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Research Article

A New Methodology for Solving Piecewise Quadratic Fuzzy Cooperative Continuous Static Games

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This paper deals with n-players fuzzy cooperative continuous static games (FCCSGs). The cost function coefficients are characterized by piecewise quadratic fuzzy numbers. One of the best approximate intervals, namely, the inexact interval of the piecewise quadratic fuzzy number is used. Furthermore, we proposed a new methodology based on the weighted Tchebycheff method to solve CCSG with n-players. The advantages of the approach are the ability to enable the decision-maker to have satisfactory solution and applied for different real-world problems with various types of fuzzy numbers. There is also a stability set of the first kind without differentiability for the optimal compromise solution that was found. In the future, the proposed methodology could be used in different types of real-world problems and multiple decision-makers. This proposed work can also be extended to hypersoft set, fuzzy hypersoft sets, intuitionistic hypersoft sets, bipolar hypersoft sets, and pythagorean hypersoft sets. At the end, a numerical example is given to demonstrate the computational efficiency of the proposed method.

1. Introduction

Game theory has enormous applications in real-world problems as in economics, engineering, biology, etc. The crucial types of games are differential games, matrix games, and continuous static games. Matrix games are named after the discrete relationship between a finite or countable set of alternative decisions and the resulting costs. In terms of a matrix (or two-player games), one player's decision corresponds to the selection of a row, and the other player's decision relates to the selection of a column, with the accompanying entries signifying the costs. It is evident that cooperative games do not necessitate the use of decision probabilities. As a result, there is no interplay between costs and decisions in games that are purely static. Differential games are distinguished by a dynamic system regulated by ordinary differential equations and costs that are always changing. There are a variety of approaches to solving the problem of continuous, static games. The player's own personality also has a role on how he or she employs these notions in the context of the game. Depending on the circumstances, a player may or may not be able to play logically, cheat, cooperate, bargain, and so on. All of these considerations must be taken into account by a player when deciding on a control vector.

Although the mentioned approaches are very suitable, however, in the real-world problems, all or some of parameters are vague and uncertain. Therefore, these techniques cannot handle CSG with uncertain problem. There are numerous works in the field of fuzzy and fuzzy extension set optimization; for example, see [1–15]. However, these models cannot solve CSG.

Vincent and Grantham [16] introduced different formulations in continuous static games (CSG). This game uses three essential concepts: min-max solutions (MMS), Nash equilibrium solution, (NES), and Pareto minimum solutions (PMS). Vincent and Leitmann [17] investigated the control-space features of cooperative solutions for all types of games. Mallozzi and Morgan [18] introduced ε-mixed approaches for CSG. El Shafei [19] proposed a new formulation of large scale CSG and explained how they can solve the mentioned problem by the concept of PMS and in [20] suggested an interactive compromise programming for a kind of Cooperative CSG (CCSG). Kenneth et al. [21] designated some methods for solving all solutions of polynomial systems and then using these compute the equilibrium manifold of a kind of CSG, see also [22–27].

Continuous static games with fuzzy parameters can be solved using the Stackelberg leader and min-max follower's solution presented by Osman et al. [28]. Osman et al. [29] also created the Nash equilibrium solution for large-scale continuous static games with parameters in all cost functions and constraints, where players are autonomous and do not participate with any other players, and each player strives to minimize their cost functions. In addition, the information that is available to every player contains the cost functions and constraints. Khalifa and Zeineldin [30] introduced a fuzzy version of CSG and using α-level sets, and reference attainable point technique suggested a solution for it. Kenneth et al. [21] using the solution of multiobjective nonlinear programming problems proposes a solution for CCSG. She also in [31] studied a CCSG with k players in fuzzy environment and presented an algorithmic approach for it. Elnaga et al. [32] focused on hybrid CSGs that contain several players playing autonomously using the NES and others playing under a secure concept using MMS in fuzzy environment, see also [33–40]. Khalifa et al. [41, 42] studied continuous static games and applied different approaches for solving this problem. Garg et al. [43] have introduced CCSG having possibilistic parameters in the cost functions.

In this paper, we proposed a new methodology based on the weighted Tchebycheff method to solve CCSG with n -players that have piecewise quadratic fuzzy number (PQFN) in the cost functions of the players. Moreover, the stability set of the first kind corresponding to the α-optimal compromise solution has been determined. One of the main advantages of our approach is that this method enables the decision-maker to have satisfactory solution and therefore can applied it for different real-world problems with various types of fuzzy numbers.

2. Research Gap and Motivation

(i) The phrase" pentagonal fuzzy number" is actually meant for dispensing the fuzzy value to each attribute/subattribute in the domain of singleargument/multiargument approximate function

(1) Many researchers discussed the fuzzy set-like structures under soft set environment with fuzzy set-like settings

(2) Along these lines, another construction requests its place in writing for tending to such obstacle; so, fuzzy set is conceptualized to handle such situations

The rest of the paper is arranged as follows: Section 3 offers some necessary prerequisites for this work. The mathematical model for continuous cooperative static games is presented in Section 4. Section 5 presents a method for finding the best compromise solution. Section 6 illustrates the concept with a numerical example. A comparison of existing algorithms and our suggested technique is shown in Section 7. Finally, in section 8, some findings are presented.

3. Basic Concepts

Here, we study some basic concepts that is need for other sections; for more details, see [44, 45].

Definition 1. (Zadeh [44]). A fuzzy set $\tilde{\mathrm{W}}$ characterized by real line $\mathfrak{R}$ is referred as fuzzy number, provided the function: $\mu_{\tilde{\mathrm{Q}}}(\mathrm{x})$: $\mathfrak{R} \longrightarrow [0, 1]$ and confirms the below conditions:

(1) The mapping $\mu_{\tilde{\mathrm{W}}}(\mathrm{x})$ is an upper semicontinuous

(2) The set $\tilde{W}$ is convex, i.e., $\mu_{\tilde{W}}(\delta\,\mathrm{x} + (1-\delta)\,\mathrm{y}) \geq \min\{\mu_{\tilde{W}}(x), \mu_{\tilde{W}}(\mathrm{y})\} \forall \mathrm{x}, \mathrm{y} \in \mathfrak{R}\,; 0 \leq \delta \leq 1$

(3) The set $\tilde{W}$ is normal, i.e., there exists a point $x_0 \in \mathfrak{R}$, so that $\mu_{\tilde{W}}(x_0)$ equals to 1

(4) Supp $(\tilde{W}) = \{x \in \mathfrak{R} : \mu_{\tilde{Q}}(x) > 0\}$ is treated as support of $\tilde{W}$, and the set "closure cl(Supp($\tilde{W}$))" is compact

Definition 2. (Jain [45]). A PQFN is denoted by $\tilde{W}_{PQ} = (w_1, w_2, w_3, w_4, w_5)$, where $w_1 \leq w_2 \leq w_3 \leq w_4 \leq w_5$ are real numbers, and is defined by if its membership function $\mu_{\tilde{\mathrm{W}}_{\mathrm{PQ}}}$ is given by

$$\mu_{\tilde{W}_{PQ}} = \begin{cases} 0, x < w_1\,; \\ \dfrac{1}{2}\dfrac{1}{(w_2 - w_1)^2}(x-u_1)^2, w_1 \leq x \leq w_2\,; \\ \dfrac{1}{2}\dfrac{1}{(w_3 - w_2)^2}(x-w_2)^2 + 1, w_2 \leq x \leq w_3\,; \\ \dfrac{1}{2}\dfrac{1}{(w_4 - w_3)^2}(x-w_3)^2 + 1, w_3 \leq x \leq w_4\,; \\ \dfrac{1}{2}\dfrac{1}{(w_5 - w_4)^2}(x-w_4)^2, w_4 \leq x \leq w_5\,; \\ 0, x > w_5. \end{cases}$$

$$\mu_{\tilde{\mathrm{W}}_{\mathrm{PQ}}} \tag{1}$$

Figure 1 shows a graphical view of PQFN.

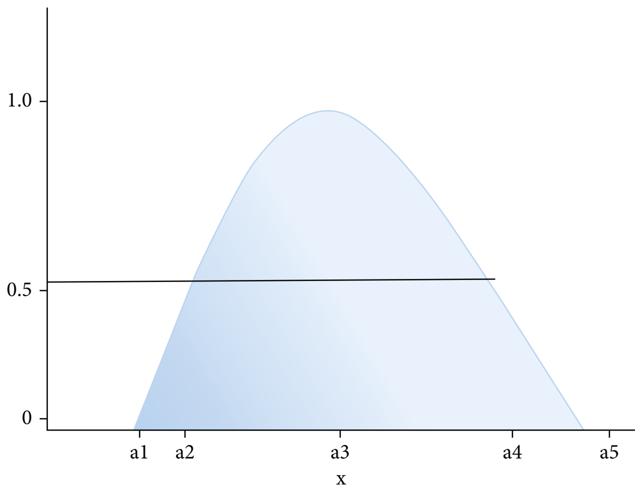

Figure 1: Graphical representation of PQFN.

Definition 3. (Jain [45]). Let $\tilde{U}_{PQ} = (u_1, u_2, u_3, u_4, u_5)$ and $\tilde{V}_{PQ} = (v_1, v_2, v_3, v_4, v_5)$ be two piecewise quadratic fuzzy numbers. The arithmetic operations on $\tilde{U}_{PQ}$ and $\tilde{V}_{PQ}$ are as follows:

(i) Addition: $\tilde{U}_{PQ}(+)\tilde{V}_{PQ} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5)i)$

(ii) Subtraction: $\tilde{U}_{PQ}(-)\tilde{V}_{PQ} = (u_1 - v_5, u_2 - v_4, u_3 - v_3, u_4 - v_2, u_5 - v_1)$

(iii) Scalar multiplication:

$$k\tilde{U}_{PQ} = \begin{cases} (ku_1, ku_2, ku_3, ku_4, ku_5), k > 0, \\ (ku_5, ku_4, ku_3, ku_2, ku_1), k < 0. \end{cases} \quad (2)$$

Definition 4. (Jain [45]). For the close interval approximation of PQFN of $[U] = [U_\alpha^-, U_\alpha^+]$, we called $\widehat{U} = \mathrm{U}_\alpha^- + \mathrm{U}_\alpha^+/2$ as the associated real number of $[U]$.

Definition 5. (Jain [45]). For $[U] = [U_\alpha^-, U_\alpha^+]$, and $[V] = [V_\alpha^-, V_\alpha^+]$, we have the following properties:

(1) Addition: $[U](+)[V] = [U_\alpha^- + Vb_\alpha^-, U_\alpha^+ + V_\alpha^+]$

(2) Subtraction: $[U](-)[V] = [U_\alpha^- - V_\alpha^+, U_\alpha^+ - V_\alpha^-]$

(3) Scalar multiplication: $\mathrm{k[U]} = \begin{cases} [kU_\alpha^-, kU_\alpha^+], k > 0 \\ [kU_\alpha^+, kU_\alpha^-], k < 0 \end{cases}$

(4) Multiplication: $[U](\times)[V]$

$$\left[\frac{\mathrm{U}_\alpha^+\mathrm{V}_\alpha^- + \mathrm{U}_\alpha^-\mathrm{V}_\alpha^+}{2}, \frac{\mathrm{U}_\alpha^-\mathrm{Vb}_\alpha^- + \mathrm{U}_\alpha^+\mathrm{V}_\alpha^+}{2}\right]. \quad (3)$$

(5) Division: $[U](\div)[V]$

$$\begin{cases} \left[2\left(\frac{U_\alpha^-}{V_\alpha^- + V_\alpha^+}\right), 2\left(\frac{U_\alpha^+}{V_\alpha^- + V_\alpha^+}\right)\right], [V] > 0, V_\alpha^- + V_\alpha^+ \neq 0, \\ \left[2\left(\frac{U_\alpha^+}{V_\alpha^- + V_\alpha^+}\right), 2\left(\frac{U_\alpha^-}{V_\alpha^- + V_\alpha^+}\right)\right], [V] < 0, V_\alpha^- + V_\alpha^+ \neq 0. \end{cases} \quad (4)$$

(6) The order relations:

(i) $[U](\leq)[V]$ if $U_\alpha^- \leq V_\alpha^-$ and $U_\alpha^+ \leq V_\alpha^+$ or $U_\alpha^- + U_\alpha^+ \leq V_\alpha^- + V_\alpha^+$

(ii) $[U]$ is preferred to $[V]$ if and only if $U_\alpha^- \geq V_\alpha^-, U_\alpha^+ \geq V_\alpha^+$

4. Problem Formulation and Solution Concepts

A fuzzy cooperative continuous static game (F-CCSG) with $n-$ players having piecewise quadratic fuzzy parameters in the cost functions of the players can be formulated as

$$(F - \mathrm{CCSG}) \quad G_1(b, ,\xi, \tilde{a}_1), G_2(b, \xi, \tilde{a}_1), \cdots, G_m(b, \xi, \tilde{a}_m), \quad \text{Subject to} \quad (5)$$

$$g_j(b, \xi) = 0, j = \overline{1, n}, \quad (6)$$

$$\xi \in \Omega = \left\{\xi \in \mathfrak{R}^s : h_l(b, \xi) \geq 0, l = \overline{1, r}\right\}, \quad (7)$$

where $G_i(b, \xi, \tilde{a}_i), ji = \overline{1, m}$ are convex functions on $\mathfrak{R}^n \times \mathfrak{R}^s$, $h_l(b, \xi), l = \overline{1, r}$ are concave functions on $\mathfrak{R}^n \times \mathfrak{R}^s$, and $g_j(b, \xi), j = \overline{1, n}$ are convex functions on $\mathfrak{R}^n \times \mathfrak{R}^s$. Assume that there exists a function $b = f(\xi)$, if the function $\mathrm{g_j(b, \xi) = 0}$ is of class $\mathfrak{C}^{(1)}$, then the Jacobian $|\partial \mathrm{g_j(b, \xi)}/\partial \mathrm{b_q}| \neq 0, \mathrm{j\,;\,q} = \overline{1, \mathrm{n}}$ in the neighborhood of a solution point (b, ξ) to (6), $b = f(\xi)$, is the solution to (6) generated by $\xi \in \Omega$; differentiability assumptions are not needed her for all the functions $G_i(b, \gamma, \tilde{a}_i), i = \overline{1, n}$, and $h_l(b, \xi)$, Ω is a regular and compact set. $\tilde{a}_i, i = \overline{1, m}$ represents a vector of PQFNs. Let $\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_m\,; \mu_{\tilde{a}_1}(a_1), \mu_{\tilde{a}_2}(a_2) \cdots, \mu_{\tilde{a}_m}(a_m)$ be the PQFNs in F-CCSG problem with convex membership functions, respectively.

The following fuzzy form [46, 47] can be used to rewrite the F-CCSG problem:

$$(\alpha - \mathrm{CCSG}) \quad G_1(b, \xi, a_1), G_2(b, \xi, a_2), \cdots, G_n(b, \gamma, a_m), \quad s.t: \quad (8)$$

$$g_j(b, \xi) = 0, j = 1, 2, \cdots, n, \quad (9)$$

$$\Omega = \left\{\xi \in \mathfrak{R}^{\mathrm{s}} : h_l(b, \xi) \geq 0, \mathrm{l} = \overline{1, r}\right\}, \quad (10)$$

$$a_i \in L_\alpha(\tilde{a}_i), i = 1, \bar{m} \tag{11}$$

Definition 6. Let $b = f(\xi)$ be the solution to (9) generated by $\xi \in \Omega$. A point $\xi^* \in \Omega$, is called a $\alpha -$ Pareto optimal solution to the α-CCSG problem, if and only if there does not exist $(\xi, a) \in \Omega \times L_\alpha(\tilde{a}_i)$ such that

$$G_i(f(\xi), \xi, a_i) \le G_i(f(\xi^*), \xi^*, a_i^*);\forall i = 1, \bar{m} \text{ and } G_i(f(\xi), \xi, a_i) < G_i\left(f\left(\bar{\xi}\right), \xi^*, a_i^*\right) \text{ for some } i \in \{1, 2, \cdots, m\}. \tag{12}$$

Based on the optimality of α-CCSG problem concept, we can show that a point $\xi^* \in \Omega$ is a solution to the α-CCSG problem if and only if $\xi^*$is solution to the following $\alpha -$ multiobjective optimization problem:

$$(\alpha - \text{MOP}) \quad \min\left(\bar{G}_1(\xi, a_1), \bar{G}_2(\xi, a_2), \cdots, \bar{G}_m(\xi, a_m)\right)^T, \quad \text{Subject to} \tag{13}$$

$$\Omega = \left\{\xi \in \mathfrak{R}^s : \bar{h}_l(b, \xi) \ge 0, l = 1, \bar{r}\right\}, \tag{14}$$

$$a_i \in L_\alpha(\tilde{a}_i), i = 1, \bar{m}, \tag{15}$$

where $\bar{h}_l(\xi), l = 1, \bar{r}$ is concave functions on $\mathfrak{R}^s$, $\bar{G}_i(\xi, a_i), i = 1, \bar{m}$ are convex functions on $\mathfrak{R}^n \times \mathfrak{R}^t$,$\bar{G}_i(\xi, a_i) = G_i(f(\xi), \xi, a_i)$, and $\bar{h}_l(\xi) = h_l(f(\xi), \xi)$. Assume that the α-MOP is to be stable [48], problem (13) will be solved by the weighting Tchebycheff method:

$$\min_{\xi \in \Omega a_i \in L_\alpha(\tilde{a}_i)} \max_{1 \le i \le m} \left\{ w_i\left(\bar{G}_i(\xi, a_i) - \bar{G}_i\left(\xi^*, a_i^*\right)\right), a_i \in L_\alpha(\tilde{a}_i), i = 1, \bar{m} \right\}, \tag{16}$$

$$\min\left\{\lambda : w_i\left(\bar{G}_i(\xi, a_i) - \bar{G}_i\left(\xi^*, a_i^*\right)\right) \le \lambda, \xi \in \Omega, a_i \in L_\alpha(\tilde{a}_i), i = 1, \bar{m}\right\}, \tag{17}$$

where $w_i \ge$, $i = 1, \bar{m}$, and $\bar{G}_i(\xi^*, a_i^*)$, $i = 1, \bar{m}$ are the ideal targets. It is noted that stability of (α-MOP) implies to the stability of problem (17).

In addition, problem (13) can be treated using the weighting method as

$$\min\left\{\sum_{i=1}^{m} w_i \bar{G}_i(\xi, a_i) : x \in \Omega, a_i \in L_\alpha(\tilde{a}_i), i = 1, \bar{m}\right\}, \text{where } w \ge 0, w \ne 0. \tag{18}$$

We can see that if there is $w^* \ge 0$ such that (ξ^*, a^*) is the unique optimal solution of issue (18) corresponding to the $\alpha -$level, then, (ξ^*, a^*) is an $\alpha -$ Pareto optimal solution of Eq. (13).

Remark 7. The stability of Eqs. (17) and (18) is inextricably linked to the stability of Eq. (13).

5. Solution Procedure

The solution method based on determining the to the $\alpha -$ best compromise solution within the inexact interval of PQFNs has the minimum deviation from the$\bar{G}_i(\xi^*, a_i^*)$, where

$$\bar{G}_i\left(\xi^*, a_i^*\right) = \min_{\xi \in \Omega, a_i \in L_\alpha(\tilde{a}_i)} \bar{G}_i(\xi, a_i), i = 1, \bar{m}. \tag{19}$$

Step 1. Calculate $\bar{G}_i^{\min}$, and $\bar{G}_i^{\max}$ (i.e., individual minimum and maximum) at $\alpha = 0$ and $\alpha = 1$; separately.

Step 2. Calculate the weight from the following:

$$w_i = \frac{\bar{G}_i^{\max} - \bar{G}_i^{\min}}{\sum_{i=1}^{m}\left(\bar{G}_i^{\max} - \bar{G}_i^{\min}\right)}. \tag{20}$$

Step 3. Formulate and solve Eq. (21).

$$\min \lambda, \quad \text{Subject to} \tag{21}$$

$$W_i\left(\bar{G}_i(\xi, a_i) - \bar{G}_i\left(\xi^*, a_i^*\right)\right) \le \lambda, i = 1, \bar{m}, \tag{22}$$

$$\xi \in \Omega, a_i = \left[(a_i)_\alpha^-, (a_i)_\alpha^+\right], i = 1, \bar{m}, \tag{23}$$

where $W_i \ge 0, i = 1, \bar{m}$, $\sum_{i=1}^{m} w_i = 1$, $[(a_{1i})_\alpha^-, (a_{2i})_\alpha^+] = L_\alpha(\tilde{a}_i)$, $i = 1, \bar{m}$

Let (ξ°, a_i°) be the $\alpha -$ optimal compromise solution.

Step 4. Determine $S(\xi^\circ, a_i^\circ)$

Let $d = (d_1, d_2) \in \mathfrak{R}^{2m}$, where $d_1 = (d_{11}, \cdots, d_{im})^T$, $d_2 = (d_{21}, \cdots, d_{2m})^T$. Assume that problem (21) can be solved for $(w^\circ, d^\circ) \in \mathfrak{R}^{3m}$ and that an $\alpha -$ Pareto optimum solution (ξ°, a_i°) can be found, then $S(\xi^\circ, a_i^\circ)$ is determined by applying the following conditions:

$$\begin{gathered} \zeta_i^\circ\left(a_i^\circ - d_{2i}\right) = 0, i = 1, \bar{m}, \\ \eta_i^\circ(d_{1i} - a_i^\circ) = 0, i = 1, \bar{m}, \\ \zeta_i^\circ, \eta_i^\circ \ge 0, d_{1i}, d_{2i} \in \mathfrak{R}, \left[(a_{1i})_\alpha^-, (a_{2i})_\alpha^+\right] = L_\alpha(\tilde{a}_i), i = 1, \bar{m} \end{gathered} \tag{24}$$

6. A Numerical Example

Consider the following two-player game with

$$\begin{aligned} \bar{G}_1(\xi, \tilde{a}_1) &= (\xi_1 - \tilde{a}_1)^2 + (\xi_2 - 1)^2, \\ \bar{G}_2(\xi, \tilde{a}_2) &= (\xi_1 - 1)^2 + \tilde{a}_2(\xi_2 - 2)^2, \end{aligned} \tag{25}$$

where player 1 controls $\xi_1 \in \mathfrak{R}$, and player 2 controls $\xi_2 \in \mathfrak{R}$ with

$$\xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0. \tag{26}$$

Let $\tilde{a}_1 = (1, 2, 3, 4, 5,)$ and $\tilde{a}_1 = (1, 3, 5, 9, 10)$ with the close interval approximation be $[(\tilde{a}_1)_\alpha] = [2, 4]$ and $[(\tilde{a}_2)_\alpha] = [3, 9]$.

Step 1. Solve the following:

$$\begin{gathered}\min\ (\xi_1 - 1)^2 + (\xi_2 - 1)^2 \\ \text{Subject to} \\ \xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0, \mu_{\tilde{a}_1}(a_1) = 0, \mu_{\tilde{a}_2}(a_2) = 0.\end{gathered}, \tag{27}$$

Let $(\xi_1, \xi_2, a_1 = 1) = (1, 1, 1)$ with $\bar{G}_1^{\min} = 0$.

Solve

$$\begin{gathered}\min\ (\xi_1 - 1)^2 + 10(\xi_2 - 2)^2 \\ \text{Subject to} \\ \xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0, \mu_{\tilde{a}_1}(a_1) = 0, \mu_{\tilde{a}_2}(a_2) = 0.\end{gathered}, \tag{28}$$

Let $(\xi_1, \xi_2, a_2 = 1) = (1, 2, 1)$ with $\bar{G}_2^{\min} = 0$.

Solve

$$\begin{gathered}\max\ (\xi_1 - 3)^2 + (\xi_2 - 1)^2 \\ \text{Subject to} \\ \xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0, \mu_{\tilde{a}_1}(a_1) = 1, \mu_{\tilde{a}_2}(a_2) = 1.\end{gathered}, \tag{29}$$

Let $(\xi_1, \xi_2, a_1 = 3) = (0, 4, 3)$ with $\bar{G}_1^{\max} = 18$.

Solve

$$\begin{gathered}\max\ (\xi_1 - 1)^2 + 5(\xi_2 - 2)^2 \\ \text{Subject to} \\ \xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0, \mu_{\tilde{a}_1}(a_1) = 1, \mu_{\tilde{a}_2}(a_2) = 1.\end{gathered}, \tag{30}$$

Let $(\xi_1, \xi_2, a_2 = 5) = (4, 0, 5)$ with $\bar{G}_2^{\max} = 29$.

Step 2. $w_1 = \bar{G}_1^{\max} - \bar{G}_1^{\min}/(\bar{G}_1^{\max} - \bar{G}_1^{\min}) + (\bar{G}_2^{\max} - \bar{G}_2^{\min}) = 0.383$ and $w_2 = \bar{G}_2^{\max} - \bar{G}_2^{\min}/(\bar{G}_1^{\max} - \bar{G}_1^{\min}) + (\bar{G}_2^{\max} - \bar{G}_2^{\min}) = 0.617$.

Step 3. Solve the following:

$$\begin{gathered}\min\ \lambda \\ \text{Subject to} \\ (\xi_1 - a_1)^2 + (\xi_2 - 1)^2 - \frac{47}{18}\lambda \le 0, \\ (\xi_1 - 1)^2 + a_2(\xi_2 - 2)^2 - \frac{47}{29}\lambda \le 0, \\ 2 \le a_1 \le 4, = [2, 4], \text{and } 3 \le a_2 \le 9, \\ \xi_1 - 4 \le 0, \xi_2 - 4 \le 0, -\xi_1 \le 0, -\xi_2 \le 0,\end{gathered} \tag{31}$$

and yields $\xi_1^\circ = 1.440665, \xi_2^\circ = 1, a_1^\circ = 2, a_2^\circ = 3$ and $\lambda^\circ = 0.1198169$.

Step 4. Determine $S(1.440665, 1, 2, 3)$ by applying the following conditions:

$$\begin{gathered}\zeta_1^\circ(2 - d_{21}) = 0, \zeta_2^\circ(3 - d_{22}) = 0, \\ \eta_1^\circ(d_{11} - 2) = 0, \eta_2^\circ(3 - d_{12}) = 0, \\ \zeta_1^\circ, \zeta_2^\circ; \eta_1^\circ, \eta_2^\circ \ge 0, [c_{1i}, c_{2i}] = \mathrm{L}_\alpha(\tilde{a}_i), i = 1, 2.\end{gathered} \tag{32}$$

We have $J_{1k}; J_{2k} \subseteq \{1, 2\}$, for $J_{11} = \{1\}, \zeta_1^\circ, > 0\ \zeta_2^\circ = 0$. For $J_{21} = \{2\}, \eta_1^\circ = 0, \eta_2^\circ = 0$, then

$$S_{J_{11},J_{21}}(1.440665, 1, 2, 3) = \{(d_1, d_2) \in \mathfrak{R}^4 : d_{21} = 2, d_{22} \ge 3, d_{11} \le 2, d_{12} = 3\}. \tag{33}$$

For $J_{12} = \{2\}, \zeta_1^\circ = 0, \zeta_2^\circ > 0$. For $J_{22} = \{1\}, \eta_1^\circ > 0, \eta_2^\circ = 0$, then

$$S_{J_{12},J_{22}}(1.440665, 1, 2, 3) = \{(d_1, d_2) \in \mathfrak{R}^4 : d_{21} \ge 2, d_{22} = 3, d_{11} = 2, d_{12} \le 3\}. \tag{34}$$

For $J_{13} = \{1, 2\}, \zeta_1^\circ > 0, \zeta_2^\circ > 0$. For $J_{23} = \emptyset, \eta_1^\circ = 0, \eta_2^\circ = 0$, then

$$S_{J_{13},J_{23}}(1.440665, 1, 2, 3) = \{(d_1, d_2) \in \mathfrak{R}^4 : d_{21} = 2, d_{22} = 3, d_{11} \le 2, d_{12} \le 3\}. \tag{35}$$

For $J_{14} = \emptyset, \zeta_1^\circ = 0, \zeta_2^\circ = 0$. For $J_{24} = \{1, 2\}, \eta_1^\circ > 0, \eta_2^\circ > 0$, then

$$S_{J_{14},J_{24}}(1.440665, 1, 2, 3) = \{(d_1, d_2) \in \mathfrak{R}^4 : d_{21} \ge 2, d_{22} \ge 3, d_{11} = 2, d_{12} = 3\}. \tag{36}$$

Hence,

$$S(1.440665, 1, 2, 3) = \bigcup_{k=1}^{4} S_{J_{1k},J_{2k}}(1.440665, 1, 2, 3). \tag{37}$$

7. Comparative Study

In order to highlight the merits of the proposed approach, Table 1 compares the suggested strategy to some current literature.

8. Conclusions and Future Works

In this paper, the weighted Tchebycheff method has applied to solve cooperative continuous static games with piecewise quadratic fuzzy numbers, and then the stability set of the

Table 1: Comparisons of the contributions of various researchers.

Author's name	Weighted Tchebycheff method	α – Pareto optimal solution	Optimal compromise solution	Parametric study	Environment
Zaichenko [49]	↓	↓	↑	↓	Fuzzy
Donahue et al. [50]	↓	↓	↓	↓	Crisp
Zhou et al. [51]	↓	↓	↑	↓	Fuzzy
Our investigation	↑	↑	↑	↑	Fuzzy

The symbols "↓" and "↑" shown in Table 1 represent whether the associated feature satisfy or not.

first kind corresponding to the $\alpha-$optimal compromise solution has determined. The advantages of the approach are the ability to enable the decision-maker to have satisfactory solution and applied for different real-world problems with various types of fuzzy numbers. The key features of this work can be summarized as follows:

(i) The fundamental theory of fuzzy set is developed and its decision constructed. A real-world problem is discussed with the support of proposed algorithm and decision support of fuzzy set

(ii) The rudiments of f fuzzy set are characterized and

(iii) The proposed model and its decision-making based system are developed. A real-life problem is studied with the help of proposed algorithm, and decision system of fuzzy set is compared professionally via strategy with some existing relevant models keeping in view important evaluating features

(iv) The particular cases of proposed models of fuzzy set are discussed with the generalization of these structures

(v) As the proposed model is inadequate with the situation in the domain of multiargument approximate function, it is mandatory. Therefore, future work may include the addressing of this limitation and the determination

Data Availability

No data were used to support this study.

Consent

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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