British Journal of Mathematics & Computer Science
6(3): 215-232, 2015, Article no.BJMCS.2015.074
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

A New Approach to H∞ Filtering for Switched T-S Fuzzy Systems with Interval Time Varying Delays and Parameter Uncertainties

Ahmida Fatima[1*] and El Houssaine Tissir[1]

[1]Department of Physics, Faculty of Sciences B.P. 1796 Fes-Atlas, Morocco.

Article Information

DOI: 10.9734/BJMCS/2015/13916
Editor(s):
(1) Radko Mesiar, Dept. of Mathematics, Faculty of Civil Engineering, Slovak University of Technology Bratislava, Slovakia.
Reviewers:
(1) Anonymous, China.
(2) Anonymous, China.
(3) Anonymous, China.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=733&id=6&aid=7609

Original Research Article

Received: 9 September 2014
Accepted: 12 November 2014
Published: 03 January 2014

Abstract

This paper deals with the problem of delay-dependent robust H∞ filtering for switched T-S fuzzy systems with interval time-varying delays and parameter uncertainties. This represents a novelty in this research domain since to the best of our knowledge there is no paper, in the literature, dealing with the delay-dependent robust H∞ filtering for switched T-S fuzzy systems time-varying delays. Our attention is focused on the design of a full order filter that guarantees the filtering error system to be robustly stable with a prescribed H∞ performance. By choosing a new Lyapunov function and using the convexity property, new sufficient conditions are derived for stability robustness of the filtering error systems and are expressed in terms of linear matrix inequalities. Finally, three examples are provided to demonstrate the effectiveness and the superiority of the proposed design methods.

Keywords: T-S fuzzy systems; switched systems; interval time-delay systems; H∞ filter; linear matrix inequality (LMI).

1 Introduction

Over the past few years, the state estimation of dynamic systems has been extensively investigated [1,2,3]. Compared with the traditional Kalman filtering, the H_∞ filter technique has several advantages. First, the noise sources in the H_∞ filtering setting are arbitrary signals with bounded energy or average power, and no exact statistics are required to be known [4]. Second,

**Corresponding author: fatima_ahmida@yahoo.fr;*

the H_∞ filter has been shown to be much more robust to parameter uncertainty in a control system. This advantage renders the H_∞ filtering approach very appropriate in some pratical applications. The $L_2 - L_\infty$ filtering technique is used in [5,6].

In recent years, T-S fuzzy model approach has been extended to H_∞ filter or controller design [7,8,9]. The problem of delay dependent robust H_∞ filtering for T-S fuzzy time delay systems with exponential stability was studied in [10]. In [11] the H∞ filtering of time-delay T-S fuzzy systems was addressed by employing a piecewise Lyapunov-Krasovskii functional. In the case when parameter uncertainty appears in T-S fuzzy system, the robust stability H_∞ filter for a class of Takagi-Sugeno (T-S) fuzzy descriptor was addressed in [12].

Many practical models in manufacturing, communication networks, automotive engine control, chemical processes etc can be described by switched models [13-15]. The idea of switching controller is introduced in order to overcome the shortcomings of the single controller and improve system performance [16–18]. Some methods have been used in the study of switched systems such as the multiple Lyapunov function [19]. Controller switching strategy is also motivated by increasing performance requirements in control, especially in the presence of large uncertainties or disturbances [20]. In [21], the exponential stability and guaranteed cost of switched linear systems with mixed varying delays are studied. A few results are encountered in the literature for T-S fuzzy switched systems with time delays. The delay dependent exponential stability is studied for systems with norm bounded uncertainties in [22], and in [23] the stability robustness of T-S fuzzy systems with two additive time varying delays is investigated by using the free weighting matrices approach. In addition, the state estimation problem has been investigated for switched systems with different performance indexes including H∞ filtering design [24,25], Sufficient conditions for the existence of the filter parameters are derived by introducing slack variables to eliminate the cross coupling of system matrices and Lyapunov matrices and formulated in terms of LMIs [24]. In [25], H∞ filtering design is developed by using switched Lyapunov functional approach for discrete time switched systems with time delay. In [26], the problem of exponential H∞ filtering for a class of continuous-time switched linear system with interval time-varying delay are studied based on the free-weighting matrix approach. Although we have cited many references dealing with H∞ filtering and stability, the problem of delay-dependent robust H∞ filtering for T-S fuzzy switched systems with interval time delays has not been investigated.

For uncertain switched T-S fuzzy time delay systems, this paper discusses the design methods of delay dependent robust H∞ filter. The parameter uncertainties are assumed to be time varying but norm-bounded. The delay is time-varying with bounded magnitude and time derivative. Based on Lyapunov-Krasovskii approach, we propose a design method of a filter that makes the filtering error dynamics asymptotically stable for all admissible uncertainties and achieves a prescribed H∞ norm performance level. Sufficient conditions for the existence of the filter are derived and formulated in terms of LMIs. Finally, three numerical examples are given to show the effectiveness and the superiority of the proposed design methods.

2 Preliminaries and Problem Formulation

Consider a class of switched systems with T-S fuzzy subsystems as follows

R^i:If η_1 (t) is W_1^i, η_2 (t) is W_2^i, . . . , η_s(t) is W_s^i

Then

$$\begin{cases}\dot{x}(t) = \left(A_{i\sigma} + \Delta A_{i\sigma}(t)\right)x(t) + \left(A_{di\sigma} + \Delta A_{di\sigma}(t)\right)x\left(t-\tau(t)\right) + \left(B_{wi\sigma} + \Delta B_{wi\sigma}(t)\right)\omega(t) \\ y(t) = \left(C_{i\sigma} + \Delta C_{i\sigma}(t)\right)x(t) + (C_{di\sigma} + \Delta C_{di\sigma}(t))x\left(t-\tau(t)\right) + (D_{wi\sigma} + \Delta D_{wi\sigma}(t))\omega(t) \\ z(t) = \left(L_{i\sigma} + \Delta L_{i\sigma}(t)\right)x(t) + (L_{di\sigma} + \Delta L_{di\sigma}(t))x\left(t-\tau(t)\right) + (G_{wi\sigma} + \Delta G_{wi\sigma}(t))\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \ldots, r\end{cases} \tag{2.1}$$

Where x(t) $\in R^n$ is the state vector; ω(t) $\in R^p$ is the disturbance input vector that belongs to L_2 [0,∞); y(t) $\in R^m$ is the measured output vector, z(t) $\in R^q$ is the signal vector to be estimated, ϕ $\in C([-h_b, 0], R^n)$ is a compatible vector-valued initial function, with norm $\|\phi\| = \sup_{\theta\in[-h_b,0]}\|\phi(\theta)\|$. $\eta_1(t), \eta_2(t), \ldots \eta_s(t)$ are the premise variables and W_l^i ,l=1,…,s are the fuzzy sets. r is the number of IF-THEN rules, the piecewise constant function σ denoting σ(t): [0, ∞) → Ƒ= {1,…,N} is a switching signal to specify, at each time instant t, the index σ ∈ Ƒ of the active subsystem, i.e. σ = j means that the j^{th} subsystem is activated. In this case, the system matrices are denoted A_{ij} , A_{dij}, $B_{\omega ij}$, C_{ij}, C_{dij}, $D_{\omega ij}$, L_{ij} , L_{dij} and $G_{\omega ij}$ for constant matrices and, $\Delta A_{ij}(t)$, $\Delta A_{dij}(t)$, $\Delta B_{\omega ij}(t)$, $\Delta C_{ij}(t)$, $\Delta C_{dij}(t)$, $\Delta D_{\omega ij}(t)$, $\Delta L_{ij}(t)$, $\Delta L_{dij}(t)$, $\Delta G_{\omega ij}(t)$ for parameter uncertainties, which are assumed to be of the form:

$$\begin{aligned}&[\Delta A_{ij}(t) \quad \Delta A_{dij}(t) \quad \Delta B_{\omega ij}(t)] = D_{1ij}F_{ij}(t)\,[E_{10ij} \quad E_{11ij} \quad E_{12ij}] \\ &[\Delta C_{ij}(t) \quad \Delta C_{dij}(t) \quad \Delta D_{\omega ij}(t)] = D_{2ij}F_{ij}(t)\,[E_{20ij} \quad E_{21ij} \quad E_{22ij}] \\ &[\Delta L_{ij}(t) \quad \Delta L_{dij}(t) \quad \Delta G_{\omega ij}(t)] = D_{3ij}F_{ij}(t)\,[E_{30ij} \quad E_{31ij} \quad E_{32ij}],\end{aligned} \tag{2.2}$$

where E_{10ij} , E_{11ij} , E_{12ij}, E_{20ij} , E_{21ij} , E_{22ij}, E_{30ij} , E_{31ij} , E_{32ij}, D_{1ij} , D_{2ij} and D_{3ij} are known constant matrices with appropriate dimensions, F_{ij} (t) are unknown time-varying matrices with Lebesgue measurable elements bounded by:

$$F_{ij}^T(t)\, F_{ij}(t) \leq I \tag{2.3}$$

τ(t) is a bounded time varying delay, assumed to be differentiable, $\dot{\tau}(t) = d$, and satisfies:

$$\begin{aligned}&0 \leq h_a \leq \tau(t) \leq h_b \\ &d_1 \leq d \leq d_2\end{aligned} \tag{2.4}$$

By using the center-average defuzzifier, product inference and singleton fuzzifier, the global dynamics of T–S fuzzy system (2.1) can be inferred as

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^r \mu_i(\eta(t))\left[\left(A_{ij} + \Delta A_{ij}(t)\right)x(t) + \left(A_{dij} + \Delta A_{dij}(t)\right)x\left(t-\tau(t)\right) + \left(B_{\omega ij} + \Delta B_{\omega ij}(t)\right)\omega(t)\right] \\ y(t) = \sum_{i=1}^r \mu_i(\eta(t))\left[\left(C_{ij} + \Delta C_{ij}(t)\right)x(t) + (C_{dij} + \Delta C_{dij}(t))x\left(t-\tau(t)\right) + \left(D_{\omega ij} + \Delta D_{\omega ij}(t)\right)\omega(t)\right] \\ z(t) = \sum_{i=1}^r \mu_i(\eta(t))\left[\left(L_{ij} + \Delta L_{ij}(t)\right)x(t) + (L_{dij} + \Delta L_{dij}(t))x\left(t-\tau(t)\right) + \left(G_{\omega ij} + \Delta G_{\omega ij}(t)\right)\omega(t)\right] \\ x(t) = \phi(t) \qquad t \in [-h_b, 0]\end{cases} \tag{2.5}$$

Where $\mu_i(\eta(t)) = w_i(\eta(t))/\sum_{i=1}^r w_i(\eta(t))$, $w_i(\eta(t)) = \prod_{v=1}^s W_v^i\left(\eta_v(t)\right)$

$W_v^i\left(\eta_v(t)\right)$ is the grade of the membership function of $\eta_v(t)$ in W_v^i, some basic properties of $\mu_i(\eta(t))$ are: $\mu_i(\eta(t)) \geq 0$, $\sum_{i=1}^r \mu_i(\eta(t)) = 1$.

In this paper, we consider the following fuzzy filter:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\eta(t))\left[A_{fij}\hat{x}(t) + B_{fij}y(t)\right] \\ \hat{z}(t) = \sum_{i=1}^{r} \mu_i(\eta(t))\left[C_{fij}\hat{x}(t) + D_{fij}y(t)\right] \\ \hat{x}(0) = 0 \end{cases} \tag{2.6}$$

Where $\hat{x}(t) \in R^n$ is the filter state, $\hat{z}(t) \in R^q$ is the estimation vector of z(t) in fuzzy system (2.1), $A_{fij}, B_{fij}, C_{fij}$ and D_{fij} are the filter matrices with appropriate dimensions, which are to be designed.
From (2.5) and (2.6), we can obtain the filtering error system:

$$\begin{cases} \dot{\tilde{x}}(t) = \left(\tilde{A}_j + \Delta\tilde{A}_j(t)\right)\tilde{x}(t) + (\tilde{A}_{dj} + \Delta\tilde{A}_{dj}(t))\tilde{x}(t-\tau(t)) + \left(\tilde{B}_{\omega j} + \Delta\tilde{B}_{\omega j}(t)\right)\omega(t) \\ e(t) = \left(\tilde{C}_j + \Delta\tilde{C}_j(t)\right)\tilde{x}(t) + (\tilde{C}_{dj} + \Delta\tilde{C}_{dj}(t))\tilde{x}(t-\tau(t)) + \left(\tilde{D}_{\omega j} + \Delta\tilde{D}_{\omega j}(t)\right)\omega(t) \\ \tilde{x}(t) = \left[\Phi^T(t) \quad 0\right]^T, \ \forall\, t \in [-h_b, 0], \end{cases} \tag{2.7}$$

Where $\tilde{x}(t) := \text{col}\{x(t) \quad \hat{x}(t)\}$, e(t):= z(t) - $\hat{z}(t)$

$$\tilde{A}_j = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} A_{ij} & 0 \\ B_{fij}C_{kj} & A_{fij} \end{bmatrix};$$

$$\tilde{A}_{dj} = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} A_{dij} & 0 \\ B_{fij}C_{dkj} & 0 \end{bmatrix};$$

$$\tilde{B}_{wj} = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} B_{wij} & 0 \\ B_{fij}D_{wkj} & 0 \end{bmatrix}; \tag{2.8}$$

$$\tilde{C}_j = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[L_{ij} - D_{fij}C_{kj} \quad -C_{fij}];$$

$$\tilde{C}_{dj} = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[L_{dij} - D_{fij}C_{dkj}];$$

$$\tilde{D}_{\omega j} = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[G_{\omega ij} - D_{fij}D_{\omega kj}];$$

And $\Delta\tilde{A}_j(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} \Delta A_{ij}(t) & 0 \\ B_{fij}\Delta C_{kj}(t)) & 0 \end{bmatrix};$

$$\Delta\tilde{A}_{dj}(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} \Delta A_{dij}(t) & 0 \\ B_{fij}\Delta C_{dkj}(t) & 0 \end{bmatrix};$$

$$\Delta\tilde{B}_{\omega j}(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \begin{bmatrix} \Delta B_{\omega ij}(t) & 0 \\ B_{fij}\Delta D_{\omega kj}(t) & 0 \end{bmatrix};$$

$$\Delta\tilde{C}_j(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[\Delta L_{ij}(t) - D_{fij}\Delta C_{kj}(t) \quad 0];$$

$$\Delta\tilde{C}_{dj}(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[\Delta L_{dij}(t) - D_{fij}\Delta C_{dkj}(t)]; \tag{2.9}$$

$$\Delta\tilde{D}_{\omega j}(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))[\Delta G_{\omega ij}(t) - D_{fij}\,\Delta D_{\omega kj}(t))];$$

The robust H∞ filtering problem to be addressed in this paper is formulated as follows:

Given the switched T-S fuzzy time-delay system (2.5) and a prescribed level of noise attenuation γ > 0, determine a filter in the form of (2.6) such that the following requirements are satisfied:

(a) The system (2.7) with ω (t) = 0 is asymptotically stable,
(b) Under zero initial conditions, for any nonzero ω (t) ∈ L_2 [0, ∞) and all admissible uncertainties, (2.7) satisfies

$$\|e\|_2 < \gamma\|\omega(t)\|_2 \tag{2.10}$$

The following lemmas will be useful are introduced.
Lemma 2.2 [27]: Let $Q = Q^T$, Ω, E, and F (t) satisfying $F^T(t)F(t) \leq I$ are appropriately dimensioned matrices, the following inequality:

$$Q + \Omega F(t)E + E^T F^T(t)\Omega^T < 0$$

Is true, if and only if the following inequality holds for any scalar $\varepsilon > 0$

$$Q + \varepsilon^{-1}\Omega\Omega^T + \varepsilon E^T E < 0$$

Lemma 2.3 [28]: For any two matrices X and Y, we have

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

Where $X \in R^{m\times n}$, $Y \in R^{m\times n}$, and ε is any positive constant.

3 Fuzzy H_∞ Performance Analysis

In this section, we propose sufficient stability criteria with an H∞ norm bound performance index for the filter error system (2.10).

Remark 3.1. The nominal system of (2.7), i.e., $\Delta A_{ij}(t) = \Delta A_{dij}(t) = \Delta B_{\omega ij}(t) = \Delta C_{kj}(t) = \Delta C_{dkj}(t) = \Delta D_{\omega kj}(t) = \Delta L_{ij}(t) = \Delta L_{dij}(t) = \Delta G_{\omega ij}(t) = 0$ is reduced to the system as follows

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_j\tilde{x}(t) + \tilde{A}_{dj}\tilde{x}\big(t-\tau(t)\big) + \tilde{B}_{\omega j}w(t) \\ e(t) = \tilde{C}_j\tilde{x}(t) + \tilde{C}_{dj}\tilde{x}\big(t-\tau(t)\big) + \tilde{D}_{\omega j}\, w(t) \\ \tilde{x}(t) = [\Phi^T(t) \quad 0]^T, \ \forall\, t \in [-h_b, 0] \end{cases} \tag{3.1}$$

Where $\tilde{A}_j, \tilde{A}_{dj}, \tilde{B}_{\omega j}, \tilde{C}_j, \tilde{C}_{dj}$ and $\tilde{D}_{\omega j}$ are defined in (2.8). In this case, we investigate the asysmptotic stability and we can state the following theorem

Theorem 3.1 : Given scalars $0 \leq h_a \leq h_b$, $d_1 \leq d_2$ and γ>0, the filter error system (3.1), is asymptotically stable and has a prescribed H_∞ performance level γ, for all differentiable delay τ (t) ∈ $[h_a, h_b]$ if there exist positive definite matrices $R_0, R_\lambda, Q_0, Q_\lambda, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, R_d, P_d$ and real matrices $\chi_{pq}, (p = 1,2; q = 1,2,\dots,6)$ with appropriate dimensions such that the following LMIs (3.2) are feasible where $d = d_1, or\ d_2$

$$\Sigma_{pik}^j = \begin{bmatrix} \Pi_{ik}^j + \left[-I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T\right] & h_a \Upsilon_{1j}^T Q_0 & \delta_h \Upsilon_{1j}^T Q_\lambda & \lambda_h \chi_p & \Upsilon_{3j}^T \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\lambda & 0 & 0 \\ * & * & * & -Q_\lambda & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.2}$$

Where

$$\Pi_{ik}^{j}=\begin{bmatrix} \varphi_{11j} & \varphi_{12j} & Q_0 & 0 & \varphi_{15j} & \varphi_{16j} \\ * & \varphi_{22j} & 0 & 0 & \varphi_{25j} & \varphi_{26j} \\ * & * & R_d-R_0+R_\lambda-Q_0-Q_\lambda & 0 & Q_\lambda & 0 \\ * & * & * & -R_\lambda-Q_\lambda-P_d & Q_\lambda & 0 \\ * & * & * & * & -(1-d)(R_d-P_d)-2Q_\lambda & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\varphi_{11j}=P_1A_{ij}+A_{ij}^TP_1+P_2B_{fij}C_{kj}+C_{kj}^TB_{fij}^TP_2^T+R_0\text{-}Q_0$$

$$\varphi_{12j}=P_2A_{fij}+A_{ij}^TP_2^T+C_{kj}^TB_{fij}^TP_3;\ \varphi_{22j}=P_3A_{fij}+A_{fij}^TP_3$$

$$\varphi_{15j}=P_1A_{dij}+P_2B_{fij}C_{dkj};\ \varphi_{25j}=P_2^TA_{dij}+P_3B_{fij}C_{dkj} \tag{3.3}$$

$$\varphi_{16j}=P_1B_{\omega ij}+P_2B_{fij}D_{\omega kj};\quad \varphi_{26j}=P_2^TB_{\omega ij}+P_3B_{fij}D_{\omega kj}$$

$$I_1=[0\quad 0\quad 0\quad -I\quad I\quad 0];\ I_2=[0\quad 0\quad I\quad 0\quad -I\quad 0];$$

$$\chi_p:=\{\chi_{p1}\quad \chi_{p2}\quad \chi_{p3}\quad \chi_{p4}\quad \chi_{p5}\quad \chi_{p6}\},\ \text{p=1,2}$$

$$\Upsilon_{1j}:=[A_{ij}\quad 0\quad 0\quad 0\quad A_{dij}\quad B_{wij}] \tag{3.4}$$

$$\Upsilon_{3j}:=\left[L_{ij}-D_{fij}C_{kj}\quad -C_{fij}\quad 0\quad 0\quad L_{dij}-D_{fij}C_{dkj}\quad G_{\omega ij}-D_{fij}D_{\omega kj}\right]$$

Proof: We introduce the following Lyapunov Krasovskii functional:

$$V(t,\tilde{x}_t)=V_p(t,\tilde{x}_t)+V_h(t,x_t) \tag{3.5}$$

Where $\tilde{x}_t$ denotes the function $\tilde{x}(s)$ defined on [t-h_b,t],

$$V_p(t,\tilde{x}_t)=\tilde{x}^T(t)P\tilde{x}(t) \tag{3.6}$$

$$V_h(t,x_t)=\int_{t-h_a}^{t}x^T(s)\,R_0x(s)ds+\int_{t-h_b}^{t-h_a}x^T(s)\,R_\lambda x(s)ds+\int_{t-\tau(t)}^{t-h_a}x^T(s)\,R_dx(s)ds+ \tag{3.7}$$

$$\int_{t-h_b}^{t-\tau(t)}x^T(s)\,P_dx(s)ds+h_a\int_{-h_a}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\,Q_0\dot{x}(s)dsd\theta+\lambda_h\int_{-h_b}^{-h_a}\int_{t+\theta}^{t}\dot{x}^T(s)\,Q_\lambda\dot{x}(s)dsd\theta$$

Where R_λ, Q_λ, R_0, R_d, P_d, Q_0, and P=$\begin{bmatrix}P_1 & P_2\\ * & P_3\end{bmatrix}$ are symmetric positive definite matrices with appropriate dimensions and $\lambda_h=h_b-h_a$.

Let,

$$\xi:=\text{col}\{x(t)\quad \hat{x}(t)\quad x(t-h_a)\quad x(t-h_b)\quad x(t-\tau(t))\quad \omega(t)\} \tag{3.8}$$

Then, the error system (2.7) can be written as,

$$\begin{cases}\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\mu_i(\eta(t))\sum_{k=1}^{r}\mu_k(\eta(t))\begin{bmatrix}\Upsilon_{1j}\\ \Upsilon_{2j}\end{bmatrix}\xi\\ e(t)=\sum_{i=1}^{r}\mu_i(\eta(t))\sum_{k=1}^{r}\mu_k(\eta(t))\,\Upsilon_{3j}\xi\\ \tilde{x}(t)=[\Phi^T(t)\quad 0]\end{cases} \tag{3.9}$$

Where Υ_{1j} are defined in (3.4) and $\Upsilon_{2j}:=\left[B_{fij}C_{kj}\quad A_{fij}\quad 0\quad 0\quad B_{fij}C_{dkj}\quad B_{fij}D_{\omega kj}\right]$.

Using the expressions in (3.5) and taking the time-derivative of $V(t,\tilde{x}_t)$ along the solution of (3.1) we obtain

$$\dot{V}_p(t,\tilde{x}_t) = 2\sum_{i=1}^{r}\mu_i(\eta(t))\sum_{k=1}^{r}\mu_k(\eta(t))\,\tilde{x}^T(t)P\begin{bmatrix}\Upsilon_{1j}\\ \Upsilon_{2j}\end{bmatrix}\xi \quad (3.10)$$

$$\dot{V}_h(t,x_t) = x^T(t)R_0x(t) - x^T(t-h_a)R_0\,x(t-h_a) + x^T(t-h_a)R_\lambda\,x(t-h_a) - x^T(t-h_b)R_\lambda\,x(t-h_b) + x^T(t-h_a)\,R_d\,x(t-h_a) - (1-d)x^T(t-\tau(t))\,R_d\,x(t-\tau(t))$$

$$+ (1-d)x^T(t-\tau(t))\,P_d\,x(t-\tau(t)) - x^T(t-h_b)P_d\,x(t-h_b) + h_a^2\dot{x}^T(t)\,Q_0\dot{x}(t) - h_a\int_{t-h_a}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds + \lambda_h^2\dot{x}^T(t)\,Q_\lambda\dot{x}(t) - \lambda_h\int_{t-h_b}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds \quad (3.11)$$

Since $\tau(t)\in[h_a,h_b]$, we define $\rho(t) = (h_b-\tau(t))/\lambda_h$, $\lambda_h = h_b-h_a$, and we use lemma 2.3. The following inequalities are obtained,

$$-h_a\int_{t-h_a}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds \le \begin{bmatrix}x(t)\\ x(t-h_a)\end{bmatrix}^T\begin{bmatrix}-Q_0 & Q_0\\ * & -Q_0\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h_a)\end{bmatrix} \quad (3.12)$$

$$-\lambda_h\int_{t-h_b}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds = -\lambda_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds - \lambda_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds$$

$$= -(h_b-\tau(t))\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds - (\tau(t)-h_a)\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds$$

$$-(1-\rho(t))\lambda_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds - \rho(t)\lambda_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds$$

$$\le \begin{bmatrix}x(t-h_a)\\ x(t-h_b)\\ x(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}-Q_\lambda & 0 & Q_\lambda\\ * & -Q_\lambda & Q_\lambda\\ * & * & -2Q_\lambda\end{bmatrix}\begin{bmatrix}x(t-h_a)\\ x(t-h_b)\\ x(t-\tau(t))\end{bmatrix}$$

$$-(1-\rho(t))\,\lambda_h\int_{t-h_b}^{t-\tau(t)}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds - \rho(t)\lambda_h\int_{t-\tau(t)}^{t-h_a}\dot{x}^T(s)Q_\lambda\dot{x}(s)ds \quad (3.13)$$

In addition, by the Leibniz- Newton formula, the following equations are true for any real matrices χ_{pq}, p= 1, 2; q=1, 2,…, 6 with appropriate dimensions:

$$0 = 2\lambda_h(1-\rho(t))\xi^T\chi_1\left[x(t-\tau(t)) - x(t-h_b) - \int_{t-h_b}^{t-\tau(t)}\dot{x}(s)ds\right],$$
$$0 = 2\lambda_h\rho(t)\xi^T\chi_2\left[x(t-h_a) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_a}\dot{x}(s)ds\right], \quad (3.14)$$

$$\chi_p := col\{\chi_{p1} \quad \chi_{p2} \quad \chi_{p3} \quad \chi_{p4} \quad \chi_{p5} \quad \chi_{p6}\},$$

Tacking account of (3.10)-(3.12) and (3.13)-(3.14), we can write,

$$\dot{V}(t,x_t) - \gamma^2\omega^T(t)\omega(t) \le \sum_{i=1}^{r}\mu_i(\eta(t))\sum_{k=1}^{r}\mu_k(\eta(t))\{\xi^T\Pi_{dj}\xi$$

$$-\lambda_h\,(1-\rho(t))\int_{t-h_b}^{t-\tau(t)}(\xi^T\chi_1 + \dot{x}^T(t)Q_\lambda)Q_\lambda^{-1}(\chi_1^T\xi + Q_\lambda\dot{x}(s))ds - \lambda_h\,\rho(t))\int_{t-\tau(t)}^{t-h_a}(\xi^T\chi_2 + \dot{x}^T(t)Q_\lambda)Q_\delta^{-1}(\chi_2^T\xi + Q_\lambda\dot{x}(s))ds\} \quad (3.15)$$

Where

$$\Pi_{dj} = (1-\rho(t))\Pi_{1ik}^{j} + \rho(t)\Pi_{2ik}^{j}$$

$$\Pi_{pik}^{j} := \Pi_{ik}^{j} + \left[-I_p^T Q_\lambda I_p + \lambda_h \chi_p + \delta_h I_p^T \chi_p^T\right] + h_a^2 Y_{1j}^T Q_0 Y_{1j} + \delta_h^2 Y_{1j}^T Q_\lambda Y_{1j} + \lambda_h^2 \chi_p^T Q_\delta^{-1} \chi_p \qquad (3.16)$$

With $\chi_p, (p=1,2)$, Π_{ik}^{j}, I_1 and I_2 are defined in (3.3).

Notice that, since $Q_\lambda > 0$, ρ (t) ∈ $[0,1]$, (3.15) implies the following:

$$\dot{V}(t,x_t) - \gamma^2 \omega^T(t)\omega(t) \le \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))\, \xi^T \Pi_{dj} \xi \qquad (3.17)$$

Due to ρ (t) ∈ $[0,1]$, Π_{dj} <0 if $\Pi_{pik}^{j} < 0$, p=1, 2.

Applying the Schur Complement $\Pi_{pik}^{j} < 0$ is equivalent to the following LMIs:

$$\Psi_{pik}^{j} = \begin{bmatrix} \Pi_{ik}^{j} + \left[-I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T\right] & h_a Y_{1j}^T Q_0 & \lambda_h Y_{1j}^T Q_\lambda & \delta_h \chi_p \\ * & -Q_0 & 0 & 0 \\ * & * & -Q_\lambda & 0 \\ * & * & * & -Q_\lambda \end{bmatrix} < 0,\ \text{p=1,2} \qquad (3.18)$$

And $\Psi_{pik}^{j} < 0$ leads for $d = d_1\ or\ d_2$, to the following:

$$\Psi_{pik}^{jl} = \Psi_{pik}^{j}\Big|_{d=d_l} < 0;\ l = 1, 2 \qquad (3.19)$$

Notice that

$$\Psi_{pik}^{j} = \frac{d_2 - d}{d_2 - d_1}\Psi_{pik}^{j1} + \frac{d - d_1}{d_2 - d_1}\Psi_{pik}^{j2} \qquad (3.20)$$

Therefore, the LMIs (3.19) imply (3.18), and Ψ_{pik}^{j} is convex in $d \in [d_1, d_2]$, for p=1,2.

Consequently, if the two LMIs (3.18) are feasible, then $\Pi_{dj} < 0$. It follows from (3.17) that

$$\dot{V}(t,x_t) - \gamma^2 \omega^T(t)\omega(t) < 0$$

From the above analysis, it can be easily seen that the asymptotic stability of the error system can be obtained for $\omega(t) = 0$, that is $\dot{V}(t,x_t) < 0$.

Next, assuming that $\tilde{x}(t) = 0$, for all $t \in [-h_b, 0]$, we prove that for prescribed performance level γ>0, the H∞ performance $\|e\|_2 < \gamma\|\omega(t)\|_2$ is guaranteed for all nonzero ω(t) ∈ $L_2[0,\infty)$.

Since $\sum_{i=1}^{r} \mu_i(\eta(t)) = 1$, we can write $e^T(t)e(t) = \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t))\, \xi^T Y_{3j}^T Y_{3j} \xi$, and (3.17) lead to the following,

$$\dot{V}(t,x_t) \le \sum_{i=1}^{r} \mu_i(\eta(t)) \sum_{k=1}^{r} \mu_k(\eta(t)) \left\{\xi^T \hat{\Pi}_{dj} \xi - e^T(t)e(t) + \gamma^2 \omega^T(t)\omega(t)\right\}, \qquad (3.21)$$

Where

$$\hat{\Pi}_{dj} = (1-\rho(t))\hat{\Pi}_{1ik}^{j} + \rho(t)\hat{\Pi}_{2ik}^{j}$$

$$\widehat{\Pi}_{pik}^{j} = \Pi_{ik}^{j} + \left[-I_p^T Q_\lambda I_p + \lambda_h \chi_p + \lambda_h I_p^T \chi_p^T\right] + h_a^2 \Upsilon_{1j}^T Q_0 \Upsilon_{1j} + \lambda_h^2 \Upsilon_{1j}^T Q_\lambda \Upsilon_{1j} + \lambda_h^2 \chi_p^T Q_\lambda^{-1} \chi_p + \Upsilon_{3j}^T \Upsilon_{3j} \quad (3.22)$$

If the condition (3.2) holds, then by using the Schur Complement we obtain $\widehat{\Pi}_{pik}^{j} < 0$ for p=1,2 and consequently $\widehat{\Pi}_{dj} < 0$ because of ρ(t) ∈ $[0,1]$. In the same way as above, we can show that $\Psi_{pik}^{j} < 0, (p = 1,2)$ are also convex in d ∈ $[d_1, d_2]$. Thus, we have,

$$\dot{V}(t, x_t) \leq -e^T(t)e(t) + \gamma^2 \omega^T(t)\omega(t) \quad (3.23)$$

Integrating both sides of (3.23) from 0 to ∞ on t, and considering the zero initial condition, we obtains

$$\int_0^\infty e^T(t)e(t) < \gamma^2 \int_0^\infty \omega^T(t)\omega(t) \quad (3.24)$$

That is $\|e\|_2 < \gamma\|\omega\|_2$,

Let $\tilde{\Sigma}_i^j = \Pi_{ik}^j - I_p^T Q_\lambda I_p + \lambda_h \chi_p I_p + \lambda_h I_p^T \chi_p^T$; $\hat{\Sigma}_i^j = \begin{bmatrix} h_a \Upsilon_{1j}^T Q_0 & \lambda_h \Upsilon_{1j}^T Q_\lambda & \lambda_h \chi_p & \Upsilon_{3j}^T \end{bmatrix}$;
$\tilde{Q} = diag\,\{Q_0, Q_\lambda, Q_\lambda, I\}$; $\Theta_i^j = \tilde{\Sigma}_i^j + \hat{\Sigma}_i^j \tilde{Q}^{-1} \hat{\Sigma}_i^{j^T}$; and,
$S_j = \left\{(x^T, y^T, z^T, u^T, v^T, q^T)^T / (x^T, y^T, z^T, u^T, v^T, q^T)\Theta_i^j (x^T, y^T, z^T, u^T, v^T, q^T)^T < 0\right\}$

Then $\cup_{j=1}^{N} S_j = \mathbb{R}^{5n+p}/\{0\}$. Construct the following sets:

$$\bar{S}_1 = S_1,\ \bar{S}_2 = S_2 - \bar{S}_1,\ \ldots,$$

$$\bar{S}_i = S_i - \cup_{j=1}^{i-1} \bar{S}_j,\ \ldots,\ \bar{S}_N = S_N - \cup_{j=1}^{N-1} \bar{S}_j$$

We get $\cup_{j=1}^{N-1} \bar{S}_j = \mathbb{R}^{5n+p}/\{0\}$, $\bar{S}_i \cap \bar{S}_j = \emptyset \;\; i \neq j$ (3.25)

Now we construct the switching rule SR as,

σ = j when $\xi = (x^T(t), \hat{x}^T(t), x^T(t-h_a),\ x^T(t-h_b), x^T(t-\tau), \omega^T(t))^T \in \bar{S}_j$, j∈ {1,2,,…,N}. So, when ξ ∈ $\bar{S}_j$, the j^{th} subsystem is activated. This completes the proof. ■

Remark 3.2: Theorem 3.1 presents a sufficient condition of asymptotic stability of the error system with time delay and guarantees an H∞ norm bound performance level γ. If we consider the non switched system of (3.1), our result reduces to that obtained in [29]. Clearly, the results of [29] cannot be used for switched fuzzy systems. This shows that our results may be viewed a generalization of the results of [29].

Next, we will investigate the system subject to the uncertainties. In this case we are interested with the robust stability and a criterion is stated in the following corollary.

Corollary 3.1 : Given a scalars $0 \leq h_a \leq h_b$, $d_1 \leq d_2$ and γ>0, the filtering error system (2.7) is robustly stable with a prescribed H_∞ performance level γ , for all differentiable delay τ(t) ∈[h_a, h_b], if there exist positive definite matrices $R_0,\ R_\lambda,\ Q_0,\ Q_\lambda,\ P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $R_d,\ P_d$, real matrices $X_{pq}, (p = 1,2; q = 1,2, \ldots, 6)$ with appropriate dimensions, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the following LMIs are feasible where $d = d_1, d_2$

$$\Sigma_{pik}^{jc} = \begin{bmatrix} \Sigma_{pik}^{j} & H_1 & \varepsilon_1 U_1 & H_2 & \varepsilon_2 U_2 & H_3 & \varepsilon_3 U_3 \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \qquad (3.26)$$

Where Σ_{pik}^{j} are define in (3.2), (3.3), (3.4) and

$$\begin{aligned} H_1 &= \begin{bmatrix} D_{1ij}^T P_1 & D_{1ij}^T P_2^T & 0 & 0 & 0 & 0 & h_a D_{1ij}^T Q_0 & \delta_h D_{1ij}^T Q_\lambda & 0 & 0 \end{bmatrix}^T \\ H_2 &= \begin{bmatrix} D_{2ij}^T B_{fij}^T P_2^T & D_{2ij}^T B_{fij}^T P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -D_{2ij}^T D_{fij}^T \end{bmatrix}^T \\ H_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{3ij}^T \end{bmatrix}^T \\ U_i &= \begin{bmatrix} E_{k0ij} & 0 & 0 & 0 & E_{k1ij} & E_{k2ij} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \end{aligned} \qquad (3.27)$$

Proof: Replacing $A_{ij}, A_{dij}, B_{\omega ij}, , C_{kj}, C_{dkj}, D_{\omega kj}, L_{ij}, L_{dij}$ *and* $G_{\omega ij}$ by $A_{ij} + \Delta A_{ij}(t)$, $A_{dij} + \Delta A_{dij}(t)$, $B_{\omega ij} + \Delta B_{\omega ij}(t)$, $C_{kj} + \Delta C_{kj}(t)$, $C_{dkj} + \Delta C_{dkj}(t)$, $D_{\omega kj} + \Delta D_{\omega kj}(t)$, $L_{ij} + \Delta L_{ij}(t)$, $L_{dij} + \Delta L_{dij}(t)$ and $G_{\omega ij} + \Delta G_{\omega ij}(t)$ in (3.2) respectively, using lemma 2.2, lemma 2.3 and by Schur Complement we obtain the LMI defined in (3.26).
This completes the proof.

4 Fuzzy H∞ Filter Design

In the sequel, the following theorem provides sufficient conditions for the existence of a filter in the form (2.6) assuring an H∞ performance for system (3.1) and shows how to determine the parameter matrices of the filter.

Theorem 4.1 : Given scalars $0 \le h_a \le h_b$, $d_1 \le d_2$ and γ>0, the filter error system (3.1), , is asymptotically stable and has a prescribed H_∞ performance level γ, for all differentiable delay τ (t) ∈$[h_a, h_b]$ if there exist positive definite matrices P_1, U, R_0, R_λ, Q_0, Q_λ, P_λ, R_λ, and real matrices N_{1ij}, N_{2ij}, N_{3ij}, N_{4ij} , $(i = 1, \dots, r;\ j = 1, \dots, N)$, and $\hat{\chi}_p^i = col\{\chi_{p1}^i \quad \hat{\chi}_{p2}^i \quad \chi_{p3}^i \quad \chi_{p4}^i \quad \chi_{p5}^i \quad \chi_{p6}^i\}$, $(p = 1,2; i = 1, \dots, r)$, with appropriate dimensions such that the following LMIs are feasible where $d = d_1, d_2$

i. U-$P_1 < 0$ (4.1)

ii. $$\Pi_p^j(m,n) = \begin{bmatrix} \hat{\Pi}_{mn}^j + \left[-I_p^T Q_\lambda I_p + \lambda_h \hat{\chi}_p^m I_p + \lambda_h I_p^T (\hat{\chi}_p^m)^T\right] & h_a \left(\Upsilon_{1j}^m\right)^T Q_0 & \lambda_h \left(\Upsilon_{1j}^m\right)^T Q_\lambda & \lambda_h \hat{\chi}_p^m & \left(\Upsilon_{3j}^m\right)^T \\ * & -Q_0 & 0 & 0 & 0 \\ * & * & -Q_\lambda & 0 & 0 \\ * & * & * & -Q_\lambda & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (4.2)$$

Where

$$\hat{\Pi}_{mn}^j = \begin{bmatrix} \hat{\varphi}_{11j} & \hat{\varphi}_{12j} & Q_0 & 0 & \hat{\varphi}_{15j} & \hat{\varphi}_{16j} \\ * & \hat{\varphi}_{22j} & 0 & 0 & \hat{\varphi}_{25j} & \hat{\varphi}_{26j} \\ * & * & R_d - R_0 + R_\lambda - Q_0 - Q_\lambda & 0 & Q_\lambda & 0 \\ * & * & * & -R_\lambda - Q_\lambda - P_d & Q_\lambda & 0 \\ * & * & * & * & -(1-d)(R_d - P_d) - 2Q_\lambda & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\hat{\varphi}_{11j} = P_1 A_{mj} + A_{mj}^T P_1 + N_{2nj} C_{mj} + C_{mj}^T N_{2nj}^T + R_0 - Q_0$$

$$\hat{\varphi}_{12j} = N_{1nj} + A_{mj}^T U + C_{mj}^T N_{2nj}^T; \qquad \hat{\varphi}_{22j} = N_{1nj} + N_{1nj}^T$$

$$\hat{\varphi}_{15j} = P_1 A_{dm} + N_{2nj} C_{dmj}; \qquad \hat{\varphi}_{25j} = U A_{dmj} + N_{2nj} C_{mj} \tag{4.3}$$

$$\hat{\varphi}_{16j} = P_1 B_{wmj} + N_{2nj} D_{wmj}; \qquad \hat{\varphi}_{26j} = U B_{wmj} + N_{2nj} D_{wmj}$$

$$\Upsilon_{1j}^m := [A_{mj} \quad 0 \quad 0 \quad 0 \quad A_{\tau mj} \quad B_{wmj}]$$

$$\Upsilon_{3j}^m := \left[L_{mj} - N_{4nj} C_{mj} \quad -N_{3nj} \quad 0 \quad 0 \quad L_{dmj} - N_{4nj} C_{dmj} \quad G_{wmj} - N_{4nj} D_{wmj}\right]$$

Moreover, the filter matrices can be constructed by

$$A_{fij} = N_{1ij} U^{-1}, B_{fij} = N_{2ij}, C_{fij} = N_{3ij} U^{-1}, D_{fij} = N_{4ij} \quad \text{(i=1,2,…,r; j=1,2,…,N)} \tag{4.4}$$

Proof: Since U>0, there exist a nonsingular real n×n matrix P_2 and a real n×n matrix $P_3 > 0$ such that U=$P_2 P_3^{-1} P_2^T$. Define,

$$\text{J:=diag } \{\text{I}, P_2^{-T} P_3, I, I, I, I, I, I, I\} \tag{4.5}$$

Multiplying $\Pi_p^j\,(i,k), p = 1,2$ defined in (4.2) by J^T on the left and by J on the right, taking,

$$\chi_{p2} := P_3 P_2^{-1} \hat{\chi}_{p2}, \ \bar{A}_{ij} = P_2^{-1} N_{1ij} U^{-1} P_2, \ \bar{B}_{ij} = P_2^{-1} N_{2ij}, \bar{C}_{ij} = N_{3ij} U^{-1} P_2, \ \bar{D}_{ij} = N_{4ij}, \tag{4.6}$$

and replacing $(A_{fij}, B_{fij}, C_{fij}, D_{fij})$ in Σ_{pik}^j , defined in (3.2) by $(\bar{A}_{ij}, \bar{B}_{ij}, \bar{C}_{ij}, \bar{D}_{ij})$, we obtain,

$$\Sigma_{pik}^j = J^T \Pi_p^j\,(i,k) J \text{ , p=1,2} \tag{4.7}$$

So $\Pi_p^j\,(i,k) < 0$ if $\Sigma_{pik}^j < 0$. On the other hand, it is clear from (4.1) that

P_1 -U= $P_1 - P_2 P_3^{-1} P_2^T > 0$. Applying Schur Complement yields $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$>0. Therefore, we can conclude from theorem 3.1 that the filter described by,

$$\begin{cases} \dot{\bar{x}}(t) = \sum_{i=1}^{r} \mu_i(\eta(t))\,(\bar{A}_{ij}\bar{x}(t) + \bar{B}_{ij} y(t)) \\ \bar{z}(t) = \sum_{i=1}^{r} \mu_i(\eta(t))\,(\bar{C}_{ij}\bar{x}(t) + \bar{D}_{ij} y(t)) \end{cases} \tag{4.8}$$

Where $(\bar{A}_{ij}, \bar{B}_{ij}, \bar{C}_{ij}, \bar{D}_{ij})$ are defined in (4.6), guarantees the asymptotically stable with the prescribed H∞ noise attenuation level γ of the filtering error system (3.1).

Next, let $\hat{x}(t) = P_2 \bar{x}(t)$ then from (4.8) we obtain the filter system (2.6) with the matrices given by (4.4).

Now, let $\tilde{\Sigma}_i^j = \hat{\Pi}_{ik}^j - I_p^T Q_\lambda I_p + \lambda_h \hat{\chi}_p^i I_p + \lambda_h I_p^T \hat{\chi}_p^{i\,T}$, $\hat{\Sigma}_i^j = \left[h_a \Upsilon_{1ij}^T Q_0 \quad \lambda_h \Upsilon_{1ij}^T Q_\lambda \quad \lambda_h \hat{\chi}_p \quad \Upsilon_{3ij}^T\right]$
$\tilde{Q} = diag\,\{Q_0, Q_\lambda, Q_\lambda, I\}$, $\Theta_{ik}^j = \tilde{\Sigma}_{ik}^j + \hat{\Sigma}_{ik}^j \tilde{Q}^{-1} \hat{\Sigma}_{ik}^{j\,T}$, and

$$S_j = \left\{(x^T, y^T, z^T, u^T, v^T, q^T)^T / (x^T, y^T, z^T, u^T, v^T, q^T) \Theta_{ik}^j (x^T, y^T, z^T, u^T, v^T, q^T)^T < 0\right\}$$

Then $\cup_{j=1}^N S_j = \mathbb{R}^{5n+p}/\{0\}$. Constructing the sets $\bar{S}_j$, j=1,…,N, and the switching rule in the same way as in the previous section completes the proof.

Remark 4.2. In theorem 4.1, the problem of constructing the matrices of the filter system of the form (2.6) that guarantees the asymptotic stability with H∞ norm bound attenuation of the error system was investigated. The H∞ filtering problem was investigated in[30] and [26] for switched systems and in [31] for fuzzy systems. To the best of our knowledge, the H∞ filtering problem has not been investigated for TS fuzzy switched systems. So, our approch gives an insight into stability and H∞ filtering analysis and synthesis. For lack of existence of results in the subject studied in this paper, we will compare, through numerical examples, our approach with that of references [26,30] considering only switched systems, and with [31] considering T-S fuzzy systems.

Next, similar to corollary 3.1, the main result for uncertain T-S fuzzy switched systems is presented in the following corollary

Corollary 4.1: Given scalars $0 \le h_a \le h_b$, $d_1 \le d_2$ and γ>0, the filtering error system (2.7), is robustly stable with a prescribed H_∞ performance level γ, for all time varying delays τ (t) ∈ $[h_a, h_b]$ if there exist positive definite matrices P_1 , U, R_0, R_λ, Q_0, Q_λ, P_d, R_d, real matrices N_{1ij}, N_{2ij}, N_{3ij}, N_{4ij} , $(i = 1, \dots, r;\ j = 1, \dots, N)$, and $\hat{\chi}_p^i = col\{\chi_{p1}^i \quad \hat{\chi}_{p2}^i \quad \chi_{p3}^i \quad \chi_{p4}^i \quad \chi_{p5}^i \quad \chi_{p6}^i\}$, $(p = 1,2; i = 1, \dots, r)$, with appropriate dimensions, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ such that the following LMIs are feasible where $d = d_1, d_2$

i. U-P_1<0 (4.9)

ii. $$\Pi_{pik}^{jc} = \begin{bmatrix} \Pi_p^j\,(i,k) & H_1 & \varepsilon_1 U_1 & H_2 & \varepsilon_2 U_2 & H_3 & \varepsilon_3 U_3 \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (4.10)$$

Where $\Pi_p^j\,(i,k)$,H_i and U_i are defined in (4.2) and (3.27) respectively. Moreover, the filter matrices can be constructed by (4.4).

5 Examples

In this section, three examples are given to show the effectiveness of the proposed method and to compare with the literature results.

Example 5.1 ([30], [31]): Consider the following time-delay fuzzy system

Rule 1: If η_1(t) is W_1

$$\text{Then} \begin{cases} \dot{x}(t) = A_{11}x(t) + A_{d11}x(t-\tau(t)) + B_{w11}w(t) \\ y(t) = C_{11}x(t) + C_{d11}x(t-\tau(t)) + D_{w11}w(t) \\ z(t) = L_{11}x(t) + L_{d11}x(t-\tau(t)) + G_{w11}w(t) \\ x(t) = \phi(t) \qquad t \in [-h_b, 0] \end{cases}$$

And

Rule 2: If η_2(t) is W_2

Then $\begin{cases} \dot{x}(t) = A_{21}x(t) + A_{d21}x(t-\tau(t)) + B_{w21}w(t) \\ y(t) = C_{21}x(t) + C_{d21}x(t-\tau(t)) + D_{w21}w(t) \\ z(t) = L_{21}x(t) + L_{d21}x(t-\tau(t)) + G_{w21}w(t) \\ x(t) = \phi(t) \qquad t \in [-h_b, 0] \end{cases}$

With $A_{11} = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}$; $A_{21} = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}$; $A_{d11} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}$;
$A_{d21} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$; $B_{w11} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}$; $B_{w21} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$; $C_{11} = [1 \quad 0]$; $C_{21} = [0.5 \quad -0.6]$;
$C_{d11} = [-0.8 \quad 0.6]$; $C_{d21} = [-0.2 \quad 1]$;$D_{w11} = 0.3$; $D_{w21} = -0.6$; $L_{11} = [1 \quad -0.5]$;
$L_{21} = [-0.2 \quad 0.3]$; $L_{d11} = [0.1 \quad 0]$; $L_{d21} = [0 \quad 0.2]$; $G_{w11} = 0$; $G_{w21} = 0$;
Our purpose is to determine the minimum H∞ prescribed attenuation level γ for which the error system is asymptotically stable. Taking $h_a = 0.3$, we compute γ for different values of h_b. The results are summarized in Table 1. In table 2 we present the results for various values of h_b when $h_a = 0.2$. It is clear that the results obtained with our method are better than those obtained by applying the existing methods [30,31].

Table 1. Minimum values of H∞ performance γ for given delay h_b with $h_a = 0.3$

	h_b=1	h_b=0.8	h_b=0.6	h_b=0.5
γ [29]	0.37	0.35	0.34	0.34
γ [30]	0.3481	0.3405	0.3322	0.3264
γ (theorem 4.1)	0.2116	0.2007	0.1853	0.1750

Table 2. Minimum values of H∞ performance γ for given delay h_b with $h_a = 0.2$

	h_b=1	h_b=0.8	h_b=0.6	h_b=0.5
γ [30]	0.3307	0.3244	0.3158	0.3148
γ (theorem 4.1)	0.2124	0.2014	0.1857	0.1751

For $h_a = 0.3, h_b = 1$, the filter parameters as given as follows:

$A_{f11} = \begin{bmatrix} -7.9117 & -3.7469 \\ 5.5546 & 0.9479 \end{bmatrix}$;$A_{f21} = \begin{bmatrix} -2.2407 & -0.3109 \\ 0.0384 & -1.9337 \end{bmatrix}$; $B_{f11} = \begin{bmatrix} -0.4551 \\ 0.3802 \end{bmatrix}$;
$B_{f21} = \begin{bmatrix} -0.1321 \\ 0.2398 \end{bmatrix}$;$C_{f11} = [-7.7611 \quad -2.3596]$; $C_{f21} = [0.0481 \quad -1.9574]$;

Example 5.2 [26]: In this example two cases are presented to demonstrate the effectiveness of the proposed approach. Namely system with constant delay and system with time varying delay.

Case 1: consider a time varying delay switched nominal system (3.1), where $h_a = 0.2$, $d_1 = 0$, $d_2 = 0.4$, γ=2 with the following two subsystems

Subsystem 1 is described by

$A_{11} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}$; $A_{d11} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; $B_{w11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C_{11} = [1 \quad 0]$; $C_{d11} = [0 \quad 1]$;
$D_{w11} = 1$; $L_{11} = [1 \quad 2]$; $L_{d11} = [1 \quad 0]$; $G_{w11} = 0.1$;

And subsystem 2 is described by

$A_{12} = \begin{bmatrix} -2 & 0 \\ 0 & -0.7 \end{bmatrix}$; $A_{d12} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$; $B_{w12} = \begin{bmatrix} -0.5 \\ 2 \end{bmatrix}$; $C_{12} = [0 \quad 1]$; $C_{d12} = [1 \quad 2]$;
$D_{w12} = 1$; $L_{12} = [2 \quad 1]$; $L_{d12} = [0 \quad 1]$; $G_{w12} = 0.2$;

Applying the results of [26], the maximum time delay h_b is found to be $h_b = 0.6$. Applying theorem 4.1 of this paper we obtain $h_b = 09030$. It is easy to verify that the results in this paper are less conservative than the ones obtained in [26]. The filter parameters are given by,

$$A_{f11} = 10^3 \times \begin{bmatrix} 0.5513 & -7.4723 \\ 0.2242 & -3.0191 \end{bmatrix}; \quad B_{f11} = \begin{bmatrix} -4.2492 \\ -1.1140 \end{bmatrix}$$
$$C_{f11} = 10^3 \times [0.1523 \quad -2.0473]; \quad D_{f11} = -0.2272$$

$$A_{f12} = 10^4 \times \begin{bmatrix} 0.0948 & -1.2768 \\ 0.0408 & -0.5489 \end{bmatrix}; \quad B_{f12} = \begin{bmatrix} -3.3350 \\ -0.9112 \end{bmatrix}$$
$$C_{f12} = 10^3 \times [0.2306 \quad -3.0986]; \quad D_{f12} = 0.6106$$

Case 2: consider a time delay switched nominal system (3.1), where $h_a = 0.3$, $d_1 = 0$, $d_2 = 0$, γ=8, with the following two subsystems

Subsystem 1 is described by

$$A_{11} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.8 \end{bmatrix}; A_{d11} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}; B_{w11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C_{11} = [1 \quad 0]; C_{d11} = [0 \quad 1];$$

$$D_{w11} = 1;\ L_{11} = [1 \quad 0]; L_{d11} = [1 \quad 0]; G_{w11} = 0.1;$$

And subsystem 2 is described by

$$A_{12} = \begin{bmatrix} -0.1 & 0 \\ 0 & -1 \end{bmatrix}; A_{d12} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; B_{w12} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}; C_{12} = [0 \quad 1]; C_{d12} = [1 \quad 0];$$

$$D_{w12} = 1;\ L_{12} = [1 \quad 1]; L_{d12} = [0 \quad 1]; G_{w12} = 0.1;$$

Computing the upper bound h_b of time delay, we obtain by applying our results and those of [26], h_b =1.4270 and h_b =1 respectively. It is easy to verify that the results in this paper are less conservative than the ones obtained in [26]. The filter parameters are given by,

$$A_{f11} = \begin{bmatrix} 0 - 3.2600 & 1.5006 \\ -1.2327 & 0.0318 \end{bmatrix}; \quad B_{f11} = \begin{bmatrix} -0.2092 \\ 1.1343 \end{bmatrix}$$
$$C_{f11} = [-1.8983 \quad 0.4018]; \quad D_{f11} = 0.0290$$

$$A_{f12} = \begin{bmatrix} -1.0478 & 0.8190 \\ 2.3136 & -3.5177 \end{bmatrix}; \quad B_{f12} = \begin{bmatrix} 1.0905 \\ -0.8014 \end{bmatrix}$$
$$C_{f12} = [-0.3187 \quad -0.5311]; \quad D_{f12} = -0.6148$$

Example 5.3: In this section, we provide a numerical example to illustrate the developed H∞ filter design approach. The uncertain T–S fuzzy switched time-delay system is described by two rules:

For the subsystem 1

Plant Rule 1:If η_1 (t) is W_1

$$\begin{cases} \dot{x}(t) = \big(A_{21} + \Delta A_{21}(t)\big)x(t) + \big(A_{d21} + \Delta A_{d21}(t)\big)x\big(t - \tau(t)\big) + \big(B_{w21} + \Delta B_{w21}(t)\big)\omega(t) \\ y(t) = \big(C_{21} + \Delta C_{21}(t)\big)x(t) + (C_{d21} + \Delta C_{d21}(t))x\big(t - \tau(t)\big) + (D_{w21} + \Delta D_{w21}(t))\omega(t) \\ z(t) = \big(L_{21} + \Delta L_{21}(t)\big)x(t) + (L_{d21} + \Delta L_{d21}(t))x\big(t - \tau(t)\big) + \big(G_{w21} + \Delta G_{w21}(t)\big)\omega(t) \\ x(t) = \phi(t) \qquad t \in [-h_b, 0] \quad i = 1, \dots, r \end{cases}$$

$$\begin{cases} \dot{x}(t) = \left(A_{11} + \Delta A_{11}(t)\right)x(t) + \left(A_{d11} + \Delta A_{d11}(t)\right)x\left(t-\tau(t)\right) + \left(B_{w11} + \Delta B_{w11}(t)\right)\omega(t) \\ y(t) = \left(C_{11} + \Delta C_{11}(t)\right)x(t) + \left(C_{d11} + \Delta C_{d11}(t)\right)x\left(t-\tau(t)\right) + \left(D_{w11} + \Delta D_{w11}(t)\right)\omega(t) \\ z(t) = \left(L_{11} + \Delta L_{11}(t)\right)x(t) + \left(L_{d11} + \Delta L_{d11}(t)\right)x\left(t-\tau(t)\right) + \left(G_{w11} + \Delta G_{w11}(t)\right)\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \ldots, r \end{cases}$$

Plant Rule 2: If η_2(t) is W_2

$$\begin{cases} \dot{x}(t) = \left(A_{21} + \Delta A_{21}(t)\right)x(t) + \left(A_{d21} + \Delta A_{d21}(t)\right)x\left(t-\tau(t)\right) + \left(B_{w21} + \Delta B_{w21}(t)\right)\omega(t) \\ y(t) = \left(C_{21} + \Delta C_{21}(t)\right)x(t) + \left(C_{d21} + \Delta C_{d21}(t)\right)x\left(t-\tau(t)\right) + \left(D_{w21} + \Delta D_{w21}(t)\right)\omega(t) \\ z(t) = \left(L_{21} + \Delta L_{21}(t)\right)x(t) + \left(L_{d21} + \Delta L_{d21}(t)\right)x\left(t-\tau(t)\right) + \left(G_{w21} + \Delta G_{w21}(t)\right)\omega(t) \\ x(t) = \phi(t) \quad t \in [-h_b, 0] \quad i = 1, \ldots, r \end{cases}$$

And for the subsystem 2

Plant Rule 1: If η_1(t) is W_1

$$\begin{cases} \dot{x}(t) = \left(A_{12} + \Delta A_{12}(t)\right)x(t) + \left(A_{d12} + \Delta A_{\tau 12}(t)\right)x\left(t-\tau(t)\right) + \left(B_{w12} + \Delta B_{w12}(t)\right)\omega(t) \\ y(t) = \left(C_{12} + \Delta C_{12}(t)\right)x(t) + \left(C_{d12} + \Delta C_{d12}(t)\right)x\left(t-\tau(t)\right) + \left(D_{w12} + \Delta D_{w12}(t)\right)\omega(t) \\ z(t) = \left(L_{12} + \Delta L_{12}(t)\right)x(t) + \left(L_{d12} + \Delta L_{d12}(t)\right)x\left(t-\tau(t)\right) + \left(G_{w12} + \Delta G_{w12}(t)\right)\omega(t) \\ x(t) = \phi(t) \quad t \in [-\mathrm{h}_b, 0] \quad i = 1, \ldots, r \end{cases}$$

Plant Rule 2: If η_2(t) is W_2

$$\begin{cases} \dot{x}(t) = \left(A_{22} + \Delta A_{22}(t)\right)x(t) + \left(A_{d22} + \Delta A_{d22}(t)\right)x\left(t-\tau(t)\right) + \left(B_{w22} + \Delta B_{w22}(t)\right)\omega(t) \\ y(t) = \left(C_{22} + \Delta C_{22}(t)\right)x(t) + \left(C_{d22} + \Delta C_{d22}(t)\right)x\left(t-\tau(t)\right) + \left(D_{w22} + \Delta D_{w22}(t)\right)\omega(t) \\ z(t) = \left(L_{22} + \Delta L_{22}(t)\right)x(t) + \left(L_{d22} + \Delta L_{d22}(t)\right)x\left(t-\tau(t)\right) + \left(G_{w22} + \Delta G_{w22}(t)\right)\omega(t) \\ x(t) = \phi(t) \quad t \in [-\mathrm{h}_b, 0] \quad i = 1, \ldots, r \end{cases}$$

Where $A_{11} = \begin{bmatrix} 17 & -1 \\ 0 & -9 \end{bmatrix}; A_{d11} = \begin{bmatrix} -10 & 0 \\ -0.15 & -1 \end{bmatrix}; B_{w11} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}; C_{11} = [-1 \quad 0];$
$C_{d11} = [0 \quad 1]; D_{w11} = 0.3;\ L_{11} = [-0.2 \quad 0.3]; L_{d11} = [1 \quad 0]; G_{w11} = -0.5;$
$A_{21} = \begin{bmatrix} -19 & 0 \\ 0 & -9 \end{bmatrix}; A_{d21} = \begin{bmatrix} 1 & 0 \\ 0.05 & -1 \end{bmatrix}; B_{w21} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}; C_{21} = [-0.21 \quad 0];$
$C_{d21} = [0 \quad 1];\ D_{w21} = 0.3;\ L_{21} = [-0.2 \quad 0.3];\ L_{d21} = [0 \quad 1];\ G_{w21} = -0.02;$
$A_{12} = \begin{bmatrix} -12 & 0 \\ -0.5 & -9 \end{bmatrix}; A_{d12} = \begin{bmatrix} -10 & 0 \\ -1 & -1 \end{bmatrix};\ B_{w12} = \begin{bmatrix} -0.5 \\ -2 \end{bmatrix}; C_{12} = [0 \quad 1];$
$C_{d12} = [1 \quad 2]; D_{w12} = 0.3; L_{12} = [-0.2 \quad 0.3];\ L_{d12} = [1 \quad 0];\ G_{w12} = -0.15;$
$A_{22} = \begin{bmatrix} -12 & 0 \\ 0 & -9 \end{bmatrix}; A_{d22} = \begin{bmatrix} -0.0021 & 0 \\ -1 & -1 \end{bmatrix};\ B_{w22} = \begin{bmatrix} -0.5 \\ -0.2 \end{bmatrix}; C_{22} = [0 \quad 1];$
$C_{d22} = [1 \quad 2]; D_{w22} = -0.6; L_{22} = [-0.2 \quad 0.3];\ L_{d22} = [0 \quad -1];\ G_{w22} = 0.6;$

The membership functions μ_1 and μ_2 are given by

$$\mu_1\left(\eta_1(\mathrm{t})\right) = \begin{cases} -1 & for\ z_1 < -1 \\ \frac{1}{2} + \frac{1}{2}z_1 & for\ |z_1| \le 1 \\ 1 & for\ z_1 > 1 \end{cases} \quad \text{And} \quad \mu_2(\eta_2(\mathrm{t})) = \begin{cases} 0 & for\ z_1 < -1 \\ \frac{1}{2} + \frac{1}{2}z_1 & for\ |z_1| \le 1 \\ 1 & for\ z_1 > 1 \end{cases}$$

The parameter uncertainties $\Delta A_{ij}(t), \Delta A_{dij}(t), \Delta B_{wij}(t), \Delta C_{ij}(t), \Delta C_{dij}(t), \Delta D_{wij}(t)$, $\Delta L_{ij}(t),\ \Delta L_{dij}(t)\ and\ \Delta G_{wij}(t)$, i=1,2, j=1,2 are assumed to satisfy (2.2) and (2.3) with $D_{111} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}; E_{1011} = [0 \quad 0.3];\ E_{1111} = [0.2 \quad 0];\ E_{1211} = 0.1;\ D_{211} = 0.8;$

$E_{2011} = [0 \quad 0.3]$; $E_{2111} = [0.2 \quad 0]$; $E_{2211} = 0.1$; $D_{311} = 0.002$; $E_{3011} = [0.1 \quad 0]$;
$E_{3111} = [0.1 \quad 0]$; $E_{3211} = 0.1$; $D_{121} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$; $E_{1021} = [0.5 \quad 0]$; $E_{1121} = [0 \quad -0.2]$;
$E_{1221} = 0.2$; $D_{221} = 0.6$; $E_{2021} = [0.5 \quad 0]$; $E_{2121} = [0 \quad -0.2]$; $E_{2221} = 0.2$;
$D_{321} = 0.001$; $E_{3021} = [0.2 \quad 0]$; $E_{3121} = [0.2 \quad 0]$; $E_{3221} = -0.5$; $D_{112} = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$;
$E_{1012} = [0 \quad 0.3]$; $E_{1112} = [0.2 \quad 0]$; $E_{1212} = 0.1$; $D_{212} = 0.8$;
$E_{2012} = [0 \quad 0.3]$; $E_{2112} = [0.2 \quad 0]$; $E_{2212} = 0.1$; $D_{312} = 0.004$; $E_{3012} = [0.1 \quad 0]$;
$E_{3112} = [0.1 \quad 0]$; $E_{3212} = -0.2$; $D_{122} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$; $E_{1022} = [0.5 \quad 0]$; $E_{1122} = [0 \quad -0.2]$;
$E_{1222} = 0.2$; $D_{222} = 0.6$; $E_{2122} = [0.5 \quad 0]$; $E_{2122} = [0 \quad -0.2]$; $E_{2222} = 0.2$;
$D_{322} = 0.002$; $E_{3022} = [0.2 \quad 0]$; $E_{3122} = [0.2 \quad 0]$; $E_{3222} = -0.2$;
Given $h_a = 0, d_1 = 0$, $d_2 = 0.2$ and the upper bound $h_b = 4.2$

The H∞ performance level is specified to be $0.9\,8$. Applying corollary 4.1, the filter parameters are obtained in the following.

$$A_{f11} = \begin{bmatrix} -217.0968 & -124.6463 \\ -14.7098 & -125.1392 \end{bmatrix}; \quad B_{f11} = \begin{bmatrix} 0.0147 \\ 0.0052 \end{bmatrix}$$
$C_{f11} = [-3.7125 \quad 5.7636]$; D_{f11} =-0.0025

$$A_{f21} = \begin{bmatrix} -283.1316 & -67.5379 \\ 1.6472 & -135.6281 \end{bmatrix}; \quad B_{f21} = \begin{bmatrix} 0.0038 \\ 0.0086 \end{bmatrix}$$
$C_{f21} = [1.0019 \quad -10.2763]$; D_{f21} =0.0140

$$A_{f12} = \begin{bmatrix} -13.8393 & 58.5614 \\ -0.7136 & -136.5757 \end{bmatrix}; \quad B_{f12} = \begin{bmatrix} 0.041 \\ -0.0073 \end{bmatrix}$$
$C_{f12} = [7.6017 \quad -25.5315]$; D_{f12} =-4.5541$\times 10^{-5}$

$$A_{f22} = \begin{bmatrix} -153.6803 & -20.050 \\ -4.1752 & -154.7375 \end{bmatrix}; \quad B_{f22} = \begin{bmatrix} -0.0102 \\ -0.0178 \end{bmatrix}$$
$C_{f22} = [3.5613 \quad -24.1776]$; D_{f22} =-0.0392

6 Conclusion

In this paper, we have established delay dependent conditions for asymptotic stability with H∞ norm bound attenuation level for T-S fuzzy switched systems with interval time-varying delays. The obtained conditions are extended to cover the robust stability in the case when the system is subject to time varying bounded perturbations. The results are expressed in terms of LMIs whose feasibility permits us to construct the matrices of a filter and guarantees the asymptotic or robust stability of a filtering error system. The H∞ filtering problem for the class of uncertain T-S fuzzy switched systems with interval time-varying delays has not been considered before. So our results are news and represent a starting point for the analysis and synthesis in this research domain.. The obtained results are less conservative than the existing ones. Numerical examples show the effectiveness and the less conservativeness of the proposed method.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Anderson BDO, Moore B. Optimal filtering, prentice-Hall, Englewood Cliffs, New York; 1979.

[2] Geromel C, De Oliveira MC. □$_2$ and □$_\infty$ robust filtering for convex bounded uncertain systems. IEEE Transactions on Automatic Control. 2001;46(1):100-107.

[3] Pila A, Shaked U, de Sousa C. □$_\infty$ filtering for continuous-time linear systems with delay. IEEE Transactions on Automatic Control. 1999;44(7):1412-1417.

[4] Nagpal KM, Khargonekar PP. filtering and smoothing in an □$_\infty$ setting. IEEE Transactions on Automatic Control. 1991;36:152-166.

[5] Gao H, Wang C. Delay-dependent robust H∞ and L2-L∞ filtering for a class of uncertain nonlinear time-delay systems. IEEE Transactions on Automatic Control. 2003;48(9):1661–1666.

[6] Li Z, Xu S. Fuzzy weighting-dependent approach to robust L2-L∞ filter design for delayed fuzzy systems. Signal Processing. 2009;89(4):463–471.

[7] Liu F, Wu M, Yon He R, Yokoyana. New delay dependent stability criteria for T-S fuzzy systems with time varying delay. Fuzzy Sets and Systems. 2010;161:2033-2042.

[8] Su X, Shi P, Wu L, Song YD. A novel approach to filter design for T-S fuzzy discrete time systems with time varying delay. IEEE Transactions, on fuzzy Systems. 2012;20(6).

[9] Dong H, Wang Z, Gao H. Robust H∞ filtering for a class of nonlinear networked systems with multiple stochastic communication delays and packet dropouts. IEEE Transactions on signal processing. 2010;58(4).

[10] Ma Y, Yan H. Delay-dependent robust H∞ filter for T-S fuzzy time-delay systems with exponential Stability. Ma and Yan Advances in Di□erence Equations. 2013;2013:362.

[11] Chen CL, Liu HX, Guan XP. H∞ filtering of time-delay T-S fuzzy systems based on piecewise Lyapunov-Krasovskii functional. Signal Process. 2009;89(10):1998-2005.

[12] MA Y, YAN H. Robust delay-dependent h∞ filter for t-s fuzzy descriptor time-delay systems with parameter uncertainties. Journal of Computational Information Systems. 2013;9:22:9095–9102.

[13] Liberzon D. Switching in systems and control, Boston, MA: BirkhÄauser Boston; 2003.

[14] Savkin AV, Evans RJ. Hybrid dynamical systems: Controller and sensor switching problems, Boston, USA: Birkh Äauser Boston; 2002.

[15] Sun ZD, Ge SS. Switched linear systems: Control and design, London, UK. Springer; 2005.

[16] Morse AS. Supervisory control of families of linear set-point controllers, part I: Exact matching. IEEE Transactions on Automatic Control. 1996;41:1413–1431,

[17] Hespanha JP, Morse AS. Switching between stabilizing controllers. Automatica. 2002;38:1905–1917.

[18] Zhang L, Gao H. Asynchronously switched control of switched linear systems with average dwell time. Automatica. 2010;46:953-958.

[19] Liu X. Stabilization of switched linear systems with mode dependent time varying delays. Applied Mathematics and Computation; 2010. DOI: 10.1016/j.amc. 2010.03.101.

[20] Hien LV, QPHA, PHat VN. switching design for robust exponential stability and stabilization of uncertain linear hybrid time delay systems. Applied Mathematical and Computation. 2009;210:223-231.

[21] Tissir EH. Exponential stability and guaranteed cost of switched linear systems with mixed time varying delays. International Scholars Research Network ISRN, Applied Mathematics. 2011;2011:1-14.

[22] Ahmida F, Tissir EH. Exponential stability of uncertain T-S Fuzzy switched system with time delay. International Journal of Automation and Computing. 2013;10:32-38.

[23] Ahmida F, Tissir EH. New stability conditions for switched uncertain T-S Fuzzy systems with two additive time varying delays. International Journal of Ecological Economics and Statics (IJEES). 2014;34:93-103.

[24] Zhang L, Shi P, Wang C, Gao H. Robust H_∞ filtering for switched linear discrete-time systems with polytopic uncertainties. International Journal of Adaptive Control and Signal Processing. 2006;20:291–304.

[25] Du D, Jiang B, Shi P, Zhou S. H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach. IEEE Transactions on Automatic Control. 2007;52:1520–1525.

[26] Wang D, Wang W, Shi P. Exponential H_∞ filtering for switched linear systems with interval time-varying delay. Int. J. Robust Nonlinear Control. 2009;19:532–551.

[27] Pererson IR, Hollot CV. A Riccati equation approach to the stabilization of uncertain linear system. Automatica.1986;22:397-411.

[28] Wang WJ, Tanaka K, Griffin MF. Stabilization, estimation and robustness for large scale time-delay systems. Control-Theory Adv. Technol. 1991;7:569-585.

[29] Jiyao A, Guilin W, Wei X. Improved results on Fuzzy H_∞ filter design for T-S Fuzzy systems. Hindawi Publishing Corporation Discrete Dynamics in Nature and Society; 2010.

[30] Zhao Y, Gao H, Lam J. New results on H_∞ filtering for fuzzy systems with interval time-varying delays. Information Sciences. 2011;181:2356-2369.

[31] Lin C, Wang QG, Lee TH, Chen B. H_∞ filter design for nonlinear systems with time-delay through T–S Fuzzy model approach. IEEE Trans. Fuzzy Syst. 2008;16(3):739–746.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=733&id=6&aid=7609