Asian Journal of Research in Computer Science

8(1): 21-45, 2021; Article no.AJRCOS.67438
ISSN: 2581-8260

Conventional and Improved Inclusion-Exclusion Derivations of Symbolic Expressions for the Reliability of a Multi-State Network

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Authors' contributions

This work was carried out in collaboration between the two authors. Author AMAR wrote the entire draft of the manuscript, conducted the mathematical and conceptual analyses and managed the basic literature survey. Author MHA participated in the literature search, performed the computational work, and constructed the table of results. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJRCOS/2021/v8i130191
Editor(s):
(1) Dr. Jong-Wuu Wu, National Chiayi University, Taiwan.
Reviewers:
(1) M. Shyamala Devi, Dr. Sagunthala R&D Institute of Science and Technology Avadi, India.
(2) Ramjeet Singh Yadav, Ashoka Institute of Technology and Management, India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/67438

Original Research Article

Received 16 February 2021
Accepted 22 April 2021
Published 27 April 2021

ABSTRACT

This paper deals with an emergent variant of the classical problem of computing the probability of the union of n events, or equivalently the expectation of the disjunction (ORing) of n indicator variables for these events, i.e., the probability of this disjunction being equal to one. The variant considered herein deals with multi-valued variables, in which the required probability stands for the reliability of a multi-state delivery network (MSDN), whose binary system success is a two-valued function expressed in terms of multi-valued component successes. The paper discusses a simple method for handling the afore-mentioned problem in terms of a standard example MSDN, whose success is known in minimal form as the disjunction of prime implicants or minimal paths of the pertinent network. This method utilizes the multi-state inclusion-exclusion (MS-IE) principle associated with a multi-state generalization of the idempotency property of the ANDing operation. The method discussed is illustrated with a detailed symbolic example of a real-case study, and it produces a more precise version of the same numerical value that was obtained earlier. The example demonstrates the notorious shortcomings and the extreme inefficiency that the MS-IE

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method suffers, but, on the positive side, it reveals the way to alternative methods, in which such a shortcoming is (partially) mitigated. A prominent and well known example of these methods is the construction of a multi-state probability-ready expression (MS-PRE). Another candidate method would be to apply the MS-IE principle to the union of fewer (factored or composite) paths that is converted (at minimal cost) to PRE form. A third candidate method, employed herein, is a novel method for combining the MS-PRE and MS-IE concepts together. It confines the use of MS-PRE to 'shellable' disjointing of ORed terms, and then applies MS-IE to the resulting partially orthogonalized disjunctive form. This new method makes the most of both MS-PRE and MS-IE, and bypasses the troubles caused by either of them. The method is illustrated successfully in terms of the same real-case problem used with the conventional MS-IE.

Keywords: Network reliability; inclusion-exclusion; probability-ready expression; multi-state system; symbolic expression; multi-state delivery network.

1. INTRODUCTION

The Inclusion-Exclusion Principle is a very useful principle of enumeration in combinatorics and discrete probability [1-20]. This principle computes the cardinality of the union of n sets, through a finite repetition of alternation between a usually over-generous inclusion and a usually over-compensating exclusion. This principle remains valid when set cardinalities are replaced by probabilities. In the probability context, the IE Principle is used for computing the probability of the union of n events. The IE principle proceeds to achieve this computation by first including (adding) the probabilities of the n events (corresponding to a number of terms that equals n choose $1 = \mathrm{n}$). This is followed by excluding (subtracting) the probabilities of the (n choose 2) pairwise intersections of these events. Next, the probabilities of the (n choose 3) triple-wise intersections of the n events are included (added), the probabilities of the (n choose 4) quadruple-wise intersections of the n events are excluded (subtracted), the probabilities of the (n choose 5) quintuple-wise intersections of the n events are included (added). This alternation of addition (inclusion) and subtraction (exclusion) is continued until the sole intersection of all the n events (corresponding to n choose $\mathrm{n} = 1$) is included (added) (if n is odd) or excluded (subtracted) (if n is even).

This paper deals with a fundamental application of the IE principle to the computation of multi-state reliability, specifically the computation of the expectation of the logical expression of a multi-state disjunctive normal form (DNF). Currently, the most computationally efficient method for handling this problem is an automated implementation of the method of the recursive sum of disjoint products (RSDP) [21,22]. We present a tutorial discussion and exposition of the conventional IE method and an improved IE approach for solving this problem. This paper is a part of an on-going activity [23-31] that strives to provide a pedagogical treatment and exposition of multi-state reliability problems. We aspire to establish a clear and insightful interrelationship between the two-state modeling and the multi-state one by stressing that multi-valued concepts are natural and simple extensions of two-valued ones. Moreover, we hope to mitigate the notorious shortcomings of the conventional MS-IE procedure by combining it with a 'shellable' version of the concept of the sum of disjoint products (SDP), or the more encompassing concept of a probability-ready expression (PRE)) in the multi-state domain.

The organization of the remainder of this paper is as follows. Section 2 introduces the running example used herein of a multi-state delivery network (MSDN) with multiple suppliers, borrowed from Lin et al. [22]. Section 3 introduces the multi-state inclusion-exclusion (MS-IE) principle, and hints at its special cases and improved variants. Section 4 symbolically applies MS-IE, in its standard or conventional form, to the running example. Section 5 introduces a 'shellable' version of the concept of a multi-state probability-ready expression (MS-PRE), uses it in conjunction with the MS-IE principle, and demonstrates its applicability in terms of the running example. Section 6 reports and discusses numerical results for the two solutions given in Sections 4 and 5. Section 7 concludes the paper. Appendix A presents the python listing of a program that obtains the conventional IE solution of the running example.

1.1 Specifications for a Running Example

Lin et al. [22] studied a specific multi-state delivery network (MSDN) with multiple suppliers, one market, multiple transfer centers and eight branches. They derived an expression of system success for specific data of delivery costs, probability distributions of all branches, available capacities, suppliers' production capacities, deterioration rate vector for the minimal paths obtained, demand, and budget. They presented the final multi-state success in their Table 2, which is expressed in our following formula with an appropriate translation of notation.

$$\begin{aligned} S = {} & X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\} \\ & \vee X_3\{\geq 3\}\, X_7\{\geq 3\} \\ & \vee X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\} \\ & \vee X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\} \\ & \vee X_2\{\geq 3\}\, X_7\{\geq 3\} \\ & \vee X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\} \\ & \vee X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\} \\ & \vee X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}. \qquad (1) \end{aligned}$$

Note that the expression of system success S in (1) reveals clearly that it pertains to a coherent system that enjoys causality, monotonicity and component relevancy [23-29]. The expression comprises eight distinct prime implicants, none of which subsumes (can be absorbed) in another. Each prime implicant is a product of solely upper values $X_k\{\geq j\}$ of various variables. The numerical values for the expectations of various variable instances, computed from the data given in Lin et al.[22] and used in [30,31] are listed in Table 1.

1.2 Inclusion-Exclusion Principle for Multi-State Probabilities

We note that computing the probability of the union of n events is equivalent to calculating the expectation of the disjunction (ORing) of the n indicator variables of such events. Usually these indicator variables are products of instances of the underlying variables. These products usually stand for the minimal paths of the system, which are the prime implicants P_i of system success, or for the minimal cutsets of the system, which are the prime implicants C_i of system failure. Note that the expectations of system success and failure are the reliability and unreliability of the system. With this interpretation, an application of the IE principle results in the following expression of reliability [32-36].

$$R = E\{\vee_{i=1}^{n_p} P_i\} = \sum_{i=1}^{n_p} E\{P_i\} - \sum\sum_{1\leq i<j\leq n_p} E\{P_i \wedge P_j\} + \sum\sum\sum_{1\leq i<j<k\leq n_p} E\{P_i \wedge P_j \wedge P_k\} - \ldots + (-1)^{n_p-1} E\{\wedge_{i=1}^{n_p} P_i\}. \qquad (2)$$

A formal proof of the IE formula is available in Hall [37], Feller [38] and Trivedi [39]. Cerasoli and Fedullo [9] discuss and compare the various available proofs for the IE principle. The number of terms in (2) is given by

$$\binom{n_p}{1} + \binom{n_p}{2} + \binom{n_p}{3} + \cdots + \binom{n_p}{n_p} = 2^{n_p} - 1, \qquad (3)$$

i.e., it is exponential in the number of minimal paths. To apply the IE principle to (1), which has $n_p = 8$, we need 255 terms as we will see in the sequel.

Table 1. Numerical values for the expectations of various variable instances, computed from data given in [22], and used in [30,31]

$X_1\{\geq 2\}$	0.897	$X_3\{\geq 3\}$	0.905
$X_2\{\geq 3\}$	0.892	$X_3\{\geq 2\}$	0.953
$X_2\{< 3\}$	0.108	$X_3\{2\}$	0.048
$X_2\{\geq 2\}$	0.965	$X_3\{< 3\}$	0.095
$X_2\{2\}$	0.073	$X_4\{\geq 2\}$	0.863
$X_4\{< 2\}$	0.137	$X_5\{\geq 3\}$	0.903
$X_5\{< 3\}$	0.097	$X_6\{\geq 2\}$	0.943
$X_7\{\geq 2\}$	0.945	$X_7\{\geq 3\}$	0.884
$X_7\{2\}$	0.061	$X_7\{< 3\}$	0.116
$X_8\{\geq 3\}$	0.906	$X_8\{\geq 2\}$	0.965
$X_8\{2\}$	0.059	$X_8\{< 3\}$	0.094

The IE principle is valid and applicable whether the implicants P_i and their constituting variables are two-valued or multi-valued. However, the implementation of the IE formula (2) in the multi-state case needs to be aided by simplification rules for various products of the underlying variables. The IE simplicity is manifested in the fact that the simplification rules it requires (when handling coherent success) is just a single rule, namely, the following domination rule (which generalizes the idempotency rule of AND for an uncomplemented literal ($X_k \wedge X_k = X_k$) in the two-valued case).

$$X_k(\geq j_1)\, X_k(\geq j_2) = X_k(\geq j_2) \qquad \text{for } j_2 \geq j_1, \quad (4a)$$

A similar simplification required by IE (when handling coherent failure) is the following domination rule (which is another generalization of the idempotency rule of AND for a complemented literal ($\overline{X}_k \wedge \overline{X}_k = \overline{X}_k$) in the two-valued case).

$$X_k(\leq j_1)\, X_k(\leq j_2) = X_k(\leq j_2) \qquad \text{for } j_2 \leq j_1, \quad (4b)$$

Despite the great importance of the IE principle in combinatorics and probability theory, and despite its genuine unrivalled conceptual simplicity, it does not seem to be the method of choice for evaluation of system reliability. It produces an exponential number of terms that have to be reduced subsequently via addition and cancellation. Moreover, it involves so many subtractions that make it highly sensitive to round-off errors in the ultra-reliable regime [36,40-42]. For the problem of the running example, the symbolic computations are tedious, indeed.

In passing, we note that if the paths P_i in equation (2) are mutually disjoint ($P_i \wedge P_j = 0$ for $1 \leq i < j \leq n_p$), then the complexity of this equation reduces from exponential to linear, viz.

$$R = E\{\vee_{i=1}^{n_p} P_i\} = \sum_{i=1}^{n_p} E\{P_i\}. \quad (5)$$

When many of the products P_i are mutually disjoint, then many of the pair-wise and k-tuple-wise intersections in (2) vanish, and the number of non-zero terms in (2) might decrease dramatically, thereby allowing one of the most effective IE improvements.

On the other hand, if the paths P_i in equation (2) are statistically independent, then (2) reduces to

$$R = E\{\vee_{i=1}^{n_p} P_i\} = \sum_{i=1}^{n_p} E\{P_i\} - \sum\sum_{1\leq i<j\leq n_p} E\{P_i\}E\{P_j\} + \sum\sum\sum_{1\leq i<j<k\leq n_p} E\{P_i\}E\{P_j\}E\{P_k\} - \ldots + (-1)^{n_p-1}\prod_{i=1}^{n_p} E\{P_i\} = 1 - \prod_{i=1}^{n_p}(1 - E\{P_i\}), \quad (6)$$

which might be obtained through an application of De Morgan law to the complement of the union of products P_i so as to obtain the intersection of complemented products $\overline{P_i}$, which is a probability-ready-expression since the complemented products $\overline{P_i}$ are statistically independent. Kessler [10] points out that when the products P_i are, in some sense, close to being independent, then there are many useful results bounding $E\{\vee_{i=1}^{n_p} P_i\}$, so that one might hope to obtain good estimates for it.

In short, we note that the IE complexity decreases dramatically for the two extreme cases of the products P_i being either statistically independent or mutually exclusive. There is also some appreciable improvements if these products P_i are, in some sense, close to either being statistically independent or mutually exclusive. To understand why these two cases are opposite extremes, and how to effectively utilize one of them, the interested reader might consult some of the References [43-47].

1.3 Application of the Conventional Inclusion-Exclusion Principle to the Running Example

Equation (1) might be rewritten as a disjunction of eight paths, namely

$$S = P_1 \vee P_2 \vee P_3 \vee P_4 \vee P_5 \vee P_6 \vee P_7 \vee P_8, \quad (7)$$

where

$$P_1 = X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_2 = X_3\{\geq 3\}\, X_7\{\geq 3\}$$

$$P_3 = X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_4 = X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$$

$$P_5 = X_2\{\geq 3\}\, X_7\{\geq 3\}$$

$$P_6 = X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}$$

$$P_7 = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}$$

$$P_8 = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}$$

The symbolic application of the IE formula (2) to the disjunction in (7) is very tedious, indeed. Hopefully, the reader would bear with this cumbersome computation, in which the derivation of 255 terms is involved, and through which repeated use is made of the domination rule (4a).

$P_1P_2 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_3\{\geq 3\}\, X_7\{\geq 3\}) = X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\}$

$P_1P_3 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_4 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}) = X_2\{\geq 2\}\, X_3\{\geq$
$3\; X4\geq 2X5\geq 3X7\geq 3\; X8\{\geq 3\}$

$P_1P_5 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\} X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_6 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_3\{\geq$
$3\; X4\geq 2X5\geq 3\; X6\geq 2X7\geq 2\; X8\{\geq 3\}$

$P_1P_7 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$2X3\geq 3\; X4\geq 2X5\geq 3\; X6\geq 2X7\geq 2\; X8\{\geq 3\}$

$P_1P_8 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\} X_3\{\geq$
$3X5\geq 3\; X6\geq 2X7\geq 3\; X8\{\geq 3\}$

$P_2P_3 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\}) = X_2\{\geq 3\} X_3\{\geq 3\} X_5\{\geq 3\} X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_4 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}) = X_2\{\geq 2\} X_3\{\geq 3\}\, X_4\{\geq$
$2X7\geq 3\; X8\{\geq 2\}$

$P_2P_5 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\} X_3\{\geq 3\} X_7\{\geq 2\}$

$P_2P_6 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq$
$2\; X6\geq 2\; X7\geq 3\; X8\{\geq 2\}$

$P_2P_7 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$2\; X3\geq 3X4\geq 2X6\geq 2\; X7\geq 3\; X8\{\geq 2\}$

$P_2P_8 = (X_3\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\} X_6\{\geq$
$2\; X7\{\geq 3\}$

$P_3P_4 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}) = X_2\{\geq 3\} X_3\{\geq 2\}\, X_4\{\geq$
$2X5\geq 3X7\geq 3\; X8\{\geq 3\}$

$P_3P_5 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\} X_5\{\geq 3\} X_7\{\geq 3\} X_8\{\geq 3\}$

$P_3P_6 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$3X3\geq 2\; X4\geq 2X5\geq 3\; X6\geq 2\; X7\geq 2\; X8\{\geq 3\}$

$P_3P_7 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$3\; X4\geq 2X5\geq 3X6\geq 2\; X7\geq 2\; X8\{\geq 3\}$

$P_3P_8 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq$
$2X5\geq 3X6\geq 2\; X7\{\geq 3\}\; X8\{\geq 3\}$

$P_4P_5 = (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})\, (X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq$
$3\; X8\geq 2$

$P_4P_6 = (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\} X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_4P_7 = (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\} X_3\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_4P_8 = (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\})$

$= X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_5P_6 = (X_2\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq 3\} X_3\{\geq$
2 X4≥2 X6≥2 X7≥3 X8{≥2}

$P_5P_7 = (X_2\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_4\{\geq$
2X6≥2 X7≥3 X8{≥2}

$P_5P_8 = (X_2\{\geq 3\}\, X_7\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 2\} X_6\{\geq$
2 X7{≥3}

$P_6P_7 = (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq$
2 X8{≥2})= X1≥2 X2≥2X3≥2 X4≥2X6≥2 X7≥2 X8{≥2}

$P_6P_8 = (X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq$
$3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, \mathbf{X_7\{\geq 2\}}\, X_8\{\geq 2\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, \mathbf{X_7\{\geq}$
$\mathbf{3\}}) = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, \mathbf{X_7\{\geq 3\}}\, X_8\{\geq 2\}$

$P_1P_2P_3 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})(X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_8\{\geq 3\}) = X_2\{\geq 3\} X_3\{\geq 3\}\, X_5\{\geq$
3 X7≥3X8{≥3}

$P_1P_2P_4 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})\, (X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}) = X_2\{\geq 2\} X_3\{\geq$
3 X4≥2 X5≥3 X7≥3 X8{≥3}

$P_1P_2P_5 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq$
$3\}$

$P_1P_2P_6 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_3\{\geq 3\}\ \ X_4\{\geq 2\} X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\} X_8\{\geq 3\}$

$P_1P_2P_7 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})\, (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{\geq 2\} X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\} X_7\{\geq 3\} X_8\{\geq 3\}$

$P_1P_2P_8 = (X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq$
2X3≥3 X5≥3 X6≥2X7≥3X8≥3

$P_1P_3P_4 = (X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\})(\, X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}) = X_2\{\geq 3\}\, X_3\{\geq$
3 X4≥2X5≥3X7≥3 X8{≥3}

$P_1P_3P_5 = (X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_7\{\geq 3\} X_8\{\geq$
$3\}$

$P_1P_3P_6 = (X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\} X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 3\}$

$P_1P_3P_7 = (X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\} X_5\{\geq 3\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 3\}$

$P_1P_3P_8 = (X_2\{\geq 3\}\, X_3\{\geq 3\} X_5\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq$
3 X3≥3X5≥3X6≥2 X7{≥3} X8{≥3}

$P_1P_4P_5 = (X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\} X_5\{\geq 3\} X_7\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}\, X_3\{\geq$
3 X4≥2X5≥3X7≥3 X8{≥3}

$P_1P_4P_6 = X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq$
$2\, X8\{\geq 2\})=X1\geq 2\, X2\geq 2\, X3\geq 3\, X4\geq 2X5\geq 3\, X6\geq 2X7\geq 3\, X8\{\geq 3\}$

$P_1P_4P_7 = X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$
$2\, X8\{\geq 2\})=X1\geq 2\, X2\geq 2\, X3\geq 3\, X4\geq 2X5\geq 3X6\geq 2X7\geq 3\, X8\{\geq 3\}$

$P_1P_4P_8 = X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) =$
$X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_5P_6 = (X_2\{\geq 3\}\, X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_5P_7 = (X_2\{\geq 3\}\, X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_5P_8 = (X_2\{\geq 3\}\, X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) =$
$X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_6P_7 = (X_1\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq$
$2\, X7\geq 2\, X8\{\geq 2\})=X1\geq 2\, X2\geq 2\, X3\geq 3\, X4\geq 2X5\geq 3\, X6\geq 2X7\geq 2\, X8\geq 3$

$P_1P_6P_8 = (X_1\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq$
$2\, X7\{\geq 3\})=X1\geq 2\, X2\geq 2X3\geq 3\, X4\geq 2X5\geq 3\, X6\geq 2X7\geq 3\, X8\{\geq 3\}$

$P_1P_7P_8 = (\, X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$
$2X6\geq 2\, X7\{\geq 3\})=X1\geq 2\, X2\geq 2X3\geq 3\, X4\geq 2X5\geq 3\, X6\geq 2X7\geq 3\, X8\{\geq 3\}$

$P_2P_3P_4 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})=$
$X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\})= X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq$
$3X7\geq 3\, X8\{\geq 3\}$

$P_2P_3P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\})=$
$X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_4P_5 = (X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq$
$2X7\geq 3\, X8\geq 2$

$P_2P_4P_6 = (X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_7 = (X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_8 = (X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) =$
$X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_5P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}X_7\{\geq 3\}\,)(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$3X3\geq 3\ \ X4\geq 2\, X6\geq 2\, X7\geq 3\, X8\{\geq 2\}$

$P_2P_5P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}X_7\{\geq 3\}\,)(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
$3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}X_7\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3X6\geq 2X7\geq 3$

$P_2P_6P_7 = (X_1\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\ X8\{\geq 2\}) = X1\geq 2\ X2\geq 2X3\geq 3\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_2P_6P_8 = (X_1\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_2P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 2\ X3\geq 3X4\geq 2X6\geq 2\ X7\geq 3\ X8\{\geq 2\}$

$P_3P_4P_5 = (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_2\{\geq 3\}\ X_7\{\geq 3\}) = X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2X5\geq 3X7\geq 3\ X8\geq 3$

$P_3P_4P_6 = (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\ X8\{\geq 2\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_3P_4P_7 = (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\ X8\{\geq 2\}) = X1\geq 2X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3X6\geq 2X7\geq 3\ X8\geq 3$

$P_3P_4P_8 = (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_5P_6 = (X_2\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_3P_5P_7 = (X_2\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_3P_5P_8 = (X_2\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3X3\geq 2X5\geq 3X6\geq 2\ X7\{\geq 3\}X8\{\geq 3\}$

$P_3P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2X6\geq 2\ X7\geq 2\ X8\{\geq 2\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2\ X7\geq 2\ X8\{\geq 3\}$

$P_3P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2X6\geq 2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_3P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3\ X3\geq 2\ X4\geq 2X5\geq 3X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_4P_5P_6 = (X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_4P_5P_7 = (X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_4P_5P_8 = (X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_4P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\ X7\geq 2\ X8\{\geq 2\}) = X1\geq 2\ X2\geq 2X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_4P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\ X7\geq 3 = X1\geq 2\ X2\geq 2X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_4P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_5P_6P_7 = (X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq$
$2\ X7\geq 2\ X8\{\geq 2\})=X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\{\geq 2\}$

$P_5P_6P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\{\geq 2\}$

$P_5P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$
$3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 2X3\geq 2\ X4\geq 2X6\geq 2\ X7\geq 3\ X8\{\geq 2\}$

$$P_1P_2P_3P_4 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_2\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})$$
$$= X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\}$$

$P_1P_2P_3P_5 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\})= X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq$
$3\ X7\geq 3X8\{\geq 3\}$

$$P_1P_2P_3P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{$$
$$\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \, X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$$

$P_1P_2P_3P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\})= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$$P_1P_2P_3P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{$$
$$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$$

$P_1P_2P_4P_5 = (\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}X_3\{\geq$
$3\ X4\geq 2\ X5\geq 3\ X7\geq 3\ X8\{\geq 3\}$

$P_1P_2P_4P_6 = (\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq$
$2\ X8\{\geq 2\})= X1\geq 2X2\geq 2X3\geq 3\ X4\geq 2\ X5\geq 3\ X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_1P_2P_4P_7 = (\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$
$2\ X8\{\geq 2\})= X1\geq 2X2\geq 2X3\geq 3\ X4\geq 2\ X5\geq 3X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_1P_2P_4P_8 = (\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$
$3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_2P_5P_6 =(\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_5P_7 =(\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq$
$2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_5P_8 =(\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) =$
$X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_6P_7$
$= (X_1\{\geq 2\}\, X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})\ (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{$
$\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_6P_8$
$= (X_1\{\geq 2\}\, X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 2\}\, X_3\{\geq 3\}\ \, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$
$2X6\geq 2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 2X3\geq 3\ X4\geq 2\ X5\geq 3\ X6\geq 2X7\geq 3X8\{\geq 3\}$

$P_1P_3P_4P_5 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_2\{\geq 3\}\ X_7\{\geq 3\}) = X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_3P_4P_6 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_3P_4P_7 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_3P_4P_8 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_3P_5P_6 = (X_2\{\geq 3\}\ X_3\{\geq 3\}X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_5P_7 = (X_2\{\geq 3\}\ X_3\{\geq 3\}X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_5P_8 = (X_2\{\geq 3\}\ X_3\{\geq 3\}X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\}$

$P_1P_3P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_3P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$ 2X6≥2 X7{≥3})=X1≥2 X2≥3 X3≥3 X4≥2X5≥3X6≥2 X7≥3 X8{≥3}

$P_1P_4P_5P_6 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_4P_5P_7 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_4P_5P_8 = (X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_4P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_4P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_4P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$ 2X6≥2 X7{≥3})=X1≥2 X2≥2 X3≥3 X4≥2X5≥3X6≥2X7≥3 X8{≥3}

$P_1P_5P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq$ 2X6≥2 X7≥2 X8{≥2})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_1P_5P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ 2X6≥2 X7{≥3})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_1P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_4P_5 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_2\{\geq 3\}\, X_7\{\geq 3\}) = X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq$ 2X5≥3X7≥3 X8≥3

$P_2P_3P_4P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq$ 2 X8{≥2})=X1≥2X2≥3X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8≥3

$P_2P_3P_4P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$ 2 X8{≥2})=X1≥2X2≥3X3≥3 X4≥2X5≥3X6≥2X7≥3 X8≥3

$P_2P_3P_4P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_6P_7 = (X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq$ 2X6≥2 X7≥2 X8{≥2})=X1≥2 X2≥3X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_2P_3P_6P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ 2X6≥2 X7{≥3})=X1≥2 X2≥3X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_2P_3P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_4P_5P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_5P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_6P_7 = (X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_6P_8$
$= (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\})$

$$= X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$$

$P_2P_4P_7P_8$
$= (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_2P_5P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ \ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq$
$2\ X7\geq 2\ X8\geq 2 = X1\geq 2\ X2\geq 3X3\geq 3\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_2P_5P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ \ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3X3\geq 3\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_2P_5P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3X3\geq 3\ X4\geq 2X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_2P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 2X3\geq 3\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_3P_4P_5P_6$
$= (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\})$
$= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_5P_7$
$= (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\})$
$= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq$
$2X6\geq 2\ X7\geq 2\ X8\{\geq 2\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_3P_4P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
$2X6\geq 2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_3P_4P_7P_8$
$= \ (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_5P_6P_7$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_3P_5P_6P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_3P_5P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\})$
$= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_3P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
$2X6\geq 2\ X7\{\geq 3\}) = X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2X5\geq 3\ X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_4P_5P_6P_7 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq$
$2\ X7\geq 2\ X8\{\geq 2\})=X1\geq 2\ X2\geq 3\ X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_4P_5P_6P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 3\ X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_4P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2X2\geq 3\ X3\geq 2\ X4\geq 2\ X6\geq 2X7\geq 3\ X8\geq 2$

$P_4P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 2X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\geq 2$

$P_5P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
$2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 3X3\geq 2\ X4\geq 2\ X6\geq 2\ X7\geq 3\ X8\{\geq 2\}$

$P_1P_2P_3P_4P_5 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_2\{\geq 3\}\ X_7\{\geq 3\})= X_2\{\geq 3\}X_3\{\geq$
$3\ X4\geq 2\ X5\geq 3\ X7\geq 3X8\{\geq 3\}$

$P_1P_2P_3P_4P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq$
$2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2\ X5\geq 3\ X6\geq 2\ X7\geq 3X8\{\geq 3\}$

$P_1P_2P_3P_4P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq$
$2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2\ X5\geq 3X6\geq 2\ X7\geq 3X8\{\geq 3\}$

$P_1P_2P_3P_4P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq$
$3\})= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_5P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq$
$2\})= X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_5P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq$
$2\})= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq$
$3\})= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq$
$2X6\geq 2\ X7\geq 2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2\ X5\geq 3\ \ X6\geq 2X7\geq 3X8\geq 3$

$P_1P_2P_3P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
$2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2\ X5\geq 3\ \ X6\geq 2X7\geq 3X8\geq 3$

$P_1P_2P_3P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ \ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
$2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3X3\geq 3\ \ X4\geq 2X5\geq 3X6\geq 2\ X7\geq 3X8\{\geq 3\}$

$P_1P_2P_4P_5P_6 = (\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}\ X_8\{$
$\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{$
$\geq 2\}) =\ X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_2P_4P_5P_7$
$= (\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}\ X_8\{3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{$
$\geq 2\}) =\ X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_2P_4P_5P_8 = (\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_1P_2P_4P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_2P_4P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_2P_4P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_2P_5P_6P_7 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq$ *2X6≥2 X7≥2 X8{≥2})=X1≥2 X2≥3 X3≥3 X4≥2 X5≥3 X6≥2 X7≥3X8{≥3}*

$P_1P_2P_5P_6P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ *2X6≥2 X7{≥3})=X1≥2 X2≥3 X3≥3 X4≥2 X5≥3 X6≥2 X7≥3X8{≥3}*

$P_1P_2P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_4P_5P_6 = (X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_4P_5P_7 = (X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_4P_5P_8 = (X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_4P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_4P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_4P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_3P_5P_6P_7 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_5P_6P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_3P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ *2X6≥2 X7{≥3})=X1≥2X2≥3 X3≥3 X4≥2X5≥3X6≥2 X7{≥3}X8{≥3}*

$P_1P_3P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 3\ X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2\ X7\geq 3\ X8\{\geq 3\}$

$P_1P_4P_5P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{$
$\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})$
$= X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_4P_5P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{$
$\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_4P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3\ X3\geq 3\ X4\geq 2X5\geq 3X6\geq 2X7\geq 3\ X8\{\geq 3\}$

$P_1P_4P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{$
$\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{$
$\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_1P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3\ X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\{\geq 3\}$

$P_2P_3P_4P_5P_6 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq$ $2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_2P_3P_4P_5P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$ $2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3X6\geq 2X7\geq 3\ X8\geq 3$

$P_2P_3P_4P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq$
$3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_4P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq$ $2X6\geq 2\ X7\geq 2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_2P_3P_4P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\geq 3$

$P_2P_3P_4P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3X6\geq 2X7\geq 3\ X8\geq 3$

$P_2P_3P_5P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{$
$\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{$
$\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{$
$\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$

$P_2P_3P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=(X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3X6\geq 2X7\geq 3\ X8\{\geq 3\}$

$P_2P_3P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 2\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq$ $2X6\geq 2\ X7\{\geq 3\})=X1\geq 2\ X2\geq 3X3\geq 3\ X4\geq 2X5\geq 3\ X6\geq 2X7\geq 3\ X8\{\geq 3\}$

$P_2P_4P_5P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq$ $2\ X7\geq 2\ X8\{\geq 2\})=X1\geq 2X2\geq 3X3\geq 3\ X4\geq 2\ X6\geq 2X7\geq 3\ X8\geq 2$

$P_2P_4P_5P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq$
$2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 2\}$

$P_2P_4P_5P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 3\})$
$= X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_2P_4P_6P_7P_8$
$= (X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 2\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 2\}$

$P_2P_5P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq$
2 X7{≥3})=X1≥2 X2≥3X3≥3 X4≥2 X6≥2 X7≥3 X8≥2

$P_3P_4P_5P_6P_7$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{$
$\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_5P_6P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_5P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\}$

$P_3P_4P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}\ X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
2X6≥2 X7{≥3})=X1≥2 X2≥3X3≥2 X4≥2X5≥3 X6≥2X7≥3 X8≥3

$P_3P_5P_6P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\ X_4\{\geq 2\}X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}\ X_2\{\geq 3\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}\ X_8\{\geq 2\})(\ X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq$
2X6≥2 X7{≥3})=X1≥2 X2≥3 X3≥2 X4≥2 X6≥2 X7≥3 X8≥2

$P_1P_2P_3P_4P_5P_6$
$= (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_3\{\geq 2\}\ X_4\{\geq 2\}\ X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{$
$\geq 2\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_5P_7$
$= (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{\geq 2\})$
$= X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_5P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}\ X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_6P_7$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_4\{\geq 2\}X_6\{\geq 2\}\ X_7\{\geq 2\}\ X_8\{$
$\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_6P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}\ X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\ X_2\{\geq 2\}\ X_3\{\geq 2\}X_6\{\geq 2\}\ X_7\{$
$\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\ X_4\{\geq 2\}\ X_5\{\geq 3\}X_6\{\geq 2\}\ X_7\{\geq 3\}X_8\{\geq 3\}$

$$P_1P_2P_3P_5P_6P_7 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$$

$$P_1P_2P_3P_5P_6P_8 = (X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$$

$$P_1P_2P_3P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$$

$$P_1P_2P_3P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\ \ X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$$

$$P_1P_2P_4P_5P_6P_7 = (\, X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = \ X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_2P_4P_5P_6P_8 = (\, X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = \ X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_2P_4P_5P_7P_8 = (\, X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = \ X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_2P_4P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 2\}X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_2P_5P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$$

$P_1P_3P_4P_5P_6P_7 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq$
2 X4≥2X6≥2 X7≥2 X8{≥2})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_1P_3P_4P_5P_6P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq$
2 X3≥2X6≥2 X7{≥3})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$$P_1P_3P_4P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_3P_4P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\}$$

$$P_1P_3P_5P_6P_7P_8 = (X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_3\{\geq 2\}X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}X_8\{\geq 3\}$$

$P_1P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\, X_3\{\geq 3\}\, X_4\{\geq 2\}X_5\{\geq 3\}\, X_6\{\geq 2\}X_7\{\geq 3\}\, X_8\{\geq 3\})(\, X_1\{\geq 2\}\, X_2\{\geq$
2 X3≥2X6≥2 X7{≥3})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_2P_3P_4P_5P_6P_7$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\,X_8\{\geq 3\})(\,X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_4\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 2\}\,X_8\{\geq 2\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_2P_3P_4P_5P_6P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_7\{\geq 3\}\,X_8\{\geq 3\})(\,X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_2P_3P_4P_5P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq$
2 X3≥2X6≥2 X7{≥3})=X1≥2X2≥3X3≥3 X4≥2X5≥3X6≥2X7≥3 X8≥3

$P_2P_3P_4P_6P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_2P_3P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_2P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 2\})(\,X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq$
2X6≥2 X7{≥3})=X1≥2X2≥3X3≥3 X4≥2 X6≥2X7≥3 X8≥2

$P_3P_4P_5P_6P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(\,X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 2\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_1P_2P_3P_4P_5P_6P_7$
$= (X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\})(\,X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_4\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 2\}\,X_8\{\geq 2\}) = X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_5P_6P_8$
$= (X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\})$

$P_1P_2P_3P_4P_5P_7P_8$
$= (X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_4P_6P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_3P_5P_6P_7P_8$
$= (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\;\,X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\;\,X_6\{\geq 2\}X_7\{\geq 3\}X_8\{\geq 3\}$

$P_1P_2P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}\,X_8\{\geq 3\}$

$P_1P_3P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}\,X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}\,X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq$
2 X3≥2X6≥2 X7{≥3})=X1≥2X2≥3 X3≥3 X4≥2X5≥3 X6≥2X7≥3 X8{≥3}

$P_2P_3P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}X_5\{\geq 3\}X_6\{\geq 2\}X_7\{\geq 3\}\,X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq$
2 X3≥2X6≥2 X7{≥3})=X1≥2X2≥3X3≥3 X4≥2X5≥3X6≥2X7≥3 X8≥3

$P_1P_2P_3P_4P_5P_6P_7P_8 = (X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\})(X_1\{\geq 2\}\,X_2\{\geq 2\}\,X_3\{\geq 2\}X_6\{\geq 2\}\,X_7\{\geq 3\}) = X_1\{\geq 2\}\,X_2\{\geq 3\}X_3\{\geq 3\}\,X_4\{\geq 2\}\,X_5\{\geq 3\}\,X_6\{\geq 2\}\,X_7\{\geq 3\}X_8\{\geq 3\}$

1.4 Improving IE with a 'Shellable' Version of a PRE

In this section, we deal with the introduction of orthogonality in a given sum-of-products formula (disjunctive normal form). If neither of the two terms A and B in the sum $(\mathrm{A} \vee \mathrm{B})$ subsumes the other ($\mathrm{A} \vee \mathrm{B} \neq \mathrm{A}$ and $\mathrm{A} \vee \mathrm{B} \neq \mathrm{B}$) and the two terms are not already disjoint ($\mathrm{A} \wedge \mathrm{B} \neq 0$), then B can be disjointed with A by using the formula [24,36,43,48-57]

$$\mathrm{A} \vee \mathrm{B} = \mathrm{A} \vee (\bar{Y}_1 \vee Y_1\bar{Y}_2 \vee Y_1Y_2\bar{Y}_3 \vee \ldots \vee Y_1Y_2 \ldots Y_{e-1}\bar{Y}_e)\, B, \tag{8}$$

Where the first term A still remains intact, while the second term B is replaced by e terms which are each disjoint with A and are also disjoint among themselves. Note that each $\mathrm{Y_k}$ $(1 \leq \mathrm{k} \leq \mathrm{e})$ is a literal that appears in the product A and does not appear in the product B. It stands for a disjunction of certain instances of some variable $\mathrm{X_{i(k)}}$. We are interested herein in the particular case for which $\mathrm{e} = 1$, i.e., when the two products A and B are such that there is a single literal that appears in the product A and does not appear in the product B. For this case, the disjointing formula (8) simplifies to the Reflection law [36].

$$\mathrm{A} \vee \mathrm{B} = \mathrm{A} \vee\ \bar{\mathrm{Y}}_1\, \mathrm{B}, \tag{9}$$

Which is conveniently referred to as a case of 'shellable' disjointing. We coin the name of 'shellable disjointing' to mimic the well-known term of a 'shellable disjunctive normal form (DNF)' that designates a DNF for which orthogonalization can be (most) efficiently performed, without any increase in the number of terms [58-61]. In the sequel, we will not strive to achieve complete orthogonality in a given sum-of-products formula (disjunctive normal form). Instead, we will apply shellable disjointing as much as we can. In comparison with schemes for producing a probability-ready expression, this scheme enjoys the advantages of simplicity and avoidance of increase in the number of terms, at the expense of still requiring the further use of the inclusion-exclusion (IE) principle. However, the IE use might simplify dramatically. Therefore, the net complexity of this scheme, which borrows ideas partially from the PRE and IE procedures, seems to be much faster and less error-prone, than a scheme based on the IE scheme alone or another using the PRE scheme solely.

To demonstrate the proposed scheme, we re-rearrange the 8 prime implicants in (1) as shown in (10), and further introduce as much orthogonality as possible through shellable disjointing (9), as shown by the bold literals in (10). For example, the first prime implicant in (1) has two literals $X_2\{\geq 3\}$ and $X_7\{\geq 3\}$, and none of the succeeding implicants share both these literals (for otherwise, the succeeding implicant would subsume the first implicant and get absorbed by it, contradicting the fact that it is prime). Three out of these succeeding implicants (the fourth, the seventh, and the eighth) do not share any of the afore-mentioned literals with the first implicant, and we deliberately abstain from disjointing any of them with the first implicant, since such an action would be complicated and would split each of the disjointed terms into several (here two) terms. The third implicant (alone) shares $X_2\{\geq 3\}$ with the first implicant, so that only the literal $X_7\{\geq 3\}$ appears in the first implicant but not in the third one. Therefore, we achieve shellable disjointing of the third implicant with the first by multiplying the third implicant with the complement of $\mathrm{X_7}\{\geq 3\}$, which is $\mathrm{X_7}\{< 3\}$ (shown in bold). Likewise, we attain shellable disjointing of the second, fifth, and sixth prime implicants with the first by multiplying each of them with the complement of $X_2\{\geq 3\}$, which is $X_2\{< 3\}$ (again, highlighted in bold). Shellable disjointing is also possible for the fourth implicant with the third by multiplying the fourth implicant with $\mathrm{X_2}\{< 3\}$ (shown in bold). As a result, the fourth implicant immediately becomes disjoint with the first, and turns capable of shellable disjointing with the second implicant through further multiplication with $X_7\{< 3\}$ (again shown in bold). Each of the fifth and sixth implicants is capable of shellable disjointing with the second implicant through multiplication with $X_3\{< 3\}$ (again, shown in bold). The eighth implicant is capable of shellable disjointing with the seventh implicant through multiplication with $X_2\{< 2\}$ (shown in bold). As an offshoot, the eighth implicant becomes orthogonal with four other implicants (the first, the third, the fifth, and the sixth).

$$\begin{aligned}
\mathrm{S} = {} & \mathrm{X_2}\{\geq 3\}\, \mathrm{X_7}\{\geq 3\} \vee \mathrm{X_2}\{< 3\}\, \mathrm{X_3}\{\geq 3\}\, \mathrm{X_7}\{\geq 3\} \\
& \vee\ \mathrm{X_2}\{\geq 3\}\, \mathrm{X_5}\{\geq 3\}\, \mathrm{X_7}\{< 3\}\, \mathrm{X_8}\{\geq 3\} \\
& \vee\ \mathrm{X_2}\{< 3\}\, \mathrm{X_3}\{\geq 3\}\, \mathrm{X_5}\{\geq 3\}\, \mathrm{X_7}\{< 3\}\, \mathrm{X_8}\{\geq 3\} \\
& \vee\ \mathrm{X_2}\{\geq 2\}\, \mathrm{X_2}\{< 3\}\, \mathrm{X_3}\{\geq 2\}\mathrm{X_3}\{< 3\}\, \mathrm{X_4}\{\geq 2\}\, \mathrm{X_7}\{\geq 3\}\, \mathrm{X_8}\{\geq 2\} \\
& \vee\ \mathrm{X_1}\{\geq 2\}\, \mathrm{X_2}\{\geq 2\}\mathrm{X_2}\{< 3\}\, \mathrm{X_3}\{\geq 2\}\mathrm{X_3}\{< 3\}\mathrm{X_6}\{\geq 2\}\, \mathrm{X_7}\{\geq 3\} \\
& \vee\ \mathrm{X_1}\{\geq 2\}\, \mathrm{X_2}\{\geq 2\}\, \mathrm{X_4}\{\geq 2\}\mathrm{X_6}\{\geq 2\}\, \mathrm{X_7}\{\geq 2\}\, \mathrm{X_8}\{\geq 2\} \\
& \vee\ \mathrm{X_1}\{\geq 2\}\, \mathrm{X_2}\{< 2\}\mathrm{X_3}\{\geq 2\}\, \mathrm{X_4}\{\geq 2\}\, \mathrm{X_6}\{\geq 2\}\, \mathrm{X_7}\{\geq 2\}\, \mathrm{X_8}\{\geq 2\}.
\end{aligned} \tag{10}$$

Employing the relation ($X_i\{\geq 2\}\, X_i\{< 3\} = X_i\{2\}$), we simplify Equation (10) to

$$S = X_2\{\geq 3\}\, X_7\{\geq 3\} \vee X_2\{< 3\}\, X_3\{\geq 3\}\, X_7\{\geq 3\}$$
$$\vee\ X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\}$$
$$\vee\ X_2\{< 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\}$$
$$\vee\ X_2\{2\}\, X_3\{2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$$
$$\vee\ X_1\{\geq 2\}\, X_2\{2\}\, X_3\{2\} X_6\{\geq 2\}\, X_7\{\geq 3\}$$
$$\vee\, X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}$$
$$\vee\ X_1\{\geq 2\}\, X_2\{< 2\} X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}. \quad (11a)$$

$$= \Re_1 \vee \Re_2 \vee \Re_3 \vee \Re_4 \vee \Re_5 \vee \Re_6 \vee \Re_7 \vee \Re_8. \quad (11b)$$

where

$\Re_1 = X_2\{\geq 3\}\, X_7\{\geq 3\}$
$\Re_2 = X_2\{< 3\}\, X_3\{\geq 3\}\, X_7\{\geq 3\}$

$\Re_3 = X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\}$

$\Re_4 = X_2\{< 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\}$
$\Re_5 = X_2\{2\}\, X_3\{2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$\Re_6 = X_1\{\geq 2\}\, X_2\{2\} X_3\{2\} X_6\{\geq 2\}\, X_7\{\geq 3\}$

$\Re_7 = X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq$
2 X8{≥2}

$\Re_8 = X_1\{\geq 2\}\, X_2\{< 2\} X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq$
2 X7≥2 X8{≥2}

With so many orthogonalities achieved in (11a), the IE formula (2) reduces significantly to.

$$E\{S\} = E\{\Re_1\} + E\{\Re_2\} + E\{\Re_3\} + E\{\Re_4\} + E\{\Re_5\} + E\{\Re_6\} + E\{\Re_7\} + E\{\Re_8\} - E\{\Re_1\Re_7\} - E\{\Re_2\Re_7\} -$$
$$E\Re 2\Re 8 - E\Re 3\Re 7 - E\Re 4\Re 7 - E\Re 4\Re 8 - E\Re 5\Re 6 - E\Re 5\Re 7 - E\Re 6\Re 7 + E\{\Re 5\Re 6\Re 7\}, \quad (12)$$

where the various intersections in (12) are

$\Re_1\Re_7 = (X_2\{\geq 3\}\, X_7\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) = X_1\{\geq 2\}\, X_2\{\geq$
3 X4≥2X6≥2 X7≥3 X8{≥2}

$\Re_2\Re_7 = (X_2\{< 3\}\, X_3\{\geq 3\}\, X_7\{\geq 3\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{2\} X_3\{\geq 3\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$\Re_2\Re_8 = (X_2\{< 3\}\, X_3\{\geq 3\}\, X_7\{\geq 3\})\ (X_1\{\geq 2\}\, X_2\{< 2\} X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{< 2\} X_3\{\geq 3\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$\Re_3\Re_7 = (X_2\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\})\ (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\}) =$
$X_1\{\geq 2\}\, X_2\{\geq 3\}\, X_4\{\geq 2\} X_5\{\geq 3\} X_6\{\geq 2\}\, X_7\{2\}\, X_8\{\geq 3\}$

$\Re_4\Re_7 = (\, X_2\{< 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\} X_8\{\geq 3\}) \quad (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq$
2 X8{≥2}) =X1≥2 X22X3≥3 X4≥2X5≥3X6≥2 X72 X8{≥3}

$\Re_4\Re_8 = (X_2\{< 3\}\, X_3\{\geq 3\}\, X_5\{\geq 3\}\, X_7\{< 3\}\, X_8\{\geq 3\})\ (X_1\{\geq 2\}\, X_2\{< 2\} X_3\{\geq 2\}\, X_4\{\geq 2\}\, X_6\{\geq 2\}\, X_7\{\geq$
2 X8{≥2})=X1≥2 X2<2X3≥3 X4≥2 X5≥3X6≥2 X72 X8{≥3}

$\Re_5\Re_6 = (X_2\{2\}\, X_3\{2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{2\} X_3\{2\} X_6\{\geq 2\}\, X_7\{\geq 3\}) = X_1\{\geq$
2 X22X32 X4≥2X6≥2 X7{≥3} X8{≥2}

$\Re_5\Re_7 = (X_2\{2\}\, X_3\{2\}\, X_4\{\geq 2\}\, X_7\{\geq 3\} X_8\{\geq 2\})\ (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})$
$= X_1\{\geq 2\}\, X_2\{2\} X_3\{2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$\Re_6\Re_7 = (X_1\{\geq 2\}\, X_2\{2\} X_3\{2\} X_6\{\geq 2\}\, X_7\{\geq 3\})\ (X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 2\}\, X_8\{\geq 2\})$
$= X_1\{\geq 2\}\, X_2\{2\}\, X_3\{2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\}$

$\Re_5\Re_6\Re_7 = (X_1\{\geq 2\}\, X_2\{2\} X_3\{2\}\, X_4\{\geq 2\} X_6\{\geq 2\}\, X_7\{\geq 3\}\, X_8\{\geq 2\})(X_1\{\geq 2\}\, X_2\{\geq 2\}\, X_4\{\geq 2\} X_6\{\geq$
2 X7≥2 X8{≥2}) =X1≥2 X22X32 X4≥2X6≥2 X7≥3 X8{≥2}

2. REPORT AND DISCUSSION OF NUMERICAL RESULTS

We presented two IE methods, the conventional one and an improved one, for solving the problem of our running example. Our two methods agreed on a value of 0.9819022224313, which is in agreement with the solution of Rushdi and Amashah [30,31], and also in agreement (albeit more precise) with the numerical value (0.981902) that was obtained earlier by Lin et al. [22]. Table 2 details the computations performed by the two IE methods. The table indicates clearly the improvements brought about by the second method, both in decreasing the number of operations and in diminishing the effect of round-off errors. We note that the round-off error in the first method would have been more pronounced, had we implemented the IE formula as is. Actually, we decreased the round-off significantly by performing an actual subtraction only once, as we summed all positive terms, summed all negative terms, and only then took the difference. We have to admit that we could not fix our symbolic computations via the conventional IE formula from the outset. The manual computation of 255 terms were too tedious and error-prone to be completed correctly in one trial. We evaluated the IE formula correctly via the python program in Appendix A, and then used the results of this program to fix bugs in the symbolic computations.

In passing, we note that the present work inspired us to apply the MS-IE Principle to the union of fewer (factored or composite) paths that is subsequently converted (at minimal cost) to PRE form [62]. We augmented the resulting procedure with another that uses the multi-state Boole-Shannon expansion [23,24,27,30,42,48,54,63-68]. Consequently, we were in a position to point out a liaison among inclusion-exclusion, probability-ready expressions and Boole-Shannon expansion for multi-state reliability [62].

Table 2. A comparison of computations by conventional IE and by IE improved with a 'shellable' PRE

Item	Conventional IE	IE improved with PRE
Sum of 8 expectations of single indicators	+ 5.7002040911313365	+ 1.638655916679425
− Sum of 28 expectations of pairwise ANDing of indicators	− 16.183321233651064	− 0.6589357136472941
+ Sum of 56 expectations of triple-wise ANDing of indicators	+ 28.38786762461678	+ 0.0021820193991679074
− Sum of 70 expectations of quadruple-wise ANDing of indicators	− 32.66489098333673	0
+ Sum of 56 expectations of quintuple-wise ANDing of indicators	+ 24.82589302827858	0
− Sum of 28 expectations of sextuple-wise ANDing of indicators	− 12.067134436416122	0
+ Sum of 8 expectations of septuple-wise ANDing of indicators	+ 3.4094675792097604	0
− expectation of octuple-wise ANDing of indicators	− 0.42618344740122005	0
Sum of all positive terms	62.3234323232364569	1.6408379360785929074
− Sum of all negative terms	− 61.34153010080513605	− 0.6589357136472941
Net required value	0.98190222243132085	0.9819022224312988074

3. CONCLUSIONS

This paper is a continuation of our ongoing efforts to extend concepts of reliability computations in the binary domain to the multi-state domain. The paper serves as an exposition of the inter-relationships between the multi-state concepts MS-IE and MS-PRE. This exposition was obtained by using the standard MS-IE approach and an improved MS-IE approach preceded by an efficient shellable PRE pre-processing. The two approaches were applied to the same problem of multi-state network reliability. Each of the two approaches recovered the same result obtained by the conventional RSDP method. Hopefully, the present work can guide more useful applications of the Inclusion-Exclusion Principle and its improved variants to other real-life problems.

COMPETING INTERESTS

The authors have declared that no competing interests exist.

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APPENDIX A

Listing in Python of a Program Computing the IE Solution for the Running Example

```
From itertools import combinations
from functools import reduce
# the lists below are from Lin et al. [21,Table 2, p.6689]
P1= [0,0,3,0,3,0,0,3]
P2= [0,0,3,0,0,0,3,0]
P3= [0,3,0,0,3,0,0,3]
P4= [0,2,2,2,0,0,3,2]
p5= [0,3,0,0,0,0,3,0]
p6= [2,0,2,2,0,2,2,2]
p7= [2,2,0,2,0,2,2,2]
p8= [2,2,2,0,0,2,3,0]
total=0
temp=[]     # temporary list to store digits
multi=1     # multiplier counter
Full_List = [P1, P2, P3, P4, p5, p6, p7, p8]

for r in range(1,9,1):     # r number of tuples to be compared starting from 1
Combinations = combinations(Full_List, r)
print("Finding maximum for a combination size of %d"%r)
for eachCombination in Combinations:
# print("Finding combination for tuples ", *eachCombination)
temp=list(map(max,zip(*eachCombination)))
print(temp)
if (temp[0]==2):multi=multi*0.897    # Enter the data from Table 1 p-6688
if (temp[1]==2):multi=multi*0.965    # 0 for X_1, 1 for X_2 ...etc
if (temp[1]==3):multi=multi*0.892
if (temp[2]==2):multi=multi*0.953
if (temp[2]==3):multi=multi*0.905
if (temp[3]==2):multi=multi*0.863
if (temp[4]==3):multi=multi*0.903
if (temp[5]==2):multi=multi*0.943
if (temp[6]==2):multi=multi*0.945
if (temp[6]==3):multi=multi*0.884
if (temp[7]==2):multi=multi*0.965
if (temp[7]==3):multi=multi*0.906
print(multi)
if(r%2==1):total=total+multi # if r odd we add it otherwise subtract from total
else :total=total-multi
multi=1          # reset the counter
print('the total=',total)
```


Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/67438