A Dynamic Programming Approach to Replacement of Transport Vehicles in Benin City, Nigeria

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Abstract

Every organisation has an objective to optimise the utility function of its available operational assets. For commercial vehicle transport operators, the goal is to operate the vehicles for as long as they can make net contribution to the organisation's corporate objective. Hence, when these vehicles are replaced becomes an issue for strategic decision making. Unfortunately many of the commercial bus transport companies lack the skill to undertake the required empirical evaluation necessary to provide objective data and information for making the vehicle replacement decisions. This study was therefore an effort to bridge this gap in knowledge. Only two out of the fourteen transporter companies of interest operating in Benin City, Edo State, Nigeria agreed to provide the required data for the study which covered the period 2008 to 2013 and for Toyota brand of buses only. The data was subjected to backward recursive dynamic programming analysis. The results showed that the four years fixed-age vehicle replacement policy employed by commercial bus transport companies in Benin City was optimum only for the Toyota high roof types of buses. The study thus recommends that commercial vehicle owners/operators should endeavour to keep reliable, relevant and up to records of their vehicles. While it is advocated that adequate and continuous training of key staff on equipment replacement should also be encouraged, operators of mass-transit systems can seek the assistance of Operations Research experts in order to enhance their decisions regarding vehicle replacement policies.

Keywords: Backward recursive, dynamic programming, minimisation model, toyota bus, vehicle replacement, replacement policy, replacement strategy.

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1 Introduction

The advent of machines has brought with it associated problems of repairs, maintenance and replacement. [1] observes that "The replacement of productive equipment ranks among the most important strategic decisions faced by both manufacturing and service firms". In Nigeria and the world over, road transport companies maintain fleets of vehicles which they use daily in providing mobility services to diverse customers seeking safety and comfort in their traveling experiences. The daily use of these vehicles results in wear and tear, as well as deterioration culminating in significant operating and maintenance costs. These costs significantly affect the performance, operation and profit of the transport companies. Hence, [2] submits that operating costs are vital and necessary to consider, both in their timing and in their magnitude. Consequently, managements of firms in the transport industry are faced with the decision of when and how to undertake vehicle replacement.

In Edo State, many of the private transport companies maintain fleets of vehicles of different types, sizes and ages. These vehicles are subjected to constant usage and hence they undergo wear and tear with the passage of time. This often results in increased operating and maintenance costs. This research focuses on how dynamic programming can be applied to vehicle replacement problems in some selected private transport companies in Edo State, Nigeria. Dynamic programming technique has been employed in vehicle replacement studies in Nigeria [3,4,5] and in other studies across the globe [6]. The choice of dynamic programming is based on its nature that fits well with the sequential characteristics of replacement problems as well as its flexibility and ability to generate solutions quickly as well as optimize within a wider range of options.

2 Review of Literature

Replacement is required in systems such as machines, tools, vehicles, capital assets and others and it may arise as a result of technological advancement, deterioration in efficiencies of these items over their life span, or it may be due to temporary or complete failure of these system. [7] see machinery replacement as the act of finding the adequate moment to change equipment in use, based on the analysis of a criterion or of a decision criteria group. [8] gives two basic reasons for equipment replacement. The first being degradation or deterioration and obsolescence and the second is the complete or partial failure that may occur in the original unit or units which in turn forces the decision of immediate replacement or repair of single or group units. Similarly, [9] identify some situations when replacement can take place. They include: when an old item has failed and does not work at all or the item is expected to fail soon; when an existing item deteriorates and works badly and also needs expensive maintenance; and when a better model of an item has been developed. Consequently, there are several approaches for resolving equipment replacement problems. These include total average cost, equivalent annual cost, differential equations, capital budgeting/cash flow, dynamic programming, Markovian processes and shortest path methods [8].

[10] captures dynamic programming as a procedure for finding optimal policies for sequential decision problems. [11] simply refer to dynamic programming as the process of solving sequential optimization problems where one needs to find the best decisions one after another. On their part, [12] view dynamic programming as a mathematical tool used to simplify decision problems by breaking such decisions down into a sequence of decision steps over time. Dynamic programming has also been described as "... a method that in general solves optimization problems that involve making a sequence of decisions by determining, for each decision, sub-problems that can be solved in like fashion, such that an optimal solution of the original problem can be found from optimal solutions of the sub-problems" [13]. It is important to note that a common thread runs through these definitions. The common thread is that dynamic programming deals with sequential decision processes that entail dividing the problem to be solved into smaller problems known as

sub-problems or stages. The sub-problems are solved one after the other, so that the answers to these small problems are used to solve the larger ones in order that the overall solution is optimal in relation to the original problem. [14] enumerate the main elements associated with a dynamic programming problem to include stages, states, decisions, transformations and returns. In the literature, problems that can be solved by dynamic programming exhibit the following properties:

i.   The problem can be decomposed into sub-problems or stages and a decision has to be made in each stage;
ii. Each stage has a number of possible states.
iii. The decision in each stage is to transform the current state into a state associated with the next stage.
iv. The Policy (or the best sequence of decisions) at any stage is independent of the decisions made at prior stages.
v.   A recursive relationship exists which identifies the optimal decision for stage *t*, given that stage *t* + 1 has already been solved.
vi. The history of the system must have no influence on its future behaviour

The advantages of using dynamic programming in vehicle replacement problems are that few constraints are placed on the function. It is flexible, and it has the ability to generate solutions quickly as well as optimize them within a wider range of options. Other advantages cited by some authors include the determination of absolute (global) maxima or minima and its ability to handle non-linear and discontinuous functions [15,16].

These advantages of dynamic programming have been exploited and hence the range of applications of dynamic programming is growing by the day. Several studies and researches have applied dynamic programming to equipment replacement problems. Some of the studies include [1,6,17,18,19,3,4,5] and a host of others.

[20] developed a dynamic programming model for machine replacement problem using revenue, operating cost and salvage value as the basic variables. He defines the year asstage represented by (i) where $(i) = 1(1)n$ and the state at stage (i) is the age of the machine at the start of year (i). The recursive equation is given below:

$$f_{n+1}(.) \equiv 0 \qquad \dots \qquad \dots \qquad \dots\dots\dots\dots\dots \text{ (1)}$$

$$f_i(t) = \max\big(r(0) + s(t) - I - c(0) + f_{i+1}(1)\big) \qquad \text{If REPLACE} \quad \dots\dots\dots\dots \text{ (2)}$$

$$f_i(t) = \max\big(r(t) - c(t) + f_{i+1}(t+1)\big) \qquad\qquad \text{If KEEP} \qquad \dots \text{ (3)}$$

Where,

$r(t)$ = Yearly Revenue

$s(t)$ = Salvage value

$c(t)$ = Operating cost

I = Acquisition Cost of Machine

$f_i(t)$ = maximum net income for years $i = 1(1)n$

[21] discussed the application of dynamic programming to equipment replacement. He reviewed a maximization model which used revenue, acquisition cost and maintenance cost as its basic variables.

$$f_t(i) = \max\left(R_t(i) - U_t(i) + f_{t+1}(i+1)\right) \qquad \ldots \qquad \ldots \qquad \text{KEEP} \qquad \ldots\ldots (4)$$

$$f_t(i) = \max\left(R_t(0) - U_t(0) - G_t(0) + f_{t+1}(i)\right) \qquad \ldots \quad \text{REPLACE} \ldots\ldots (5)$$

$f_t(i)$ = The maximum overall return from a machine of i years at the beginning of the   year
$$t, t+1, \ldots\ldots\ldots T$$

$R_t(i)$ = Revenue obtained from a machine at age i at the beginning of year t

$U_t(i)$ = Maintenance cost of machine of age i at the beginning of year t

$G_t(i)$ = Replacement cost of machine of age i by a new machine at the beginning of year t.

In analyzing their data regarding the study of the replacement of BRTC buses in Bangladesh, [22] employed the following dynamic programming algorithm based on the assumption that the purchase price (P) of a new machine is greater than its sale value (S_t) at the end of the t-th period. Their objective was to maximize the present value of future returns.

$$f_t = v\left(f_{t+1} + R_{t+1} - U_{t+1}\right) \qquad \ldots \qquad \text{To KEEP} \ldots \qquad \ldots \qquad \ldots \qquad (6)$$

$$f_t = v\left(S_t - P + f_o\right) \qquad\qquad \ldots \qquad \text{To REPLACE} \quad \ldots \qquad \ldots \qquad (7)$$

Where,

f_t =   Present value of future returns using an optimal strategy from the end of the t-th period.

P =   Purchase price

S_t =   Sale value at the end of the tth period

R_i =   Output

U_i =   Maintenance cost

[23] in their empirical contribution to vehicle replacement, formulated a cost minimization model with the assumptions of constant cost, existence of active resale and internal market that can be solved by dynamic programming. The model is given by

$$K(T) = \frac{\left(Q + B(t)R - S(t)R^t\right)}{R} \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots\ldots (8)$$

where,

Q = Purchase cost

$B(t) = \left(b_1 + b_2 t\right)$ = Maintenance cost

$S(t) = Q(1-d)^t$ = Salvage value

b_1 = Routine maintenance cost; representing the constant component of the maintenance cost.

b_2 = Dependent maintenance cost; representing the increase per period component of the time

d = deterioration
t = time

[4] also applied [24] economic model to the study of vehicle replacement in government parastatals in Nigeria. He employed the model given below:

$$MinV(N) = \frac{A - S(N)R^{N-1} + f(0)R^{N-1}\sum_{t=1}^{N-1} f(t)R^{t-1}}{1 - R^N} \quad \dots \quad \dots \quad \dots \dots \quad (9)$$

Where,

$V(N)$ = Annual average cost at age N

A = The acquisition cost of a new equipment

$S(N)$ = The salvage value of an equipment at age N

R = Discount factor

$f(t)$ = Maintenance cost function at period t.

They found correlation between vehicle accumulated operating and maintenance costs and its cumulative age in use. They also found that an individual vehicle policy is preferred to the age of group vehicle policy and average vehicle policy because it provides cost saving.

3 Methodology

For the purpose of this replacement analysis the vehicles were grouped according to the types of vehicle used. Data pertaining to repair and maintenance expenditure (parts and labour) and vehicle age in use were collected from two transport companies which agreed to make their records available for the study on the condition of anonymity over a four-year period. The data were collected from the vehicle log books and recorded in the data sheets. Data collected include: vehicle make and model, date of purchase, initial purchase price, and detailed repair and maintenance expenditure (including those for vehicle parts and for labour). Backward recursion method of dynamic programming algorithms were used to solve the problem using data from the three bus models.

3.1 Model Specification

The dynamic programming algorithm given in [25] was adopted as it contains all the variable obtained from the transport companies. The decisions of interest in this study are whether to keep the machine or replace it with a new one at each stage. The algorithm is given as follows:

$$V_k(i) = Min\big(C(i) + V_{k+1}(i+1)\big) \quad \dots \quad \dots \quad \text{KEEP} \quad \dots \quad \dots \dots (10)$$

$$V_k(i) = Min\big(C(0) + I - S(i) + V_{k+1}(1)\big) \dots \quad \dots \quad \text{REPLACE} \quad \dots \dots (11)$$

Where:

$C(i)$ = Operating cost of a bus that is i years old.

$C(0)$ = Operating cost of a new bus.

I = Acquisition cost of a new bus

$S(i)$ = Salvage value of an i year old bus.

$V_k(i)$ = total cost for a bus of age (i) at stage (k).

$V_{k+1}(i+1)$ = total cost for a bus of age ($i+1$) at stage ($k+1$).

$V_{k+1}(1)$ = total cost for a bus of age (1) at stage ($k+1$)

i = The state variable, (bus age at stage k)

D_k = decision at stage k .

k = stage, (Year of operation)

3.1.1 Assumptions

Several assumptions were made. These include the following:

1. Vehicle replacement decisions are made annually at the beginning of the year.
2. The vehicle is needed over a finite time horizon
3. Depreciation is charged at 25% in line with Statement of Accounting Standard (SAS) 9 – Accounting for Depreciation issued by the Nigerian Accounting Standards Board (N.A.S.B.). Depreciation was computed by straight line method.

4 Results and Discussion

Table 1 shows the year, purchase price and operating costs data collected from two transport companies in Benin City. The data on Toyota High Roof and Toyota Medium Roof buses were collected from company A while data on Toyota standard roof were collected from company B. The purchase prices for the three models of buses are recorded in column 6, while the operating cost comprises the cost of maintaining the vehicle and the cost of running the buses to provide the required service are contained in column 4 and the computed salvage values for the different buses are recorded in column 5.

The optimal replacement policy for Toyota high roof bus reported in Table 2 is Keep (K), Keep (k), keep (k) and keep (k) in stages 1 to 4 as they corresponds with the minimum cost value of (N11,548,124.00, N7,736,474.00, 3,625,181.00 and 311,650). The total cost for this optimal policy is N11,548124.00.

The optimal policy for the Toyota medium roof bus reported in Table 3 is Keep–Replace–Replace–Replace read from bottom to top that corresponds with the minimum cost. This means that the decisions to be made are keep in the first year and replace in the second, third and fourth years of the planning horizon and the total cost for this optimal policy is N11,466,668.00.

Table 1. Purchase price and operating cost data

Year	Bus Type	Age (i)	Operating and maintenance cost	Salvage value S(i)	Acquisition cost (I)
1	2	3	4	5	6
2010	Toyota high roof	0	3,811,650.00		7,000,000.00
2009	Toyota medium roof		2,866,667.00		7,000,000.00
2008	Toyota standard roof		1,838,100.00		4,700,000.00
2011	Toyota high roof	1	4,111,293.00	5,250,000.00	
2010	Toyota medium roof		3,605,555.00	5,250,000.00	
2009	Toyota standard roof		2,364,800.00	3,525,000.00	
2012	Toyota high roof	2	2,948,331.00	3,500,000.00	
2011	Toyota medium roof		2,563,889.20	3,525,000.00	
2010	Toyota standard roof		2,938,800.00	2,350,000.00	
2013	Toyota high roof	3	676,850.00	1,750,000.00	
2012	Toyota medium roof		3,222,222.20	1,725,000.00	
2011	Toyota standard roof		2,164,150.00	1,175,000.00	

Source: Official Records of Companies A and B

Table 2. Result for toyota high roof bus

Stage (K)	State (i)	Decision Keep	Replace	Minimum cost $V_k(i)$	Decision	Benefit
4	1	611293	311,650.00	311650	**Replace**	299,643.00
	2	1,198,331.00	2,061,650.00	1,198,331.00	**Keep**	-863,319.00
	3	676,850.00	3,811,650.00	676,850.00	**Keep**	-3,134,800.00
		Keep	**Replace**	$V_k(i)$		
3	1	5,309,624.00	5,873,300.00	5,309,624.00	**Keep**	-563,676.00
	2	3,625,181.00	7,623,300.00	3,625,181.00	**Keep**	-3,998,119.00
		Keep	**Replace**	$V_k(i)$		
2	1	7,736,474.00	10,871,274.00	7,736,474.00	**Keep**	-3,134,800.00
		Keep	**Replace**	$V_k(i)$		
1	0	11,548,124.00	18,548,124.00	11,548,124.00	**Keep**	-7,000,000.00

Source: Researchers' Computation (2014)

Table 3. Result for toyota medium roof bus

Stage (K)	State (i)	Decision Keep	Replace	Minimum cost $V_k(i)$	Decision	Benefit
4	1	80555	-633,333.00	-633333	**Replace**	713,888.00
	2	838,889.20	1,091,667.00	838,889.20	**Keep**	-252,777.80
	3	3,222,222.20	2,891,667.00	2,891,667.00	**Replace**	330,555.20
3		**Keep**	**Replace**	$V_k(i)$		
	1	4,444,444.20	3,983,334.00	3,983,334.00	**Replace**	461,110.20
	2	5,455,556.20	5,708,334.00	5,455,556.20	**Keep**	-252,777.80
		Keep	**Replace**	$V_k(i)$		
2	1	9,061,111.20	8,600,001.00	8,600,001.00	**Replace**	461,110.20
		Keep	**Replace**	$V_k(i)$		
1	0	11,466,668.00	18,466,668.00	11,466,668.00	**Keep**	-7,000,000.00

Source: Researchers' Computation (2014)

From Table 4 based on the decision criterion of minimum cost, Keep–Replace–Replace–Replace summarizes the optimal solution for Toyota standard roof bus. At the start of year 1, given the age of the bus (i) =0, the optimal decision is to keep the bus. Consequently, the kept bus will be 1 year old at the start of year 2 and calls for replacement. The replaced bus will be 1 year old at the start of year 3. This also calls for replacement. Therefore, at the start of year 4 the bus will be one year old and should be replaced also.

An inspection of the stage-1 (Year 1) column in Table 5 shows that transport companies should keep their new vehicles for all the bus types, hence all new buses should be kept in year 1. Consequently, the age of the three buses in service at the start of year 2 is 1. The action required for Toyota high roof in year 2 is keep while replacement is required for Toyota medium roof and standard roof respectively. In years 3 and 4, similar action as in year 2 is required for the buses. The optimal replacement policy for Toyota high roof bus is K-K-K-K which translates to keep, keep, keep and keep for the planning horizon. The companies should keep a new Toyota high roof bus of age zero in order to minimize cost as they will make a savings of N7, 000,000.00 from this decision. Also, the company should keep the bus when it is one year old as they will be saving a total sum of N3, 134,800.00 from this decision. In this same vein, if a two year old Toyota high roof bus is replaced, the company will be incurring cost totaling N3, 998,119.00. Also, when a three year old Toyota high roof is replaced the company will be incurring more cost amounting to N3,134,800.00 for not keeping it.

Table 4. Result for Toyota standard roof bus

Stage (K)	State (i)	Decision Keep	Replace	Minimum cost $V_k(i)$	Decision	Benefit
4	1	14800	-511,900.00	-511900	**Replace**	526,700.00
	2	1,763,800.00	663,100.00	663,100.00	**Replace**	1,100,700.00
	3	2,164,150.00	1,838,100.00	1,838,100.00	**Replace**	326,050.00
3		**Keep**	**Replace**	$V_k(i)$		
	1	3,027,900.00	2,501,200.00	2,501,200.00	**Replace**	526,700.00
	2	4,776,900.00	3,676,200.00	3,676,200.00	**Replace**	1,100,700.00
		Keep	**Replace**	$V_k(i)$		
2	1	6,041,000.00	5,514,300.00	5,514,300.00	**Replace**	526,700.00
		Keep	**Replace**	$V_k(i)$		
1	0	7,352,400.00	12,052,400.00	7,352,400.00	**Keep**	-4,700,000.00

Source: Researchers' Computation (2014)

Table 5. Summary of result of dynamic programming

Bus type	Sequence of optimal actions				The total optimal cost for this optimal policy
	Yr 1	**Yr 2**	**Yr 3**	**Yr 4**	**Amount ₦**
Toyota High Roof	K	K	K	K	11,548124.00
Toyota Medium Roof	K	R	R	R	11,466,668.00
Toyota Standard Roof	K	R	R	R	7,352,400.00

Source: Extracted from tables 2, 3 and 4

The optimal replacement policy for Toyota medium roof bus is K-R-R-R which translates to keep, replace, replace and replace for the planning horizon. The decisions advocates that the companies should keep a new Toyota medium roof bus (age zero) in order to minimize cost as they will make a savings of N7, 000,000.00 from this decision. Also, the company should replace their medium roof bus when it is one year old rather than keeping it as they will be incurring cost of N461, 110.00 in year 2, 3 and N713,888.00 in year 4 for keep decision. When we carefully observe the result in Table 3, if a one year old Toyota medium roof bus is kept in year 2 and replaced in the third year, the company will be incurring cost totaling N252, 777.80. Also, when a three year old Toyota high roof is kept the company will be incurring more cost amounting to N330, 555.00 for not replacing it.

The optimal replacement policy for Toyota standard roof bus is K-R-R-R which translates to keep, replace, replace and replace for the planning horizon. The companies should keep a new Toyota medium roof bus of age zero in order to minimize cost as they will make a savings of N4,700,000.00 from this decision. However, the company should replace the bus when it is one year old as they will be saving a total sum of N526, 700.00 from this decision. In the same vein, if a two year old Toyota medium roof bus is kept, the company will be incurring cost totaling N1, 100,700.00 for this decision. Also, when a three year old Toyota high roof is kept the company will be incurring more cost amounting to N326, 050.00 for not replacing it.

Our analysis shows that keeping Toyota high roof bus for 4 years is adequate while the keeping of Toyota medium and standard roofs bus for 4 years is not commendable and optimal to the company as late replacement of equipment results in high operational cost [6,26,27]. The optimal replacement policy for medium-roof and standard roof is to replace these buses yearly rather than the four years. Therefore, we conclude that the four years age replacement policy in place for Toyota high roof buses in these transport companies is optimal within the confines of our assumptions as the bus would have served out its useful years. However, the four years age replacement policies in place for Toyota medium and standard roof buses in these transport companies is not optimal as the companies are incurring high operation costs.

5 Conclusion and Recommendation

Equipment replacement decision plays a vital role in every organization that uses one form of equipment or the other. Based on the results obtained from dynamic programming, it may therefore be concluded that the 4 year fixed age replacement policy employed by transport companies in Edo State is not optimal for the medium roof and standard roof variants of Toyota buses.

The operations of transport companies are machine-based; hence, the right decisions regarding vehicle replacement would be highly valuable. In view of the foregoing, this study recommends that commercial vehicle owners/operators should endeavor to keep reliable, relevant and up to records of their vehicles.

Competing Interests

Authors have declared that no competing interests exist.

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