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Traveling Wave Solutions for a Coupled KdV Equations

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Authors' contribution

This work was carried out in collaboration between all authors. Author ST designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author YZ managed the analyses of the study. Author YZ managed the literature searches. All authors read and approved the final manuscrip

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Abstract

In this paper, we use a method in order to find exact explicit traveling solutions in the subspace of the phase space for coupled KdV equations. The key idea is removing a coupled relation for the given system so that the new systems can be solved. The existence of solitary wave solutions is obtained. It is shown that bifurcation theory of dynamical systems provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Keywords: Solitary wave solution, bifurcation theory, dynamical systems, coupled KdV equations.

1 Introduction

The study on the various physical structures of nonlinear dispersive equations has attracted much attention in connection with the important problems that arise in scientific applications. Mathematically, these physical structures have been studied by using various analytical methods, such as inverse scattering method [1], Darboux transformation method [2,3], Hirota bilinear method [4], Lie group method [5,6], sine-cosine method [7,8], tanh function method [9,10], Fan-expansion method [11,12] and so on. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations.

In 1981, Hirota [4] presented the following coupled KdV equations,

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$$\begin{cases} u_t + 6\alpha u u_x - 2bvv_x + \alpha u_{xxx} = 0, \\ v_t + 3\beta u v_x + \beta v_{xxx} = 0, \end{cases} \quad (1.1)$$

which were derived to model the interaction of water waves. Here α, β, b are constants. Liu et al. in [13] based on a subtle balance method, a given function expansion which was applied to Eq. (1.1), a series of periodic solutions, solitary wave solutions and singular solutions are obtained by aid of symbolic computation.

For $b = 3, \alpha = \beta$, Eq. (1.1) becomes to

$$\begin{cases} u_t + 6\alpha u u_x - 6vv_x + \alpha u_{xxx} = 0, \\ v_t + 3\alpha u v_x + \alpha v_{xxx} = 0, \end{cases} \quad (1.2)$$

Recently, Wang et al. [14] considered the solitary wave solutions of Eq. (1.2), by using a homogeneous balance method.

In this paper, we consider the dynamical bifurcation behavior for the traveling wave solutions of Eq. (1.2). Let $u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = x - ct$, where c is the wave speed. Then Eq. (1.2) become to

$$\begin{cases} -cu' + 6\alpha uu' - 6vv' + \alpha u''' = 0, \\ -cv' + 3\alpha uv' + \alpha v''' = 0, \end{cases} \quad (1.3)$$

where " ′ " is the derivative with respect to ξ .To remove the coupled relationship of Eq. (1.3), let $u = a + bv^2$, then substitute it into Eq. (1.3) and integrate once, we obtain

$$\begin{cases} -ac + 3\alpha a^2 + \left(6\alpha ab - bc - 3\right)v^2 + 3\alpha b^2 v^4 + 2\alpha b\left(vv'' + \left(v'\right)^2\right) = g_1, \\ -cv + \alpha v\left(3a + bv^2\right) + \alpha v'' = g_2, \end{cases} \quad (1.4)$$

where g_1 and g_2 are integral constants.

Thus, we have two Hamiltonian systems

$$\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{g_1 + ac - 3\alpha a^2 + \left(bc - 6\alpha ab + 3\right)v^2 - 3\alpha b^2 v^4 - 2\alpha b y^2}{2\alpha bv}, \quad (1.5)$$

$$\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{1}{\alpha}\left(g_2 + \left(c - 3\alpha a\right)v - \alpha bv^3\right) \quad (1.6)$$

with the first integral

$$y^2 = \frac{2\left(g_1 + ac - 3\alpha a^2\right)v^2 + \left(bc - 6\alpha ab - 3\right)v^4 - 2\alpha b^2 v^6 + 4h_1}{4\alpha b v^2}, \tag{1.7}$$

$$y^2 = \frac{4g_2 v + 2\left(c - 3\alpha a\right)v^2 - \alpha b v^4 + 2h_2}{2\alpha}, \tag{1.8}$$

and

$$H_1\left(v, y\right) = \alpha b v^2 y^2 - \left[\frac{1}{2}\left(g_1 + ac - 3\alpha a^2\right)v^2 + \frac{1}{4}\left(bc - 6\alpha ab - 3\right)v^4 - \frac{1}{2}\alpha b^2 v^6\right] = h_1, \tag{1.9}$$

$$H_2\left(v, y\right) = \alpha y^2 - \left[2g_2 v + \left(c - 3\alpha a\right)v^2 - \frac{\alpha b}{2}v^4\right] = h_2, \tag{1.10}$$

For

$$h_1 = h_2 = g_2 = 0, a = \frac{c \pm \sqrt{c^2 + 12\alpha g_1}}{6\alpha}, b = \frac{1}{c - 2\alpha a}, c^2 + 12\alpha g_1 \geq 0, \tag{1.11}$$

system (1.5) and system (1.6) are equivalent to each other. In this paper, we always assume that Eq. (1.2) satisfy (1.11). Therefore, under condition (1.11), we only consider the system (1.6) and then we have two Hamiltonian systems

$$\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{v}{2\alpha}\left(c - \sqrt{c^2 + 12\alpha g_1} - \frac{6\alpha v^2}{2c - \sqrt{c^2 + 12\alpha g_1}}\right), \tag{1.12}$$

$$\frac{dv}{d\xi} = y, \frac{dy}{d\xi} = \frac{v}{2\alpha}\left(c + \sqrt{c^2 + 12\alpha g_1} - \frac{6\alpha v^2}{2c + \sqrt{c^2 + 12\alpha g_1}}\right), \tag{1.13}$$

with the first integral

$$y^2 = \frac{1}{2\alpha}\left(c - \sqrt{c^2 + 12\alpha g_1}\right)v^2 - \frac{3v^4}{4\left(2c - \sqrt{c^2 + 12\alpha g_1}\right)}, \tag{1.14}$$

$$y^2 = \frac{1}{2\alpha}\left(c + \sqrt{c^2 + 12\alpha g_1}\right)v^2 - \frac{3v^4}{4\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}, \tag{1.15}$$

and

$$H_{21}(v, y) = y^2 - \frac{1}{2\alpha}\left(c - \sqrt{c^2 + 12\alpha g_1}\right)v^2 + \frac{3v^4}{4\left(2c - \sqrt{c^2 + 12\alpha g_1}\right)} = 0. \tag{1.16}$$

$$H_{22}(v, y) = y^2 - \frac{1}{2\alpha}\left(c + \sqrt{c^2 + 12\alpha g_1}\right)v^2 + \frac{3v^4}{4\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)} = 0. \tag{1.17}$$

2 Exact Explicit Traveling Solutions of System (1.2)

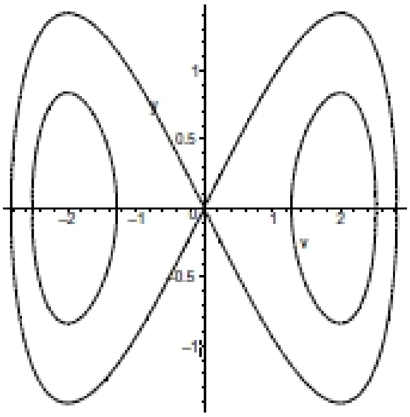
2.1 $a = \frac{c - \sqrt{c^2 + 12\alpha g_1}}{6\alpha}$

Suppose that $g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ or $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$ .Then, the origin of system (1.12) is a saddle point. Corresponding to the level curves defined by $H_{21}(v, y) = 0$; two homoclinic orbits of Eq. (1.12) have the following parametric representation (Fig. 1)

$$v = \pm\sqrt{\frac{\left(c - \sqrt{c^2 + 12\alpha g_1}\right)\left(2c - \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}}\sec h\sqrt{\frac{c - \sqrt{c^2 + 12\alpha g_1}}{2\alpha}}\xi. \tag{2.1}$$

Thus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as follows:

$$\begin{cases} u(x,t) = \dfrac{c - \sqrt{c^2 + 12\alpha g_1}}{6\alpha} + \left(\dfrac{6\alpha}{2c - \sqrt{c^2 + 12\alpha g_1}}\right)v^2, \\ v(x,t) = \pm\sqrt{\dfrac{\left(c - \sqrt{c^2 + 12\alpha g_1}\right)\left(2c - \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}}\sec h\sqrt{\dfrac{c - \sqrt{c^2 + 12\alpha g_1}}{2\alpha}}(x - ct). \end{cases} \tag{2.2}$$



$g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ ***or*** $g_1 < 0, \alpha < 0, c > 2\sqrt{\alpha g_1}$.

Fig. 1. The phase portraits of (2.12) for $H_{21}(v, y) = 0$.

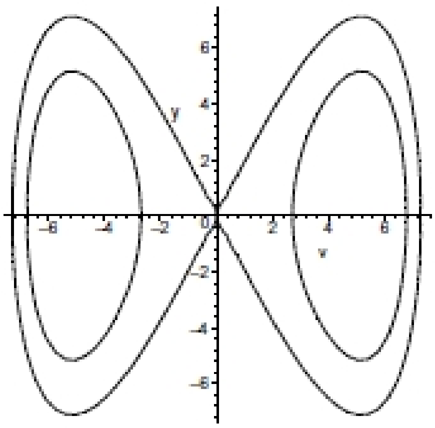
2.2 $a = \dfrac{c + \sqrt{c^2 + 12\alpha g_1}}{6\alpha}$

Suppose that $g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ or $g_1 > 0, \alpha > 0, c > 0$ .Then, the origin of system (1.13) is a saddle point. Corresponding to the level curves defined by $H_{22}(v, y) = 0$; two homoclinic orbits of Eq. (1.13) have the following parametric representation (Fig. 2)

$$v = \pm\sqrt{\frac{\left(c + \sqrt{c^2 + 12\alpha g_1}\right)\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h\sqrt{\frac{c + \sqrt{c^2 + 12\alpha g_1}}{2\alpha}}\xi. \tag{2.3}$$

Thus, we obtain the parametric representation of Eq. (1.2) a smooth solitary wave solution as follows:

$$\begin{cases} u(x,t) = \dfrac{c + \sqrt{c^2 + 12\alpha g_1}}{6\alpha} + \left(\dfrac{6\alpha}{2c + \sqrt{c^2 + 12\alpha g_1}}\right)v^2, \\ v(x,t) = \pm\sqrt{\dfrac{\left(c + \sqrt{c^2 + 12\alpha g_1}\right)\left(2c + \sqrt{c^2 + 12\alpha g_1}\right)}{3\alpha}} \sec h\sqrt{\dfrac{c + \sqrt{c^2 + 12\alpha g_1}}{2\alpha}}(x - ct). \end{cases} \tag{2.4}$$



$g_1 < 0, \alpha > 0, c > 2\sqrt{-3\alpha g_1}$ or $g_1 > 0, \alpha > 0, c > 0$.

Fig. 2. The phase portraits of (2.13) for $H_{22}(v, y) = 0.$

Remark. Obviously, solutions (2.2) and (2.4) are different from solutions (3.22) and (3.23) of [13], respectively.

3 Conclusion

In this paper, we have considered traveling wave solutions for the coupled KdV equations of Eq. (1.2) in its subspace of parameter space by using the method of dynamical systems. We obtain parametric representations for the solitary wave solutions of Eq. (1.1) in different parameter regions of the parameter space.

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Competing Interests

Authors have declared that no competing interests exist.

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