}.$$

Where $\alpha, \beta > 0$ and $\alpha + \beta < 1$.

For each $u,v\epsilon X, H \in H([0,C)$ and $\psi\epsilon\psi[0, H((C - 0))$, where $C = D$ if $D = \infty$ and $C > D$ if $D < \infty$. And

(ii) (A, M) and (B, N) are OWC.

Then A, B, M and N are having a unique common fixed point in X.

Proof. Since by (ii) (A, M) and (B, N) are each OWC (Occasionally Weakly Compatible), then there exists Two points $u, v\epsilon X$ such that $Au = Mu$ and $Bv = Nv$. We claim that $Au = Bv$. For otherwise from (i) we get that

$$\begin{aligned} L(u, v) &= \text{Max}\{\alpha[\rho(Mu, Nv) + \rho(Mu, Au) + \rho(Nv,Bv)]\} + \text{Max}\{ \beta[\rho(Mu,Bv) + \rho(Nv, Au)/2]\}, \\ &= \text{Max}\ \{\alpha[\rho(Mu, Nv) + \rho(Au, Au) + \rho(Nv, Nv)]\} + \text{Max}\ \{ \beta[\rho(Mu, Nv) + \rho(Nv, Mu)/2]\}, \\ &= \text{Max}\{\alpha[\rho(Mu, Nv) ,0]\} + \text{Max}\{\beta[\rho(Mu, Nv)]\}, \\ &= \alpha[\rho(Mu, Nv)]+ \beta[\rho(Mu, Nv)], \\ &= (\alpha + \beta)\ \rho(Mu, Nv). \qquad \ldots \quad (1). \end{aligned}$$

Then by (i) and (1) we get that

$$\begin{aligned} H((\rho(Au, Bv))) &< \psi(H(L(u, v))) \\ &= \psi(H(\alpha + \beta))\, \rho(Mu, Nv), \\ &\quad \text{since } \alpha + \beta < 1. \\ &< H(\rho(Mu, Nv)) \\ &= H(\rho(Au, Bv)), \\ &< H(\rho(Au, Bv)), \text{ and} \end{aligned}$$

which is a contradiction.

Therefore, we get that ‘Au = Bv’.

That is, ‘Au = Mu = Bv = Nv’.

Moreover, if there exists another point ‘z’ such that, ‘Az = Mz’, then using (i) and (1) we get that ‘Az = Mz = Bv = Nv’ (or) ‘Au = Bz’ and ‘w = Au = Mu’ is the unique point of coincidence of A and M. And we get by the Lemma (1.1) ‘w’ is a common fixed point of A and M. By symmetry there exists a unique point ‘z ϵ X’ such that ‘z = Bz = Mz’.

Suppose ‘w ≠ z’ by (i) and (1) we get that

$$\begin{aligned} H((\rho(w, z)) &= H(\rho(Aw, Bz)) \\ &< \psi(H(L(w, z))) \\ &< \psi(H(\rho(w, z))), \\ &< H(\rho(w,z)), \end{aligned}$$

which is a contradiction.

Therefore, ‘w = z’ and ‘w’ is a common fixed point. And by the above discussion we get that ‘w’ is unique. Therefore, ‘w’ is a unique common fixed point of A, B, M and N in X. Hence the theorem.

3. CONCLUSIONS

In this research paper we obtained generalized results and which are more general than of the results of Abbas and Rhoades [5].

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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